Mechanism Design, Implementation Theory, and Higher Order Uncertainty

Ritesh Jain

August 26, 2021

Institute of Economics, Academia Sinica

Motivating Example: Holmström and Myerson (1983)

- **Basic Environment**:
  - Agents: $I = \{1, 2\}$.
  - Payoff types: $\Theta_1 = \Theta_2 = \{\alpha, \beta\}$
  - Alternatives: $X = \{A, B, C\}$.
- **Preferences**: The following table displays the preferences

	$U_{1,\alpha}$	$U_{1,\beta}$	$U_{2,\alpha}$	$U_{2,\beta}$
A	2	0	2	2
B	1	4	1	1
C	0	9	0	-8

- **Information**: There is $p \in \Delta(\Theta)$ which is assumed to be uniform, that is $p = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\}$. Interim beliefs are described by Bayes rule. In this setting, for every $i \in I, (\theta_i, \theta_j) \in \Theta, p_i(\theta_j|\theta_i) = \frac{1}{2}$.

Motivating Example

- **Basic Environment**: $I = \{1, 2\}$, $\Theta_1 = \Theta_2 = \{\alpha, \beta\}$, and $X = \{A, B, C\}$
- **Preferences**: The following table displays the preferences

	$U_{1,\alpha}$	$U_{1,\beta}$	$U_{2,\alpha}$	$U_{2,\beta}$
A	2	0	2	2
B	1	4	1	1
C	0	9	0	-8

  - α: $A \succ_\alpha^1 B \succ_\alpha^1 C$ and $A \succ_\alpha^2 B \succ_\alpha^2 C$
  - β: At β, 2 dislike C more relative to α, $1's$ preferences reverse: $C \succ_\beta^1 B \succ_\beta^1 A$.

Motivating Example

- **Basic Environment**: $I = \{1, 2\}$, $\Theta_1 = \Theta_2 = \{\alpha, \beta\}$, and $X = \{A, B, C\}$
- **Preferences**:
  - α: $A \succ_\alpha^1 B \succ_\alpha^1 C$ and $A \succ_\alpha^2 B \succ_\alpha^2 C$
  - β: At β, 2 dislike C more relative to α, $1's$ preferences reverse: $C \succ_\beta^1 B \succ_\beta^1 A$.
- **Information**: There is $p \in \Delta(\Theta)$ which is assumed to be uniform, that is $p = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\}$. Interim beliefs are described by Bayes rule. In this setting, for every $i \in I, (\theta_i, \theta_j) \in \Theta, p_i(\theta_j|\theta_i) = \frac{1}{2}$.

Efficient SCF

The following social choice function maximizes ex-ante utilitarian welfare:

f	α	β
α	A	B
β	C	B

1. **Ex-ante**: No asymmetric information between the planner (we) and the agents.
2. **Interim**: Once every agent learns its type we have *asymmetric information* between the planner and players.

Who is a Planner?

Two perspectives:

- A physical entity with a well defined utility function. For ex. Auctioneer.
- Planner refers to the society I
  - In this interpretation, society is treated separately from its individual members.
  - SCF referents societies objective from an ex-ante perspective. For ex. design of constitutions.

Key take away:

Ex-ante and Interim stage creates information asymmetry.

Question

1. Can planner design a game/contract that provides agents incentives to reveal the truth?
2. Agents know that their announcement affects the decision $\implies$ the problem is non-trivial
3. Many senders and one receiver problem:
    - Receiver **designs** a *mechanism* and **commits** to it
    - senders send messages via the *mechanism*.
    - receiver selects an outcome for each message profile

Mechanism Design/Implementation

Mechanism:

- $\mathcal{M} = (\prod_{i \in I} M_i, g)$
- M_i: Messages for each agent and $g : M \to X$

