

# Prediction Market Alternatives for Complex Environments

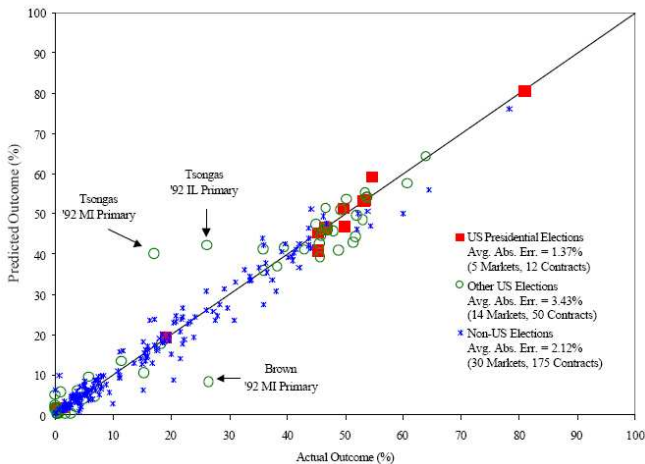
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Tulane University  
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# The Success of Prediction Markets

- Wall St. market: 1848–1940 (Rhode & Strumpf 2004)
  - 11/15 correct in mid-October, only 1 very wrong (Wilson 1916)
- Iowa Electronic Markets (Berg et al. 2003)
  - See figure...

# The Success of Prediction Markets



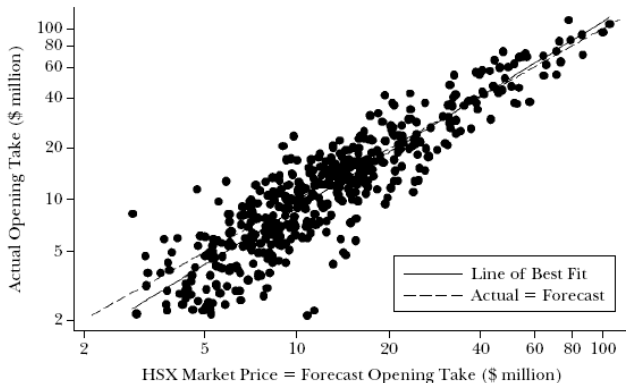
Avg. Error: 1.5% vs. 2.1%. Source: Berg, Forsythe, Nelson & Rietz (2003)

# The Success of Prediction Markets

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  - 11/15 correct in mid-October, only 1 wrong (W. Wilson)
- Iowa Electronic Markets (Berg et al. 2003)
  - See figure...
  - But... Erikson & Wlezien use *trends* in polls
- TradeSports (Tetlock, Wolfers, Zitzewitz, others...)
  - Trade volume during Davidson vs. Kansas  $\approx$  7,700 \$10 tickets
- NewsFutures, Hollywood Stock Exchange (Pennock et al. 2001)
  - See figure...

# The Success of Prediction Markets

Figure 3  
Predicting Movie Success



Source: Data from 489 movies, 2000–2003 (<http://www.hsx.com>).

Source: Wolfers & Zitzewitz (2004)

# Corporate Applications

- Predicting printer sales at Hewlett-Packard (K-Y Chen & Plott 2002)
- Companies claiming to use prediction markets:

Abbot Labs	Arcelor Mittal	Best Buy	Chrysler
Corning	Electronic Arts	Eli Lilly	Frito Lay
General Electric	Google	Hewlett-Packard	Intel
InterContinental Hotels	Masterfoods	Microsoft	Motorola
Nokia	Pfizer	Qualcomm	Siemens
TNT			

- Are they doing it 'right'? Volume? Complexity??

# The Policy Analysis Market (PAM)

- 2001–2003 DARPA (DoD) => NetExchange (Ledyard, Polk, Hanson)
- Goal: Predict events the DoD might care about
- NetExchange focus: political instability in Middle East
- A subset of the issues:
  - Correlation blows up the state space
  - Manipulation? (Camerer 98, Strumpf & Rhode 07)
  - Moral Hazard? (Hanson et. al 07)
  - Moral repugnance & P.R. (Roth 07, Hanson 07)
- Aug 03: Shut Down, DARPA audited, Poindexter 'retired'

## Questions:

- 1 Can markets actually work when the environment gets 'complicated'?
- 2 Would other mechanisms do better?

