Edgeworth Cycles: An Experimental Test of Markov Perfection

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Disclaimer: This presentation will make me look bad.

Maskin & Tirole (1988): A Theory of Dynamic Oligopoly

The setting for which Markov perfection was invented:

- Two firms.
- Stage game: Bertrand (price) competition.
- Downward-sloping market demand ($D(p) = 100 - p$)
- Lowest-priced firm serves entire market. (Tie: split 50-50)
- No capacity constraints.
- Constant marginal cost ($MC = 10$).
- Discrete price grid ($\{10, 15, \ldots, 100\}$).
- Infinitely repeated.
- **Alternating moves** (lock-in).

Monopoly price: $\max_p (p - 10) * (100 - p) \Rightarrow p^m = 55$.
Stage game equilibria: $p_1 = p_2 \in \{10, 15\}$. Zero (or small) profit.

Markov Perfect Equilibrium: A Refresher

General notation:

- p_i^t - Price (action) of player i at time t.
- $p^t = (p_1^t, p_2^t)$ - Price profile at t.
- $h^t = (p^1, \ldots, p^{t-1})$ - History at time t.
- $h^t \in \mathcal{H}^t$ - Set of all length-t histories.
- $p_i^t(h^t)$ - Strategy of i at t, depending on h^t.

Markov-Perfect Equilibrium (MPE)

- $p_i^t(\cdot)$ can only depend on 'payoff-relevant' information.
- Severely limits detection & punishment. Cognitively simple.
- Refinement of subgame perfect equilibrium (SPE)
- Invented by Maskin & Tirole (1988) to study dynamic oligopoly.
- Very convenient for empirical work & applied theory.

Markov Perfect Equilibrium

Standard 'state variable' definition:

- State variable ω^t, depends on past play (incl. nature). Should only capture 'payoff-relevant' information
- $p_i^t(\omega^t)$ - strategy at t depends only on state variable.
- Time-invariant strategy: $p_i(\omega^t)$
- MPE $=$ SPE in this restricted class of strategies.
- What is ω^t?
  - Exogenous or derived?

Markov Perfect Equilibrium

'History partition' definition (Maskin & Tirole 2001, F&T text):

- Partition of $\mathcal{H}^t$: $\Pi^t = \{\mathcal{H}^t_0, \mathcal{H}^t_1, \ldots, \mathcal{H}^t_K\}$
- $p^t_i(h^t)$ is Π^t-measurable. $h^t, \hat{h}^t \in \mathcal{H}^t_k \Rightarrow p^t_i(h^t) = p^t_i(\hat{h}^t)$
- Now ω^t is derived as $\omega^t = \mathcal{H}^t_k$.
- Markov: partition based on 'payoff-relevant' information:
  - $h^t, \hat{h}^t \in \mathcal{H}^t_k$ *iff* h^t and $\hat{h}^t$ induce same continuation game.
    - Same available actions & same payoffs in all periods going forward.
- Coarser partition: not consistent with forward-looking rationality
- Finer partition: less restrictive.
- MPE = SPE with each p^t_i restricted to be msbl. in this partition
- **Question:** What partition do people actually use?

MPE in Maskin & Tirole (1988) Dynamic Oligopoly

Return to alternating-move oligopoly:

Only part of h^t that affects i's continuation game is p_j^{t-1}.

$$\pi_i(p_i^t; h^t) = \begin{cases} 0 & \text{if} \quad p_i^t > p_j^{t-1} \\ (p_i^t - 10) * (100 - p_i^t) & \text{if} \quad p_i^t < p_j^{t-1} \\ \frac{1}{2}(p_i^t - 10) * (100 - p_i^t) & \text{if} \quad p_i^t = p_j^{t-1} \end{cases}$$

Thus, in MPE, $p_i^t(p_j^{t-1})$.