Definition

A SCF f is weakly-implementable by a mechanism $\mathcal{M}$ if there exists a Bayes Nash Equilibrium $\sigma = (\sigma_1, \sigma_2)$, such that for every $(\theta_1, \theta_2) \in \Theta$, it holds that

$$g(\sigma_1(\theta_1), \sigma_2(\theta_2)) = f(\theta_1, \theta_2) \quad (1)$$

Mechanism Design/Implementation

Direct Mechanism:

- $M_i = \Theta_i$, for every $i \in I$
- $g = f$

Weak-Implementation:

- Focus on direct mechanisms
- Focus on truth-telling strategies

Example Continued

f	α	β
α	A	B
β	C	B

1. $\sigma_i^t(\theta_i) = \theta_i$: Truth-telling is an equilibrium $\implies$ f is incentive compatible.
2. f is weakly implementable.

Revelation Principle Fails: Part 1

f	α	β
α	A	B
β	C	B

1. $\sigma_i^t(\theta_i) = \theta_i$: Truth-telling is an equilibrium $\implies$ f is incentive compatible.
2. f is weakly implementable.

but....

- $\sigma_1'(\theta_1) = \theta_1$ and $\sigma_2'(\theta_2) = \beta$ is also an equilibrium.
- σ' undermines the planners goal.
- σ': Always B is selected.

Revelation Principle Fails: Part 1

f	α	β
α	A	B
β	C	B

1. $\sigma_i^t(\theta_i) = \theta_i$: Truth-telling is an equilibrium $\implies$ f is incentive compatible.
2. f is weakly implementable.

but....

- $\sigma_1'(\theta_1) = \theta_1$ and $\sigma_2'(\theta_2) = \beta$ is also an equilibrium.
- σ' undermines the planners goal.
- σ': Always B is selected.

Implementation Theory

Definition
A SCF f is *fully*-Bayesian implementable by a mechanism $\mathcal{M}$ if for (a) BNE exists and (b) every Bayes Nash Equilibrium $\sigma = (\sigma_1, \sigma_2)$ and for every $(\theta_1, \theta_2) \in \Theta$, it holds that

$$g(\sigma_1(\theta_1), \sigma_2(\theta_2)) = f(\theta_1, \theta_2) \tag{2}$$

Example Continued

Solution: Indirect Mechanism:

g	α	β
α	A	B
β	C	B
α'	B	A
β'	B	C

Two Equilibrium.

- $\hat{\sigma}^t$: Truthtelling
- $\hat{\sigma}_1(\alpha) = \alpha', \hat{\sigma}_1(\beta) = \beta'$ and $\hat{\sigma}_2(\alpha) = \beta, \hat{\sigma}_2(\beta) = \alpha$

A Design Recipe

Augmented Revelation Principle: Mookherjee and Reichelstein (1990)

- Start with a direct mechanism: $M = \Theta$, $g = f$
- 'Augment' it
  - $\hat{M}_i = \Theta_i \cup \mathbb{C}$, $\mathbb{C}$ is some countable set
  - Extend g to $\hat{M}$ such that no bad equilibrium remains.
- Implementation theory: provides condition on f that ensure that such an augmentation is possible.

Goal: To provide a brief overview of this literature by minimizing the details.

Information Structure

1. **Incomplete Information**: Assymetric information (*a*) among the players and (*b*) between the players and the planner.
2. **Complete Information**: No Asymmetric information among the players but between the players and the planner
   2.1 For every $(\theta_1, \theta_2) \in (\Theta_1, \Theta_2)$, it holds that

   $$p(\theta_2|\theta_1) = p(\theta_1|\theta_2) = 1$$

   2.2 Every $(\theta_1, \theta_2) \in (\Theta_1, \Theta_2)$ is common knowledge among the agents

Complete Information

- Simplest non-trivial set up to analyze the implementation problem.
- Conceptually, extension to general information structures is similar.
- My view: complete information and $I \geq 3$ is a *tool* to develop conceptual grip at this literature.