## Answers:

Test markets vs. 3 other mechs in complex lab environments

- 1 Market performs poorly; incentivized, iterated polls perform better



# Behavioral Mechanism Design

- Methodology of combining experiments & theory to design better mechanisms for real-world use
- Short run goal: find a better mechanism
  - 1 Propose alternative mechanisms
    - Existing theory & behavioral data as guides
  - 2 Testbed proposed mechanisms
    - The control of the laboratory
  - 3 Tweak if necessary
- Long run goal: improve the design process
  - 1 Identify general principles while testbedding
  - 2 Add new constraints, etc., to the design problem

## Questions:

- 1 Can markets actually work when the environment gets complicated?
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## Answers:

Test markets vs. 3 other mechs in complex lab environments

- 1 Market falls apart, simple iterated polls perform better
- 2 *Why* the poll seems to do better in this environment

# Easy vs. Hard Environments

Example similar to our experiment:

- 1 **Simple:** Will UNC beat Kansas tonight?
  - Two states:  $\{\text{UNC}, \text{Kansas}\}$ , one security
- 2 **Hard:** Who will win each of the last 3 games (2 semi's and final)?
  - Three events, not independent
  - Eight states:  $\{\text{UNC}, \text{Kansas}\} \times \{\text{Memphis}, \text{UCLA}\} \times \{\text{East}, \text{West}\}$
  - “Eastern finalist wins” is correlated with other 2 events
  - Incomplete set of securities is typically used
    - TradeSports offers 6 securities (1+1+4)
  - We will use a complete set of 8

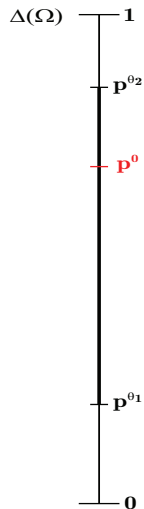
## 2-State Environment

- Two coins:  $\theta \in \Theta = \{X, Y\}$
- Two flip outcomes:  $\omega \in \Omega = \{H, T\}$
- State of the world =  $(\theta, \omega)$
- $\theta$  and  $\omega$  are correlated, but we care only about  $\omega$ 
  - $p(H|X) = 0.2$
  - $p(H|Y) = 0.4$
- One coin  $\theta$  is randomly drawn (50/50)
- Each subject sees flips of chosen coin
  - $s_j = (H, T, H, H)$
  - $\#s_j \in \{2, 3, 4\}$
- Observe sample  $\omega$ 's, infer about true  $\theta$ , predict true  $\omega$

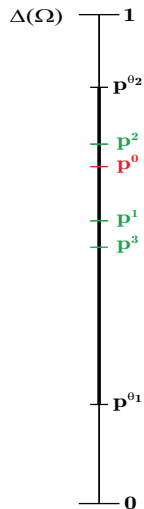
# Distributions: 2 States



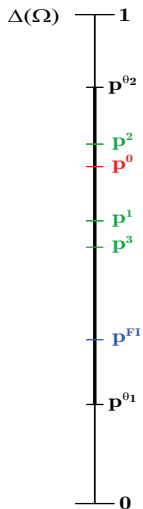
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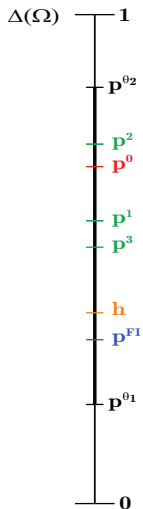


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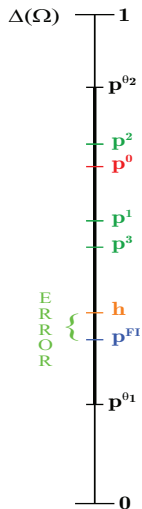




# Distributions: 2 States



# Distributions: 2 States



# 8-State Environment

- Three coins, ordered:  $\theta \in \Theta = \{XYZ, XZY, YXZ, YZX, ZXY, ZYX\}$
- Eight flip outcomes:  
 $\omega \in \Omega = \{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\}$ 
  - $\Pr[X = H] = 0.2$
  - $\Pr[Y = H] = 0.4$
  - $\Pr[Z = H] = 0.4$
  - $\Pr[Y = X] = 2/3$  (correlation distinguishes  $Y, Z$ )
- One ordering  $\theta$  is randomly drawn (uniformly)
- Each subject sees flips from chosen coin ordering
  - $s_i = (THT, THT, HHH, TTT, HTH)$
  - $\#s_i \in \{3, 5, 7\}$