Maskin & Tirole (1988): A Theory of Dynamic Oligopoly

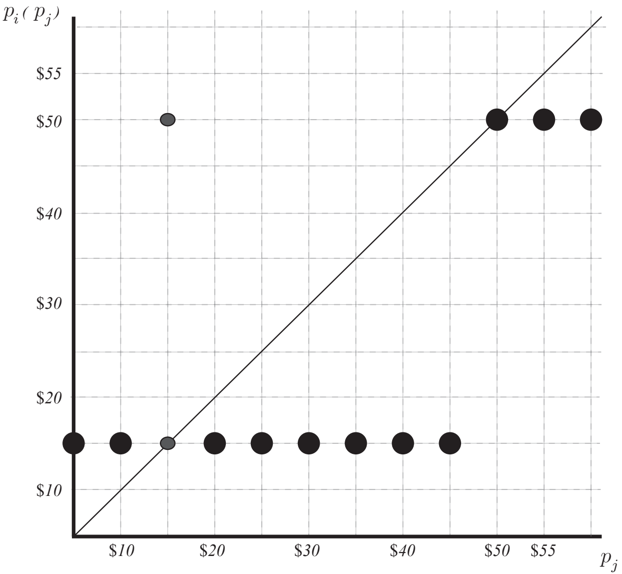
Subgame Perfect: Folk Theorem (Fudenberg & Maskin 1986)

Markov Perfect: (1) Fixed-price equil., and (2) Edgeworth cycles.

Fixed-price equilibria:

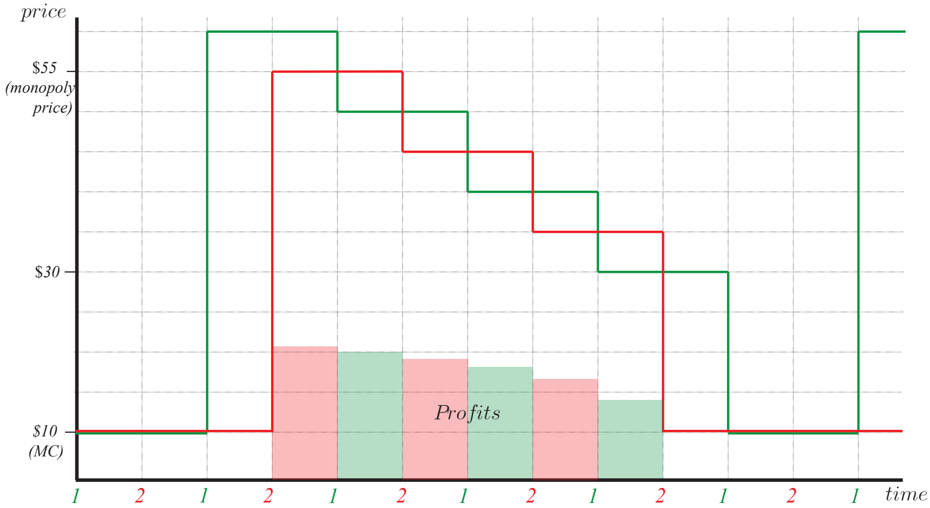
- Fix any collusive price $p^* \in \{30, 35, 40, \ldots, 75\}$.
- $p_i(p_j^{t-1}) = \begin{cases} p^* & \text{if} \quad p_j^{t-1} \geq p^* \\ 15 & \text{if} \quad p_j^{t-1} \in \{10\} \cup \{20, 25, \ldots, p^* - 5) \\ mix\{15, p^*\} & \text{if} \quad p_j^{t-1} = 15 \end{cases}$
- Note: Can't punish at 15 forever

Maskin & Tirole (1988): Fixed Price Equilibrium



Here comes the cool part...