Complete Information

- Simplest non-trivial set up to analyze the implementation problem.
- Conceptually, extension to general information structures is similar.
- My view: complete information and $I \geq 3$ is a *tool* to develop conceptual grip at this literature. **But can't escape the Bayesian setup.**

Complete Information

- Simplest non-trivial set up to analyze the implementation problem.
- Conceptually, extension to general information structures is similar.
- My view: complete information and $I \geq 3$ is a *tool* to develop conceptual grip at this literature. **But can't escape the Bayesian setup.**
- Information structure matters: Palfrey and Srivastava (1987) provide interesting examples.

Maskin's Theorem

- Basic Environment: I, Θ, and $X = \{A, B, C\}$
- Preferences: $\theta \in \Theta$: $u_i^\theta : X \to \mathbb{R}$
- Complete Information: For every $\theta \in \Theta$: Everyone knows θ, everyone know that everyone knows θ and so on...
- Example: $\Theta = \{(\alpha, \alpha), (\beta, \alpha), (\alpha, \beta), (\beta, \beta)\}$
- $\mathcal{M}$ is a game form and $(\mathcal{M}, \theta)$ is game of complete information.
    - $NE(\mathcal{M}, \theta)$: Set of Nash equilibria at θ.

Nash Implementation

Definition
A SCF f is Nash implementable by a mechanism $\mathcal{M}$ provided that for every $\theta \in \Theta$, it holds that

$$\bigcup_{m \in NE(\mathcal{M},\theta)} g(m) = f(\theta) \tag{3}$$

At every state $\theta \in \Theta$,

- $NE(\mathcal{M}, \theta)$ unique in outcomes.
- Multiple equilibria not ruled out.

Maskin Monotonicity

Definition
f satisfies Maskin monotonicity provided that for every $(\theta, \theta') \in \Theta \times \Theta$

$$\forall i \in I, L_i(f(\theta), \theta) \subseteq L_i(f(\theta), \theta') \implies f(\theta) = f(\theta')$$

Maskin Monotonicity

Definition
f satisfies Maskin monotonicity provided that for every $(\theta, \theta') \in \Theta \times \Theta$

$$\forall i \in I, L_i(f(\theta), \theta) \subseteq L_i(f(\theta), \theta') \implies f(\theta) = f(\theta')$$

Lower contour set:

$$L_i(f(\theta), \theta) = \{x \in X | u_i^{\theta}(x) \leq u_i^{\theta}(f(\theta))\}.$$

$$L_i(f(\theta), \theta') = \{x \in X | u_i^{\theta'}(x) \leq u_i^{\theta'}(f(\theta))\}.$$

Maskin Monotonicity + No Veto Power

Definition
f satisfies Maskin monotonicity provided that for every $(\theta, \theta') \in \Theta \times \Theta$

$$\forall i \in I, L_i(f(\theta), \theta) \subseteq L_i(f(\theta), \theta') \implies f(\theta) = f(\theta')$$

Definition
f satisfies No Veto Power (NVP) provided that for every $\theta \in \Theta$ if there is an alternative $x \in X$ such that $|\{i \in I | x \in \underset{z}{argmax} u_i(z, \theta)\}| \geq I - 1$, then $f(\theta) = x$

Maskin's Theorem

Theorem

1. *If f is Nash implementable, then f satisfies Maskin monotonicity*
2. $I \geq 3$. *If f satisfies Maskin monotonicity and No Veto Power, then f is Nash implementable*

Maskin's Theorem

- Maskin Monotonicity
  - becomes restrictive with the size of Θ.
  - $I \geq 2$, $X \geq 3$ and Θ is large. If f satisfies MM, then f is constant or dictatorial.
- NVP
  - not necessary but is consider 'mild'.
  - NVP is vacuous in economic environments.
  - NVP is restrictive when $I = 2$.

Maskin's Theorem

Maskin's Theorem: Suppose $I \geq 3$. *Maskin monotonicity is necessary and 'almost' sufficient for Nash implementation.*

Maskin's Theorem

Maskin's Theorem: Suppose $I \geq 3$. Maskin monotonicity is necessary and 'almost' sufficient for Nash implementation.