# Distributions: 8 States

$\Delta(\Omega)$

$\bullet p^{\theta_1}$

$\bullet p^{\theta_2}$

$\bullet p^{\theta_3}$

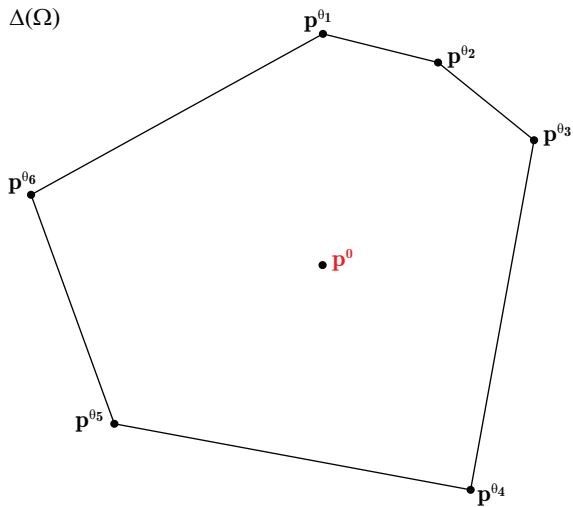
$\bullet p^{\theta_6}$

$\bullet p^0$

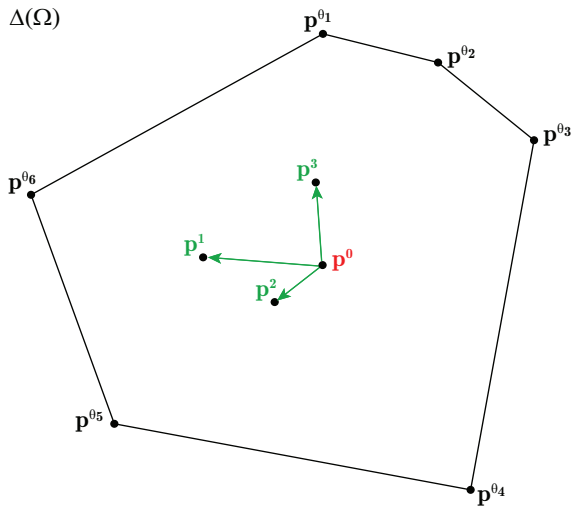
$\bullet p^{\theta_5}$

$\bullet p^{\theta_4}$

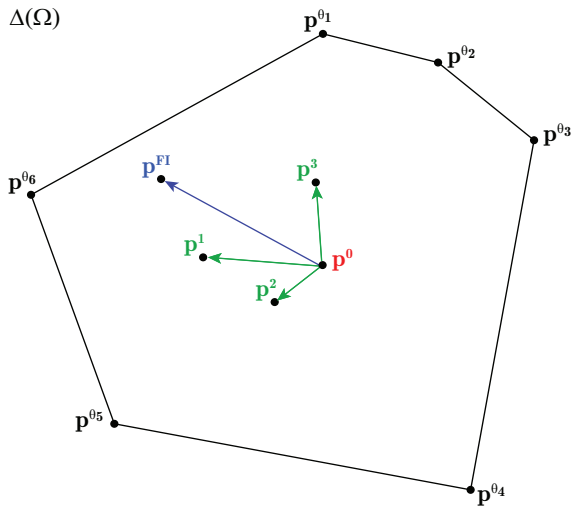
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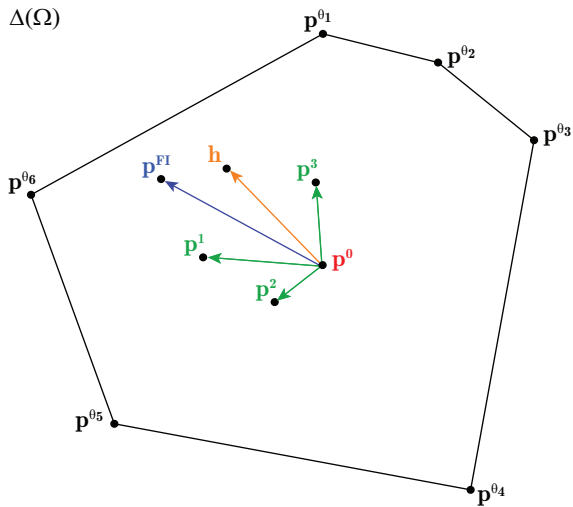
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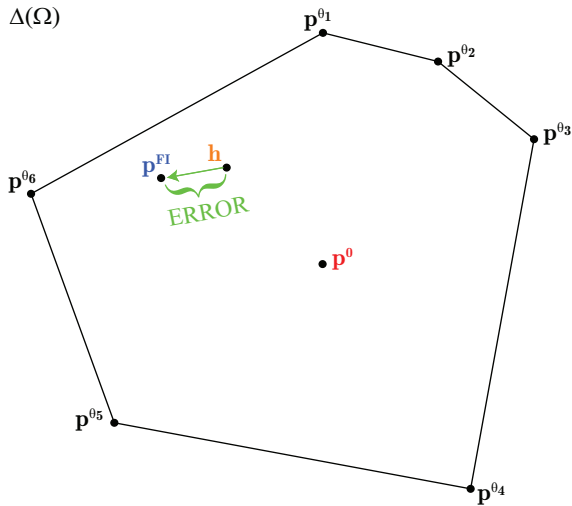


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# The Mechanisms

- 1 Double Auction (prediction market)
- 2 Pari-mutuel (horse track)
- 3 Iterated Poll ('Delphi method': RAND/USAF)
- 4 Market Scoring Rule (Hanson 2003)

# Alternative Mechanisms: Pari-Mutuel

- Bettors buy tickets on each event
  - $n_j = \#$  of tickets purchased on event  $j$
- Payoff odds of event- $j$  tickets =  $(n_j / \sum_k n_k)^{-1}$
- Still need  $2^k$  securities
- Still have a no-trade theorem