Edgeworth Cycle Equilibrium

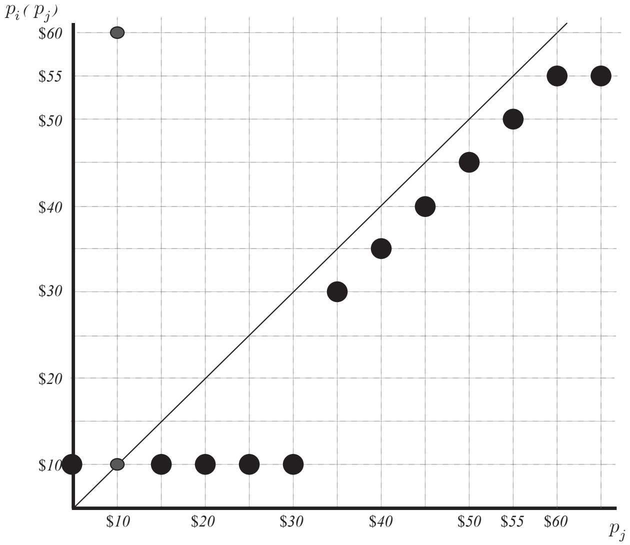


Maskin & Tirole (1988): A Theory of Dynamic Oligopoly

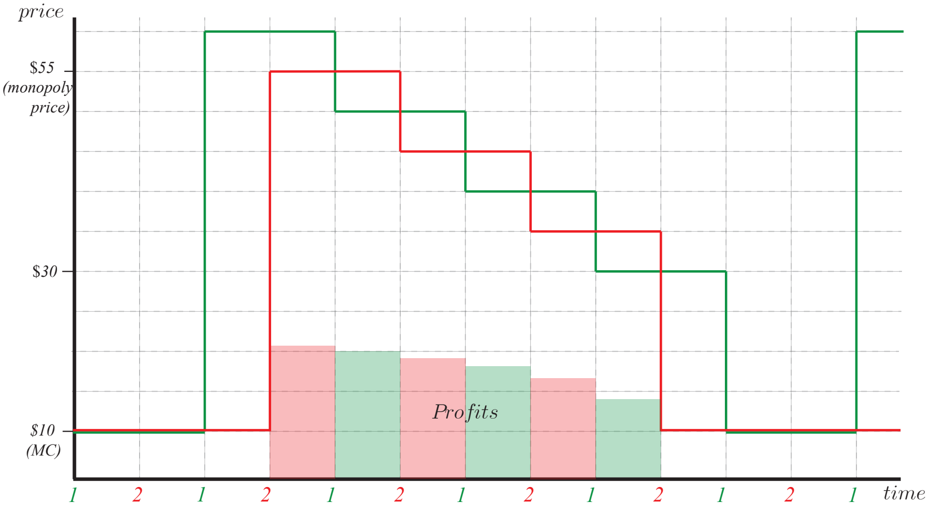
Edgeworth Cycle Equilibrium:

$$p_i(p_j^{t-1}) = \begin{cases} 55 & \text{if} \quad p_j^{t-1} \geq 60 \\ 50 & \text{if} \quad p_j^{t-1} = 55 \\ 45 & \text{if} \quad p_j^{t-1} = 50 \\ 40 & \text{if} \quad p_j^{t-1} = 45 \\ 35 & \text{if} \quad p_j^{t-1} = 40 \\ 30 & \text{if} \quad p_j^{t-1} = 35 \\ 10 & \text{if} \quad p_j^{t-1} \in \{15, 20, 25, 30\} \\ mix\{10, 60\} & \text{if} \quad p_j^{t-1} = 10 \end{cases}$$

Edgeworth Cycle Equilibrium



Edgeworth Cycle Equilibrium



Why This Game?

Markov Perfection makes strong, specific (& surprising) predictions.

Thus, lots of power in testing MPE.