- 'almost': No mathematical content.
- Unfortunate language used in this literature. I try to avoid it. Let the reader decide whether the gap in the characterization is small or not.

Literature

Bayesian Implementation:

- Bayesian Monotonicity: Postlewaite and Schmeidler (1986) Palfrey and Srivastava (1987), Palfrey and Srivastava (1989a), Jackson (1991).
- Augmented Revelation Principal: Mookherjee and Reichelstein (1990)

Nash Implementation:

- Full Characterization: Moore and Repullo (1990), Dutta and Sen (1991), Sjöström (1991).
- Full characterization for Bayesian Implementation: Open Problem.
  - Issue: Characterization will involve information structure.
  - Jackson (1991): Bayesian Monotonicity + Bayesian No Veto Power not enough.

Key Take Away

- Information structure matters.
  - Complete Information: Pareto Efficiency is Nash Implementable in complete information environments
  - Incomplete Information: Exante, Interim, Ex-post are not (Palfrey and Srivastava (1987)).
- Full *Bayesian* implementation is restrictive in *static* games

Solution $\backslash$ Game form	Static	Extensive
BNE	✓	SPNE, Sequential
Refinement	Undominated, Trembling Hand	NA

Literature: stage game mechanisms or Equilibrium Refinement

Use stage game mechanisms + Equilibrium Refinement

Subgame Perfect Implementation:

- Theory: Moore and Repullo (1988), Abreu and Sen (1990).
- Experiments: Fehr et al. (2021), Aghion et al. (2018).

Equilibrium Refinements:

- Players do not play weakly dominated strategies.
- Palfrey and Srivastava (1991), Palfrey and Srivastava (1989b).

Literature: stage game mechanisms or equilibrium refinement

Key take away:

1. Maskin monotonicity and Bayesian monotonicity can be avoided
2. Any incentive compatible social choice function is implementable
3. Augmented Revelation Mechanisms: required

Jackson's critique: Jackson (1992)

- Any f is implementable if players do not play weakly dominated strategies.
- Implementing mechanism must be 'bad' if f is not dictatorial.

Literature

Literature Surveys:

1. Papers: Maskin and Sjöström (2002), Jackson (2001), and Palfrey (2002)
2. Book: Palfrey et al. (2020)

Higher Order Uncertainty

Two Common Knowledge Assumptions

1. Equilibrium: Strategy profile is common knowledge $\implies$ no strategic uncertainty.
2. Information Structure is common knowledge $\implies$ no ‘structural’ uncertainty.

Complete Information: ‘structural’ uncertainty

1. $\theta \in \Theta$ is commonn knowledge. What if θ is ‘almost’ common knowledge?
2. Two views:
   - **weak**: Nature announces θ to everyone simultaneously but with an error (Monderer and Samet, 1989)
   - **strong**: Nature announces θ to everyone sequentially but with an error (Rubinstein, 1989)

Complete Information: ‘structural’ uncertainty

1. $\theta \in \Theta$ is common knowledge. What if θ is ‘almost’ common knowledge?
2. ’almost’ common knowledge
   - **weak**: Everyone knows θ with high probability, everyone knows that everyone knows θ with high probability and so on... (Monderer and Samet, 1989)
   - **strong**: Everyone knows θ with high probability, everyone knows that everyone knows θ with high probability and so on upto some finite but large k. (Rubinstein, 1989)

Revisit Implementation

Chung and Ely (2003)

- Robustness: Full implementation is required on all models *nearby* complete information in the sense of (Monderer and Samet, 1989).
- Result: Suppose players do not play weakly dominated strategies and implementation is required to be robust: *Maskin monotonicity is necessary*.
- The positive implementation results with refinement purely driven by complete information assumption.

Aghion et al. (2012)

- Robustness: Full SPNE-implementation is required on all models *nearby* complete information in the sense of (Monderer and Samet, 1989).
- Result: *Maskin monotonicity is necessary*.
- The positive implementation results with extensive form games purely driven by complete information assumption.