# Alternative Mechanisms: Poll

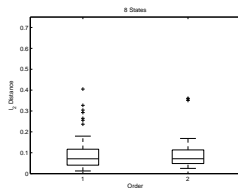
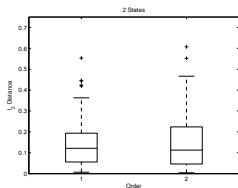
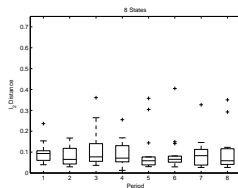
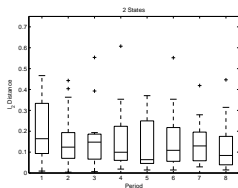
- Players announce a belief distribution  $P^i$  over the 8 events
- $\bar{P} = (1/n) \sum_i P^i$  is shown
- Repeat 5 times
- Everyone paid based on final average distribution  $\bar{P}$
- Incentive compatible scoring rule:
  - Everyone receives  $(\ln [\bar{P}_j] - \ln [1/8])$  event- $j$  securities
  - If event  $k$  is true, event- $k$  security pays \$1.
- There exist many seq. equil. with full info aggregation
- There exist babbling seq. equil. with “almost” no aggregation

# Alternative Mechanisms: Market Scoring Rule (Hanson)

- A public distribution is shown:  $(1/8, \dots, 1/8)$
- Individuals may 'move' the distribution to  $(P_1^i, \dots, P_8^i)$
- Move from  $(Q_1, \dots, Q_8)$  to  $(P_1^i, \dots, P_8^i) \implies$ 
  - Receive  $\left( \ln [P_j^i] - \ln [Q_j^i] \right)$  event- $j$  securities for each  $j$
  - Moving  $P_j$  up means buying, down means selling
  - If event  $k$  is true, event- $k$  security pays \$1
  - Incentive compatible: you should move to your best guess
  
- Subsidized  $\implies$  avoids no-trade theorem
- Incentive compatible  $\implies$  myopic players reveal truthfully
- Incentive to misrepresent? Depends on move timing...

- Run experiments using Caltech undergrads paid  $\approx$  \$35
- No experience
- Crossover design: DA-Poll, Poll-DA, MSR-Pari, Pari-MSR
- 3 subjects per group
- 8 periods with each mechanism
- No rematching

# Period & Order Effects



No significant period or order effects (good!)

## 2 States: Error

Comparison of  $l_2$  distances with 2 states:

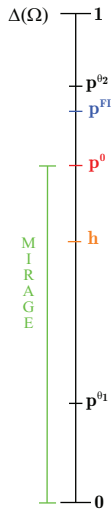
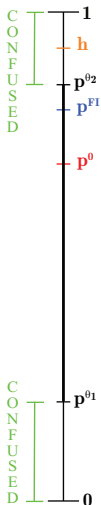
	Avg Dist.	Wilcoxon $p$ -values			
		DblAuctn	MSR	Parimutuel	Poll
Avg Dist.	—	0.262	0.419	0.295	0.266
DblAuctn	0.262	—	<b>0.092</b>	0.646	0.663
MSR	0.419	—	—	0.225	<b>0.098</b>
Parimutuel	0.295	—	—	—	0.519
Poll	0.266	—	—	—	—

$$\text{MSR} \geq \text{Parimutuel} \geq \text{Poll} \geq \text{DblAuctn}$$

$$\text{MSR} > \text{Poll} \geq \text{DblAuctn}$$



## 2 States: Confusion & Mirages



## 2 States: Catastrophes

Periods with catastrophes:

(32 pers. total)	DbIAuc	MSR	Pari	Poll
No Trade	0	1	<b>4</b>	0
Confusion	5	7	6	<b>11</b>
Mirage	13	<b>14</b>	10	12
Confused Mirage	0	1	1	<b>3</b>
None	<b>14</b>	12	13	12

## 2 States: Summary of Results

Mech	2 States				8 States			
	Err	NoTrd	Mirg	Conf	Err	NoTrd	Mirg	Conf
DbIAuc	✓	✓	✓	✓				
MSR	×	✓	×	✓				
Pari	✓	×	✓	✓				
Poll	✓	✓	✓	×				

## 8 States: Error

Comparison of  $l_2$  distances with 8 states:

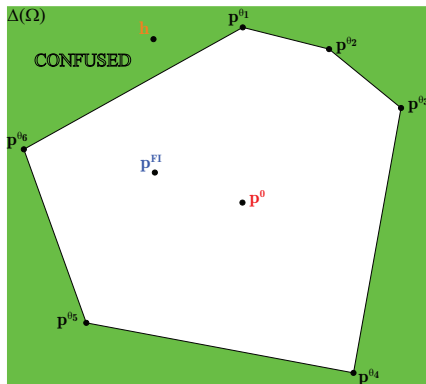
	Avg $l_2$ Dist.	Wilcoxon $p$ -values			
		DbIAuc	MSR	Parimutuel	Poll
Avg $l_2$ Dist.	—	0.696	0.527	0.605	0.418
DbIAuc	0.696	—	<b>0.002</b>	<b>0.093</b>	< <b>0.001</b>
MSR	0.527	—	—	<b>0.083</b>	0.324
Parimutuel	0.605	—	—	—	<b>0.001</b>
Poll	0.418	—	—	—	—