History of Theory of Price Cycles

Theory:

1. Bertrand (1883) (following Cournot 1838)
2. Edgeworth (1881): Non-existence if firms can't supply whole market
   - 'Adaptive' version of mixing? Not equilibrium.
3. Constrained firms: Shapley (1957), Shubik (1959), Beckman (1965), Shapley & Shubik (1969), Levitan & Shubik (1972), ...
4. Maskin & Tirole (1988)
   1. Eckert (2003): unequal 'tie-breaking' rule
   2. Noel (2008): 3 firms, product differentiation, discount factors, etc.

Robustness of Cycling

Simultaneous Moves?

- **No longer any payoff-relevant state variable!** $p_i^t(h^t) = p_i^t(\hat{h}^t)\ \forall h^t, \hat{h}^t$
- Stage game equilibria are the only MPE
- Intuition: 'instant undercutting'
- What if we allow strategies that condition on finer partition?
  - Full history? $h^t = (p_i^1, p_j^1, p_i^2, p_j^2, \ldots, p_i^{t-1}, p_j^{t-1})$
    - ⋆ Folk theorem
  - Previous 1 period? (p_i^{t-1}, p_j^{t-1})
    - ⋆ Fixed price equil? ✓. Grim trigger to $10.
    - ⋆ Cycling equil? NO. i can 'skip his turn'
  - Previous 2 periods? $(p_i^{t-2}, p_j^{t-2}, p_i^{t-1}, p_j^{t-1})$
    - ⋆ Fixed price equil? ✓. Grim trigger to $10.
    - ⋆ Cycling equil? ✓. Grim trigger to $15.

Robustness of Cycling

Continuous Time?

- Known to generate *more* cooperation in theory & lab
- Simon & Stinchcombe (1989)
  - Lots of messy issues
  - Solution: Instantaneous but sequential response
  - Problem: Not a perfect match to our experiment
- Markov: i can react to j's current price (& nothing else)... but j can respond instantly.
  - Fixed price equil? NO. Can't sustain punishment.
  - Cycling equil? NO. Instant undercutting.
  - Conjecture: Only MPE are stage game equilibria.
- Maskin & Tirole (WP): Poisson 'lock-in' time
  - Mathematically *equivalent* to discrete-time model
  - State variable: locked-in price of opponent (if any)
  - Same MPE: fixed-price and cycling
  - (Lock-in keeps cycles from instant collapse)

The Design

2×2 design: $\{$simultaneous,alternating$\} \times \{$discrete,continuous$\}$

Cycling Equilibria?

	Discrete Time	Cts. Time
Simult. Move	NO	NO
Alt. Move	YES	YES

Fixed-Price Equilibria?

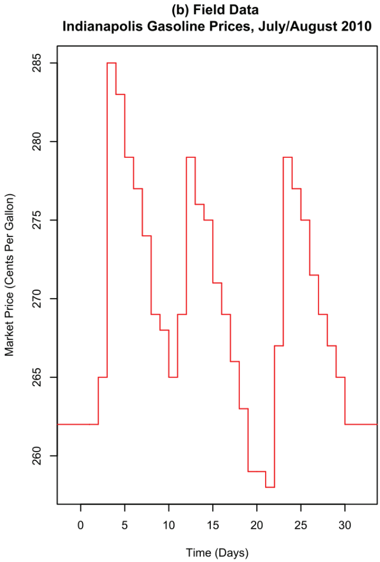
	Discrete Time	Cts. Time
Simult. Move	NO	NO
Alt. Move	YES	YES

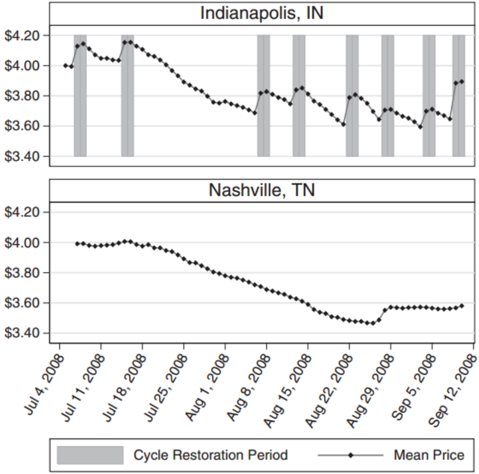
Evidence?