Revisit Implementation

Chung and Ely (2003)

- The positive implementation results with refinement purely driven by complete information assumption.

Aghion et al. (2012)

- Robustness: Full SPNE-implementation is required on all models *nearby* complete information in the sense of (Monderer and Samet, 1989).
- Result: *Maskin monotonicity is necessary*.
- The positive implementation results with extensive form games purely driven by complete information assumption.

Revisit Implementation

Chung and Ely (2003)

- The positive implementation results with refinement purely driven by complete information assumption.

Aghion et al. (2012)

- Robustness: Full SPNE-implementation is required on all models *nearby* complete information in the sense of (Monderer and Samet, 1989).
- Result: *Maskin monotonicity is necessary*.
- The positive implementation results with extensive form games purely driven by complete information assumption.

Fun fact: Aghion et al. (2012) only QJE in the literature!

Revelation Principle Fails: Part 2

Oury and Tercieux (2012)

- Relax the full implementation requirement to weak implementation, that is, mechanism design.
- Strengthen the robustness requirement to strong
- Implementing equilibrium must be robust to small deviations from the information structure (complete information here).
- This requirement is called continuous implementation

Result:

- if f is continuously implementable, then it must be fully implementable in rationalizable strategies.
- rationalizability:
  - Complete information: Bernheim (1984), Pearce (1984)
  - Incomplete Information: Battigalli and Siniscalchi (2003), and Dekel et al. (2007)

Revelation Principle Fails: Part 2

Oury and Tercieux (2012)

- This requirement is called continuous implementation

Result:

- if f is continuously implementable, then it must be fully implementable in rationalizable strategies
- revelation principle is not robust to higher order uncertainty
- Mechanism design + Higher order uncertainty $\implies$ implementation in rationalizable strategies

Two Common Knowledge Assumptions

1. Equilibrium: Strategy profile is common knowledge $\implies$ no strategic uncertainty.
2. Information Structure is common knowledge $\implies$ no 'structural' uncertainty.

Takeaway: if we want to relax (2) we need to relax (1).

Solution concept

- $\mathcal{S}_i^{\mathcal{M},\theta}$: set of rationalizable strategies for i
- $\mathcal{S}^{\mathcal{M},\theta} = \mathcal{S}_1^{\mathcal{M},\theta} \times \mathcal{S}_2^{\mathcal{M},\theta} \ldots \times \mathcal{S}_I^{\mathcal{M},\theta}$: set of rationalizable strategy profiles
- $\mathcal{S}_i^{\mathcal{M},\theta}$: Iterated elimination of never best responses

$$\mathcal{S}_i^{\mathcal{M},\theta} = \bigcap_{k \geq 1} \mathcal{S}_{i,k}^{\mathcal{M},\theta}$$

Solution concept

When $k = 0$, let $\mathcal{S}_{i,0}^{\mathcal{M},\theta} = M_i$

Never Best Response

- Step $k = 1$
  - m_i is never a 1-best response if there is no belief $\lambda_i \in \Delta(M_{-i})$ such that m_i is a best response to λ_i.
  - Remaining strategies: $\mathcal{S}_{i,0}^{\mathcal{M},\theta}$, for every i
- Step $k = 2$
  - m_i is never a 2-best response if there is no belief $\lambda_i \in \Delta(\mathcal{S}_{i,1}^{\mathcal{M},\theta})$ such that m_i is a best response to λ_i.
- Iterate!