DbIAuc > Parimutuel > MSR  $\geq$  Poll

## 8 States: Catastrophes: No Trade

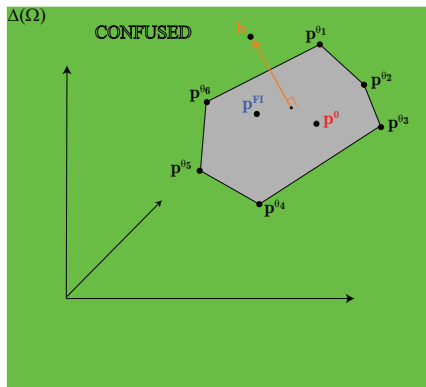
	DbIAuc	MSR	Parimutuel	Poll
Periods w/ No Trade	0	0	9/32	0

## 8 States: Confusion



$$\Pr(TTT|\theta) = 24/75 \text{ \& } \Pr(HHH|\theta) = 4/75 \quad \forall \theta$$

## 8 States: Confusion



Confusion occurs in *every* period of *every* mechanism

## 8 States: Catastrophes: Confusion

$l_2$  distance to convex hull, conditional on trade occurring:

	Avg Dist.	DbIAuc	MSR	Pari.	Poll
Avg. Dist.		0.447	0.362	0.398	0.312
# Trade Pers.		32	32	23	32
DbIAuc	0.447	—	<b>0.001</b>	0.107	< <b>0.001</b>
MSR	0.362		—	0.180	0.257
Pari	0.398			—	<b>0.008</b>
Poll	0.312				—

$$\text{DbIAuc} \geq \text{Pari} \geq \text{MSR} \geq \text{Poll}$$

$$\text{DbIAuc} > \text{MSR} \geq \text{Poll}$$

$$\text{DbIAuc} \geq \text{Pari} > \text{Poll}$$



## 8 States: Catastrophes: Mirages

Frequency of Mirages:

	Pers. w/ Trade	No. of Mirages	Frequency
DbIAuc	32	13	0.406
MSR	32	7	0.219
Pari.	23	7	0.304
Poll	32	3	0.094

DbIAuc > MSR > Poll  
Pari > Poll

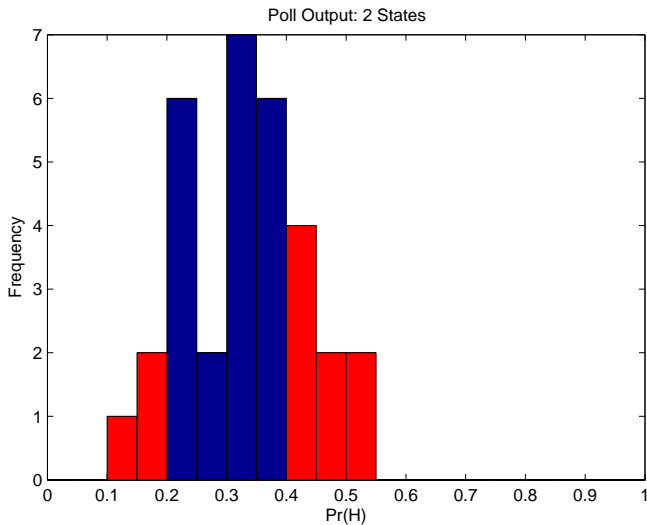
## 8 States: Summary

Mech	2 States				8 States			
	Err	NoTrd	Mirg	Conf	Err	NoTrd	Mirg	Conf
DbIAuc	✓	✓	✓	✓	×	✓	×	×
MSR	×	✓	×	✓	✓	✓	✓	✓
Pari	✓	×	✓	✓	×	×	✓	×
Poll	✓	✓	✓	×	✓	✓	✓	✓

Increased complexity: Double auction fails, MSR & Poll work

# Declaring a Winner?

Poll's only failing: confusion in 2-states. How bad is it?



# Beating the Prior

Percentage of periods where mechanism outperformed the “informed” prior:

	2 States	8 States
DblAuc	0.375	0.000
MSR	0.355	0.250
Pari	0.393	0.044
Poll	<b>0.406</b>	<b>0.313</b>

Poll looks good (relatively)...

- *Why* does the poll out-perform the market?
- **Observation 1:** Preferences are aligned in the poll, so traders have no incentive to misrepresent
- 'Misrepresenter': Move away from full info, then move toward
- Number of misrepresentors per mechanism:

DbIAuc	MSR	Pari	Poll
14	5	12	3

- **Observation 2:** Traders have an incentive to participate in the poll
- No-trade theorem in DbIAuc and Parimutuel
- MSR and poll are subsidized
  - 25.9 cents/trader/period in 2-state
  - 35.0 cents/trader/period in 8-state
- Pari-mutuel no trade: 4/32 and 9/32 pers.
- DbIAuc: 1 inactive trader in 4/64 periods
- MSR: 1 period of no trade (1st period confusion?)