Are cycles seen in the field??

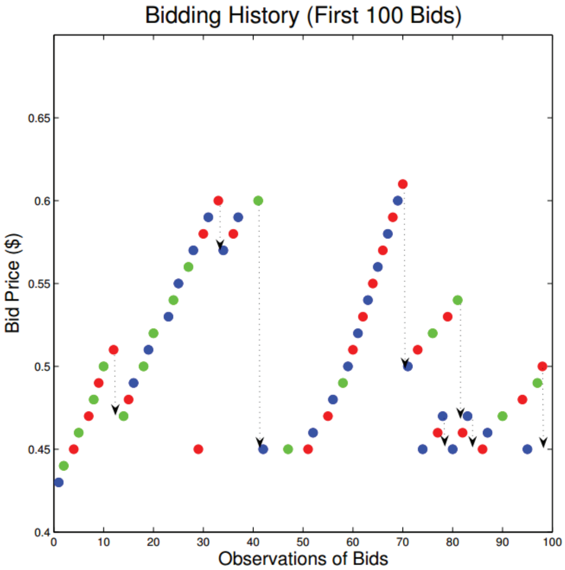
(Note: this is a tangent.)

Retail Gas Prices in the Midwest





Sponsored Search Auctions: Yahoo! 2001



Evidence??

The literature:

1. Allvine & Patterson (1974) “Highway Robbery”: U.S. gas 1960s-70s
2. Castanias & Johnson (1993): LA gas prices vs. M&T (1988)
3. Eckert (2003), Noel (2007a,b), Atkinson (2009): Canadian gas prices
4. Lewis (2009a,b): Midwestern gas prices
5. Wang (2009a,b): Perth, Australia gas prices
6. Foros & Steen (2008): Norwegian gas prices
7. Edelman & Ostrovsky (2007): Sponsored search auctions
    - Also Zhang (2005 WP)?

Notes

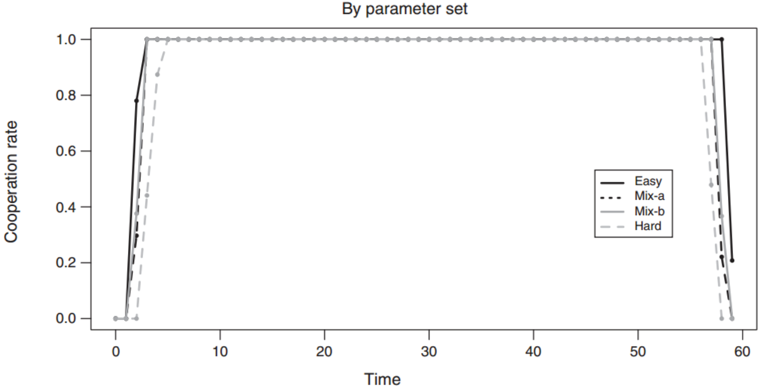
1. Big player/leader seems important (Speedway, QT)
2. Consumer can save 4% by timing purchase
3. Cycling city no better/worse for unsophisticated drivers
4. Non-cycling cities: rockets & feathers
    - Lewis (2009): Katrina price recovery

Mystery: Theory's not robust. Do gas stations *really* alternate their price-setting??? (Not a good question for the lab.)

David Byrne: runners & folk theorems & anti-trust lawsuits.

Experiments on Continuous Time

- Near-instantaneous reactions minimize benefit to deviation.
- Friendman & Oprea (2012), e.g.:



Experiments on Markov Perfection

- McKelvey & Palfrey (1995): Hold-out game (??)
- Repeated PD: Cooperation = non-Markov behavior.
- Vespa JMP: CPR game, MPE even when inefficient
- Vespa & Wilson WP: Dynamic PD, Dynamic CPR, Random PD
- Battaglini Nunnari & Palfrey: Comparative statics suggest MPE
- Most papers: applications
- Engle-Warnick; Dal-Bo & Frechette: recovering strategies

Novelty of our paper?