Solution concept

When $k = 0$, let

$$\mathcal{S}_{i,0}^{\mathcal{M},\theta} = M_i$$

When $k \geq 1$, let

$$\mathcal{S}_{i,k}^{\mathcal{M},\theta} = \left\{ m_i \in \mathcal{S}_{i,k-1}^{\mathcal{M},\theta} \;\middle|\; \begin{array}{c} \exists \lambda_i \in \Delta(\mathcal{S}_{-i,k-1}^{\mathcal{M},\theta}) \;\; \textit{such that} \\ m_i \in \underset{m_i' \in M_i}{\textit{argmax}} \displaystyle\sum_{m_{-i} \in \mathcal{S}_{-i,k-1}^{\mathcal{M},\theta}} \lambda_i(m_{-i}) u_i(g(m_i, m_{-i}), \theta) \end{array} \right\}$$

Implementation in Rationalizable strategies

Definition
A mechanism $\mathcal{M}$ implements an SCF f in rationalizable strategies if for every $\theta \in \Theta$,

1. $\mathcal{S}^{\mathcal{M},\theta} \neq \emptyset$
2. $\bigcup_{m \in \mathcal{S}^{\mathcal{M},\theta}} g(m) = f(\theta)$

Complete Information: Bergemann et al. (2011)

Definition
f satisfies Maskin monotonicity* provided that there exists a partition P_f of Θ such that f is measurable with respect to P_f for every $(\theta, \theta') \in \Theta \times \Theta$

$$\forall \hat{\theta} \in P(\theta), \exists i \in I,\ L_i(f(\theta), \hat{\theta}) \subseteq L_i(f(\theta), \theta') \implies \theta' \in P(\theta)$$

1. Maskin monotonicity* $\implies$ Maskin monotonicity
2. Maskin monotonicity $\not\Longrightarrow$ Maskin monotonicity*: Jain (2021).
   - Condorcet rule on Condorcet domain (Healy and Peress (2015)).

Complete Information: Bergemann et al. (2011)

Theorem
$I \geq 3$, f satisfies Maskin monotonicity* and NWA*, then f is implementable in rationalizable strategies.

Full characterization:

- $I \geq 3$: Xiong (2021), strict Group Monotonicity**
- $I \geq 2$: Jain et al. (2021), Iterated monotonicity (IM)
- IM endogenize the Partition that appears in MM^{**} and strict Group Monotonicity**.

Basic Set up

- **Basic Environment**:
  - Agents: $I = \{1, 2, \ldots, I\}$.
  - Payoff types: Θ_i and $\Theta = \prod_{i \in I} = \Theta$
  - Alternatives: X.
- **Preferences**:
  - $u_i : X \times \Theta \to \mathbb{R}$, expected utility
- **Information**:
  - $T = \prod_{i \in I} T_i$
  - $\mathcal{T}_i = (T_i, \hat{\theta}_i, \hat{\pi}_i)$, where $\hat{\theta}_i : T_i \to \Theta_i$, and $\hat{\pi}_i : T_i \to \Delta(\Theta_{-i})$
  - $\mathcal{T} = \prod_{i \in I} \mathcal{T}_i$

Robustness notions: Topology

The largest type space $\mathcal{T}^*$: Universal type space

- Nice topological structure
- The idea of closeness can be defined using metric spaces
- Topology on types: Dekel et al. (2005), Chen et al. (2010), and Chen et al. (2017)

Robustness notions

Baseline type space $\overline{\mathcal{T}}$:

- Bayesian Implementation: fixed type space $\overline{\mathcal{T}}$, classical literature
- Local Robust: implementation on all type spaces nearby $\overline{\mathcal{T}}$
  - Oury and Tercieux (2012) $\implies$ Interim Rationalizable implementation
- Global: implementation on all type spaces $\mathcal{T}$:
  - Bergemann and Morris $\implies$ ex-post version of rationalizability

Rationalizable Implementation

$\mathcal{M}^{Aug}$ and $M_i = \Theta_i \cup \mathbb{C}$, $\theta \in M \implies g(m) = f(\theta)$