- **Observation 3:** Attention is 'spread thin' in the DblAuc

States	Txns/Min.	Vol./Min.
2	5.00	6.48
8	2.60	14.47

- % of txns on 2 most active securities: 46%
- % of txns on 2 least active securities: 8%
- Low-hanging fruit is missed:
  - $p(TTT) = 24/75$  and  $p(HHH) = 4/75$  regardless of pvt info
  - Avg  $|p(TTT) - 24/75|$  and Avg  $|p(HHH) - 4/75|$  are greater than any other mechanism
  - Significantly greater than MSR and Poll

# Observations

- **Observation 4:** Poll averages traders' announcements, mitigating effects of a single aberrant trader
- Frequency of worse-than-average final reports & predictions

Mech	2 States		8 States	
	Last Report	Prediction	Last Report	Prediction
DbIAuc	11	11	24	24
MSR	18	18	9	9
Pari-mutuel	11	11	9	9
Poll	<b>28</b>	<b>8</b>	<b>21</b>	<b>8</b>



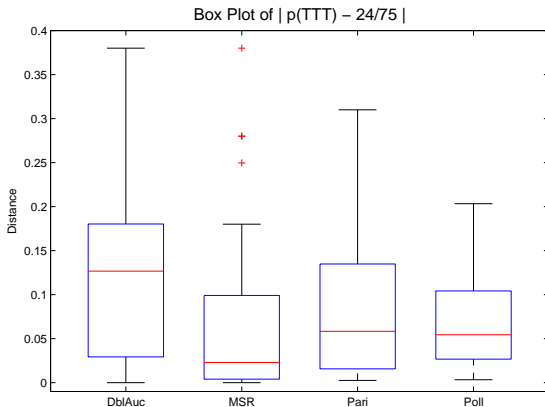
# Summary

- Double auction works fine with 2 states, not 8
  - Observation: think markets problem (focus on 2 securities)
  - Note: not market power problem
- Pari-mutuel hurt by delay and no trade
- MSR helps 'unfocus' attention, but prone to bad outcomes
  - Single 'bad' player can damage performance
- Poll performs best
  - Aligned incentives, participation incentives, averaging smooths behavior, completely 'unfocused'

The End

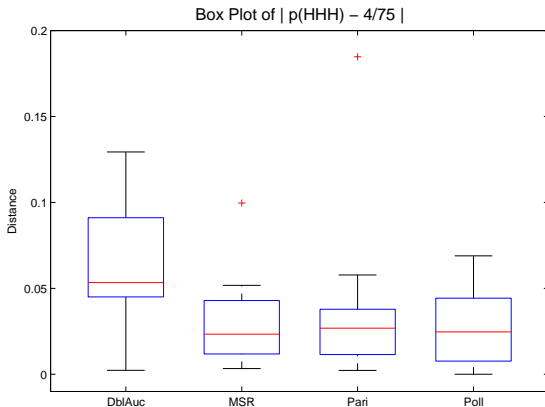
# $p(TTT)$ and $p(HHH)$

Recall that  $p(TTT) = 24/75$  and  $p(HHH) = 4/75$  regardless of signals



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  - *More* total volume per minute (14.47 units vs. 6.48 units)
- Fraction of trades in each market:

Session	TTT	TTH	THT	THH	HTT	HTH	HHT	HHH
1	0.18	0.13	0.15	<b>0.04</b>	<b>0.01</b>	0.15	0.12	0.23
2	0.25	0.17	0.07	<b>0.03</b>	0.15	0.05	0.14	0.12
3	0.34	<b>0.04</b>	0.26	0.07	0.14	<b>0.02</b>	<b>0.02</b>	0.11
4	0.16	0.07	0.23	0.12	0.17	0.09	0.07	0.09
All	0.27	0.11	0.16	0.05	0.14	0.05	0.09	0.12



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4	0.16	0.07	0.23	0.12	0.17	0.09	0.07	0.09
All	0.27	0.11	0.16	0.05	0.14	0.05	0.09	0.12

- 'FOCUS' = standard deviation of these distributions
- Focus, # transactions, and trading volume don't predict accuracy well.

- Is MSR 'trading' focused on a small number of securities?