- Edgeworth cycles & their robustness make a sharp prediction
  - Powerful test
- Can still do strategy estimation procedures.

Experiments on Cycling

- Capacity-constrained markets:
  - DA markets: Plott & Smith (1978), Davis & Holt (1994), etc.
  - Adaptive cycles sometimes seen
    Kruse (1993), Wilson (1998), Davis & Wilson (2000), Fonseca & Normann (2013), Guilllen (2004), Durham et al (2004), Cason et al (2005), Peeters & Strobel (2003), Brandts & Guillen (2007), Durham et al (2004), Davis & Wilson (2008), Kruse et al (1994).
- Bertrand competition w/ Alternating Moves:
  1. Martini (2003): Against computers playing Markov strategies
  2. Bruttel (2009): Focused on fixed-price equil. Some evidence of cycles.
  3. Leufkens & Peeters (2011): Not many cycles... $\rightarrow$

Leufkens & Peeters

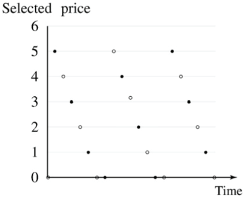
Setup:

- Test M&T's leading example (6 price levels)
- {Alternating,Simultaneous} $\times$ {Finite,Infinite}
- Algorithm identifies cycling at group level

Identified Cycling Groups:

	Alt.	Sim.
Finite	5/15	0/15
Infinite	1/15	4/14

Leufkens & Peeters



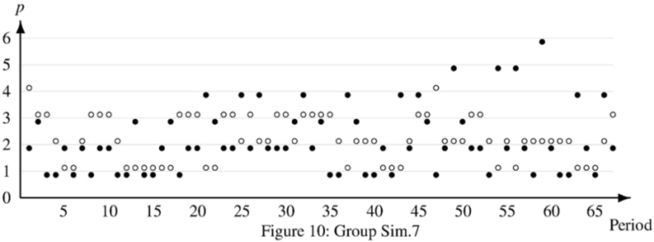


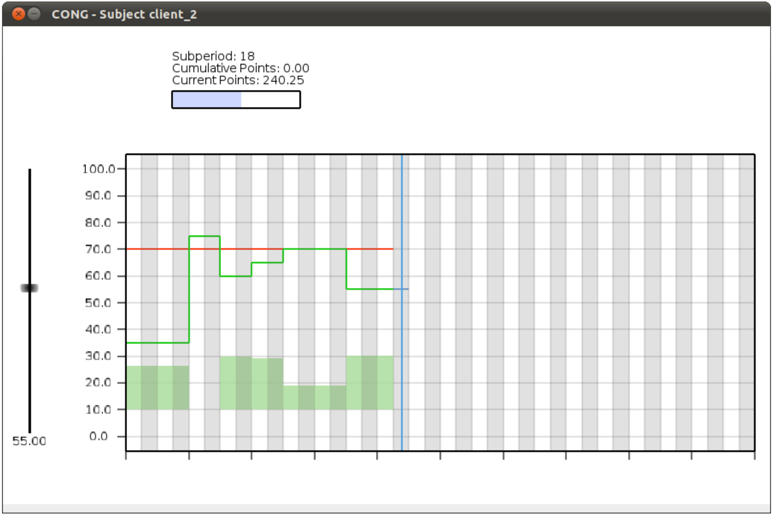
Figure 10: Group Sim.7

Lesson: We need to be careful when identifying cycles!