$\mathbb{C}\backslash$ Robustness	Robust	Interim-$\mathcal{T}$	Comp Info-$\mathcal{T}^{CI}$
No Restriction	BM(2011, GEB) KS(2020, WP) work in progress	OT(2012, ECTA) KSS(2020,WP) work in progress	BMT(2012, JET) Xiong(2021,WP) JKL(2021,WP) Jain (2021,GEB)
Finite-Virtual	BM (2009,TE)	AM(1992,WP)	AM(1992, ECTA)
Finite-Exact			CKSX(2021, GEB)
Direct	BM(2009,ReS)		HM (2012, TE)
Direct	OP(2019, AER)	OP(2019, AER)	OP(2019, AER)

BM:Bergemann-Morris; BMT:Bergemann-Morris-Tercieux; OT: Oury -Tercieux; KS: Kunimoto-Saran; KSS: Kunimoto-Saran-Serrano; AM: Abreu-Matsushima; HM: Healy-Mathevet; OP: Ollar-Penta; CKSX: Chen-Kunimoto-Sun-Xiong; JKL: Jain-Korpela-Lombardi.

Rationalizable Implementation: My favourite

$\mathcal{M}^{Aug}$ and $M_i = \Theta_i \cup \mathbb{C}$, $\theta \in M, g(m) = f(\theta)$

$\mathbb{C}\backslash$ Robustness	Robust	Interim-$\mathcal{T}$	Comp Info-$\mathcal{T}^{CI}$
No Restriction	BM(2011, GEB) KS(2020, WP) work in progress	OT(2012, ECTA) KSS(2020,WP) work in progress	BMT(2012, JET) Xiong(2021,WP) JKL(2021,WP) Jain (2021,GEB)
Finite-Virtual	BM (2009,TE)	AM(1992,WP)	AM(1992, ECTA)
Finite-Exact			CKSX(2021, GEB)
Direct	BM(2009,RES)		HM (2012, TE)
Direct	OP(2017, AER)	OP(2017, AER)	OP(2017, AER)

Oury and Tercieux (2012); Abreu and Matsushima (1992); and Ollár and Penta (2017).

Concluding Remarks

- Rationalizability is more than a solution concept
- It is a tool to perform robust equilibrium analysis.
- Battigalli and Siniscalchi (2003): ‘tractable way to implement robust Bayesian Nash analysis’
- This is just the tip of the ice-berg. Many other frameworks on which Maskin (1999) has been studied.
- Almost all models follows the same basic insight.
- Single valued rules: What about $F : \Theta \to 2^X \setminus \{\emptyset\}$?

References i

Abreu, D. and Matsushima, H. (1992). Virtual implementation in iteratively undominated strategies II: Incomplete information. Princeton University working paper.

Abreu, D. and Sen, A. (1990). Subgame perfect implementation: A necessary and almost sufficient condition. Journal of Economic theory, 50(2):285–299.

Aghion, P., Fehr, E., Holden, R., and Wilkening, T. (2018). The role of bounded rationality and imperfect information in subgame perfect implementation—an empirical investigation. Journal of the European Economic Association, 16(1):232–274.

Aghion, P., Fudenberg, D., Holden, R., Kunimoto, T., and Tercieux, O. (2012). Subgame-perfect implementation under information perturbations. The Quarterly Journal of Economics, 127(4):1843–1881.

Battigalli, P. and Siniscalchi, M. (2003). Rationalization and incomplete information. Advances in Theoretical Economics, 3(1).

Bergemann, D., Morris, S., and Tercieux, O. (2011). Rationalizable implementation. Journal of Economic Theory, 146(3):1253–1274.

Bernheim, B. D. (1984). Rationalizable strategic behavior. Econometrica: Journal of the Econometric Society, pages 1007–1028.

Chen, Y.-C., Di Tillio, A., Faingold, E., and Xiong, S. (2010). Uniform topologies on types. Theoretical Economics, 5(3):445–478.

Chen, Y.-C., Di Tillio, A., Faingold, E., and Xiong, S. (2017). Characterizing the strategic impact of misspecified beliefs. The Review of Economic Studies, 84(4):1424–1471.

Chung, K.-S. and Ely, J. C. (2003). Implementation with near-complete information. Econometrica, 71(3):857–871.