Session	TTT	TTH	THT	THH	HTT	HTH	HHT	HHH
1	0.23	0.12	0.11	0.09	0.17	0.12	0.09	0.07
2	0.36	0.14	0.17	<b>0.02</b>	0.14	0.06	<b>0.02</b>	0.08
3	0.22	0.16	0.13	0.08	0.10	<b>0.03</b>	0.07	0.21
4	0.20	0.13	0.06	0.10	0.16	0.10	0.10	0.16
All	0.24	0.13	0.11	0.08	0.14	0.08	0.08	0.14

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  - $ERROR_t = 0.007 + 0.202 \times FOCUS_t$

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  - $ERROR_t = 0.007 + 0.202 \times FOCUS_t$
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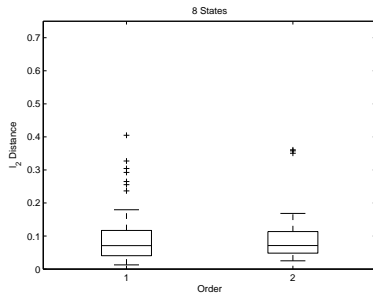
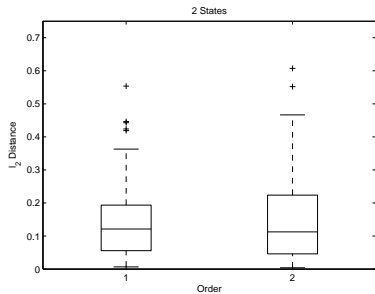
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- Step 5: Run the mechanism, which generates  $h(\omega)$  over  $\Omega$ .

# Order Effects



- Pairwise Wilcoxon tests:  $p = 0.82$  or  $0.93$
- By mechanism:  $p \geq 0.39$  or  $\geq 0.07$  (Pari. worse)

## 2 States: Catastrophes: Confusion

Average  $l_2$  distance to convex hull, conditional on confusion:

DbIAuc	0.0011
MSR	0.0340
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- Is slight confusion any better than bad confusion??

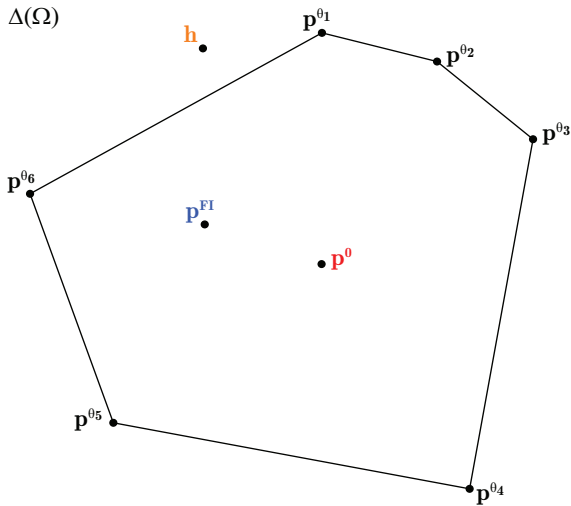
## 2 States: Mechanism Error Without Confusion

Average mechanism error, conditional on NO confusion:

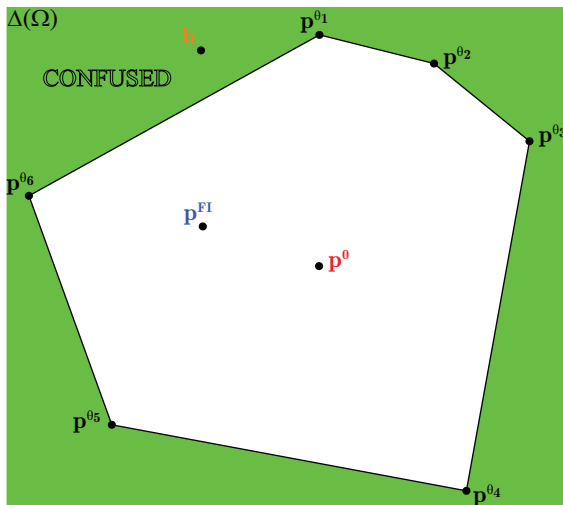
	No Confusion	All Periods
DbIAuc	0.128	0.131
MSR	0.136	0.210
Pari	0.110	0.148
Poll	0.093	0.133

No significant differences in pairwise tests

# 8 States: Catastrophes: Confusion

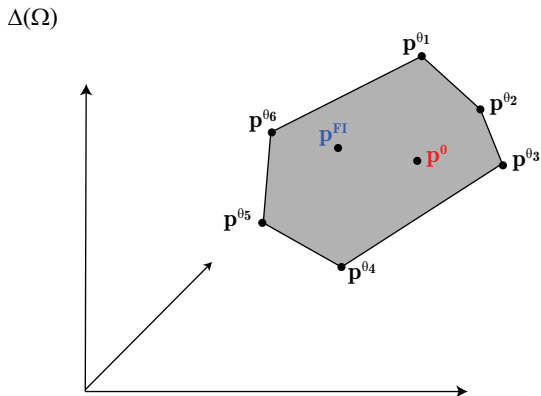


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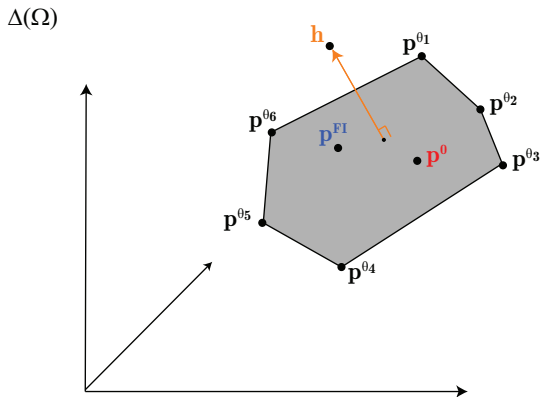




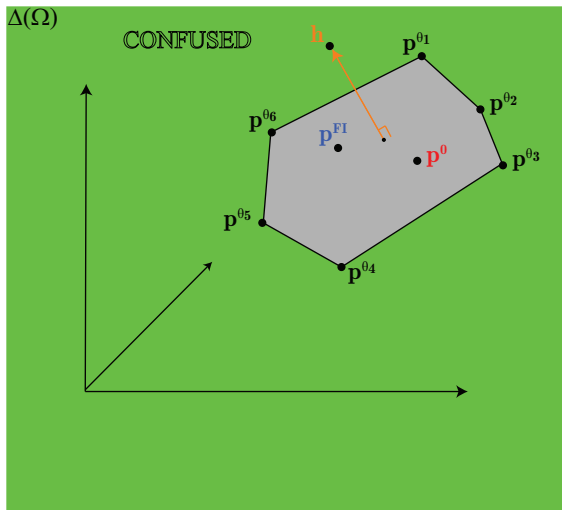
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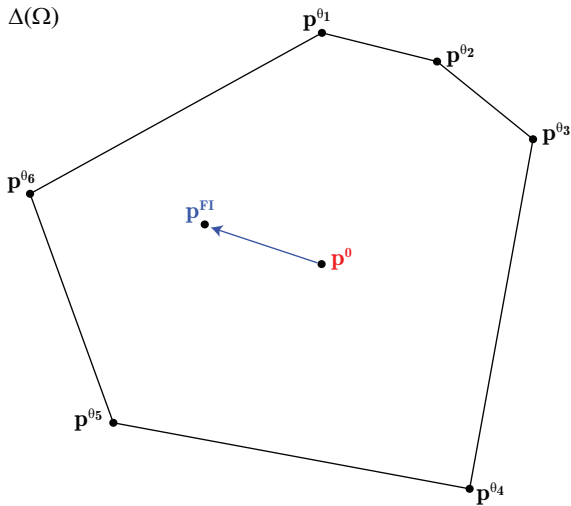
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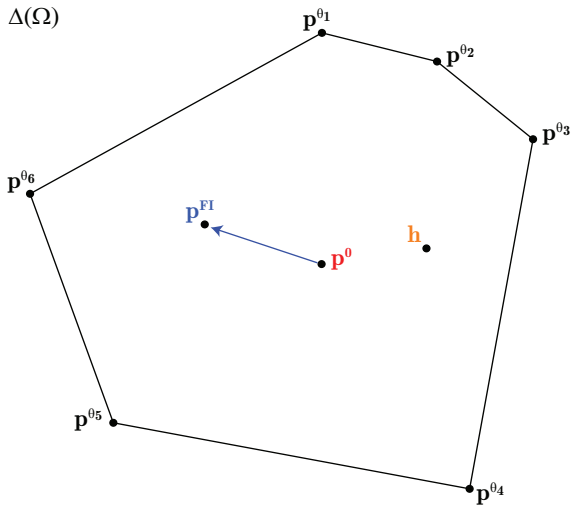
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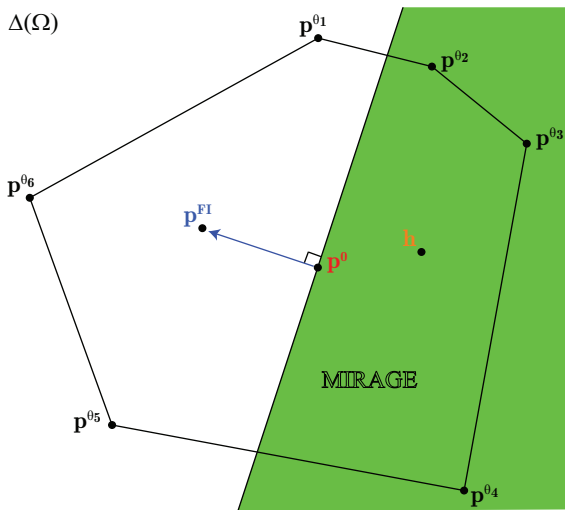
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- An alternative definition:
  - Of the 8 probabilities, 6 should move from  $p^0$
  - How many (out of 6) move the right way?

	Mean	MSR	Pari	Poll
DbIAuc	3.03	<b>0.049</b>	<b>0.046</b>	<b>0.034</b>
MSR	3.69		0.798	0.239
Pari	3.70			0.467
Poll	3.97			

Wilcoxon  $p$ -values

## 8 States: Mirages

Wilcoxon test on 'angles' between  $(h - p^0)$  and  $(p^{Fl} - p^0)$ :

	MSR	Pari	Poll
DbIAuc	<b>0.025</b>	0.490	0.290
MSR		0.180	0.773
Pari			0.286
Poll			

Wilcoxon test  $p$ -values



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