Dekel, E., Fudenberg, D., and Morris, S. (2005). Topologies on types. Harvard Institute of Economic Research Discussion Paper, (2093).

Dekel, E., Fudenberg, D., and Morris, S. (2007). Interim correlated rationalizability. Theoretical Economics.

Dutta, B. and Sen, A. (1991). A necessary and sufficient condition for two-person nash implementation. The Review of Economic Studies, 58(1):121–128.

Fehr, E., Powell, M., and Wilkening, T. (2021). Behavioral constraints on the design of subgame-perfect implementation mechanisms. American Economic Review, 111(4):1055–91.

Healy, P. J. and Peress, M. (2015). Preference domains and the monotonicity of condorcet extensions. Economics Letters, 130:21–23.

Holmström, B. and Myerson, R. B. (1983). Efficient and durable decision rules with incomplete information. Econometrica: Journal of the Econometric Society, pages 1799–1819.

Jackson, M. O. (1991). Bayesian implementation. Econometrica: Journal of the Econometric Society, pages 461–477.

Jackson, M. O. (1992). Implementation in undominated strategies: A look at bounded mechanisms. The Review of Economic Studies, 59(4):757–775.

Jackson, M. O. (2001). A crash course in implementation theory. Social choice and welfare, 18(4):655–708.

Jain, R. (2021). Rationalizable implementation of social choice correspondences. Games and Economic Behavior, 127:47–66.

Jain, R., Korpella, V., and Lombardi, M. (2021). An iterative approach to rationalizable implementation. Technical report, Institute of Economics, Academia Sinica, Taipei, Taiwan.

Maskin, E. (1999). Nash equilibrium and welfare optimality. The Review of Economic Studies, 66(1):23–38.

Maskin, E. and Sjöström, T. (2002). Implementation theory. Handbook of social Choice and Welfare, 1:237–288.

Mookherjee, D. and Reichelstein, S. (1990). Implementation via augmented revelation mechanisms. The Review of Economic Studies, 57(3):453–475.

Moore, J. and Repullo, R. (1988). Subgame perfect implementation. Econometrica: Journal of the Econometric Society, pages 1191–1220.

Moore, J. and Repullo, R. (1990). Nash implementation: a full characterization. Econometrica: Journal of the Econometric Society, pages 1083–1099.

Ollár, M. and Penta, A. (2017). Full implementation and belief restrictions. American Economic Review, 107(8):2243–77.

Oury, M. and Tercieux, O. (2012). Continuous implementation. Econometrica, 80(4):1605–1637.

Palfrey, T. R. (2002). Implementation theory. Handbook of game theory with economic applications, 3:2271–2326.

Palfrey, T. R. and Srivastava, S. (1987). On bayesian implementable allocations. The Review of Economic Studies, 54(2):193–208.

Palfrey, T. R. and Srivastava, S. (1989a). Implementation with incomplete information in exchange economies. Econometrica: Journal of the Econometric Society, pages 115–134.

Palfrey, T. R. and Srivastava, S. (1989b). Mechanism design with incomplete information: A solution to the implementation problem. Journal of Political Economy, 97(3):668–691.

Palfrey, T. R. and Srivastava, S. (1991). Nash implementation using undominated strategies. Econometrica: Journal of the Econometric Society, pages 479–501.

Palfrey, T. R., Srivastava, S., and Postlewaite, A. (2020). Bayesian implementation. CRC Press.

Pearce, D. G. (1984). Rationalizable strategic behavior and the problem of perfection. Econometrica: Journal of the Econometric Society, pages 1029–1050.

Postlewaite, A. and Schmeidler, D. (1986). Implementation in differential information economies. Journal of Economic Theory, 39(1):14–33.

Sjöström, T. (1991). On the necessary and sufficient conditions for nash implementation. Social Choice and Welfare, 8(4):333–340.

Xiong, S. (2021). Rationalizable implementation: Social choice function.