Our Design

- Sessions of 12 or 14 subjects.
- Randomly re-matched in pairs for 30 supergames.
- Profit: $(100 - p)(p - 10)$, split 50-50 if tied
- Price $\in \{10, 15, 20, 25, \ldots, 100\}$
- Indefinitely repeated (5% chance of ending each period)
- 4 Treatments:
  1. Discrete-time, alternating-move: **DA**
  2. Discrete-time, simultaneous-move (baseline): **DB**
  3. Continuous-time, 'alternating-move' (Poisson lock-in): **CA**
  4. Continuous-time, 'simultaneous-move' (no lock-in): **CB**
- Markov prediction: Cycling *iff* alternating move (DA & CA)

Interface



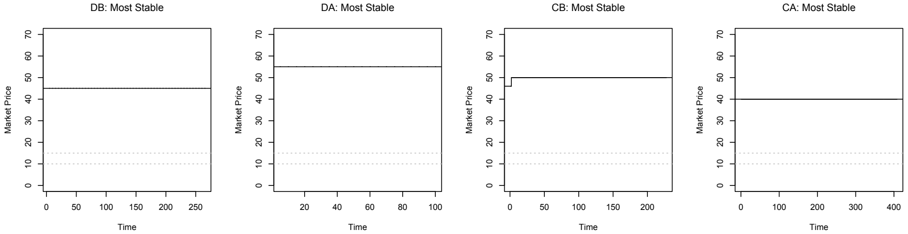
CONG - Subject client_2
Subperiod: 18
Cumulative Points: 0.00
Current Points: 240.25
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
55.00

Are You Ready?

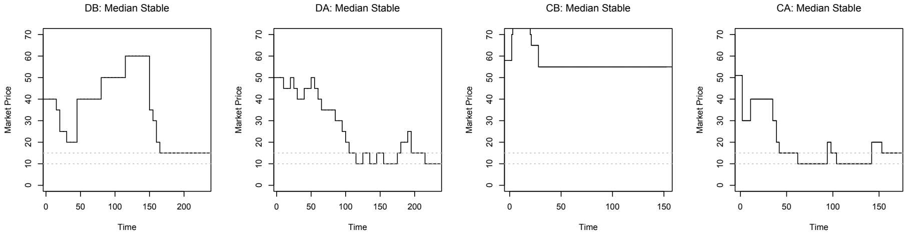
Finally... some results

Results: Example Periods, Lowest Variability

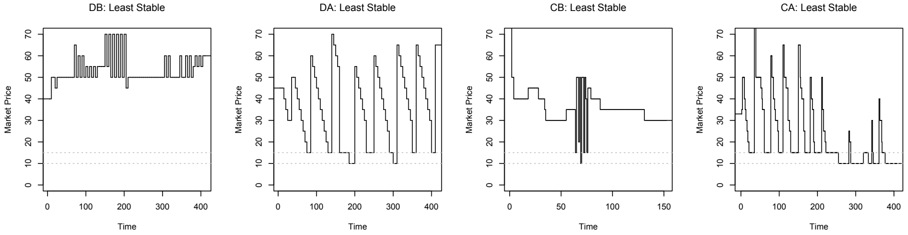
Market price time paths by treatment:



Results: Example Periods, Median Variability



Results: Example Periods, Maximum Variability



Summary of Results to Come

What I hope to convince you:

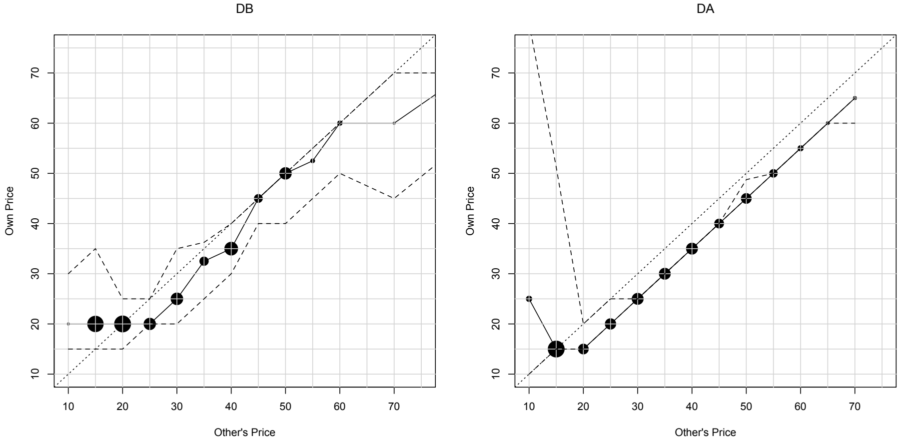
Cycling?

	Discrete Time	Cts. Time
Simult. Move	No	No
Alt. Move	Yes	Yes

Fixed (Collusive) Pricing?

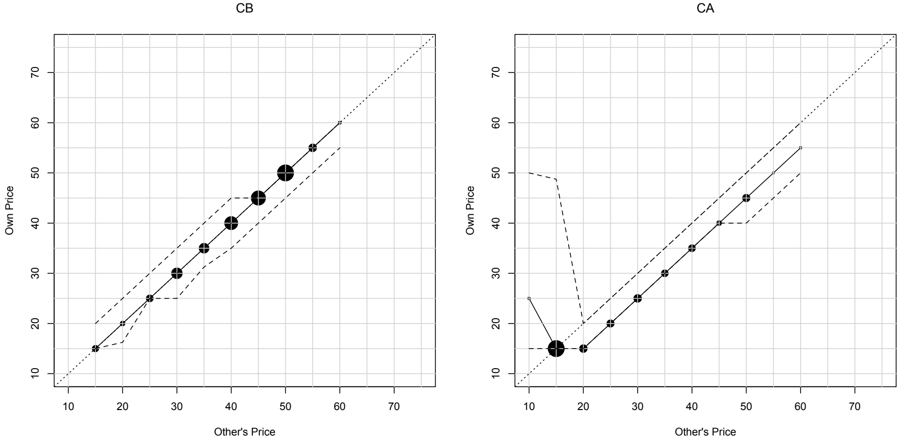
	Discrete Time	Cts. Time
Simult. Move	Not much	Yes
Alt. Move	Not much	Not much

Responses to p_j^{t-1}



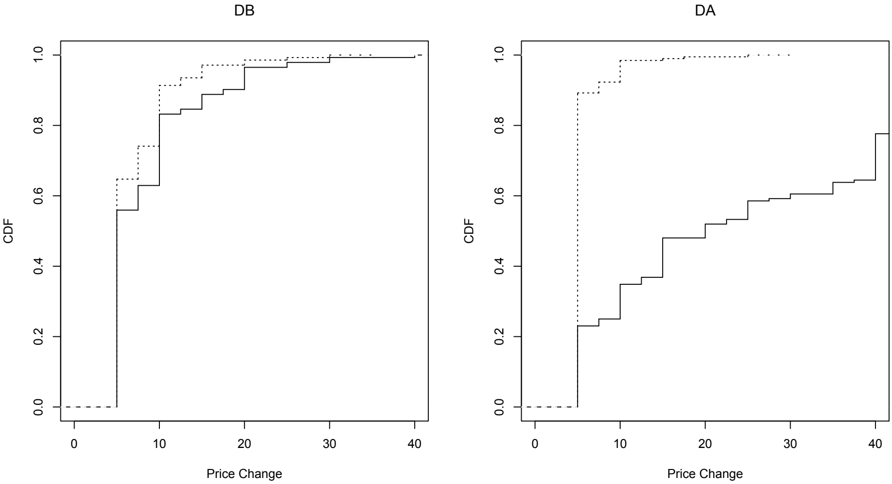
45-degree line: $p_i = p_j$

Responses to p_j^{t-1}



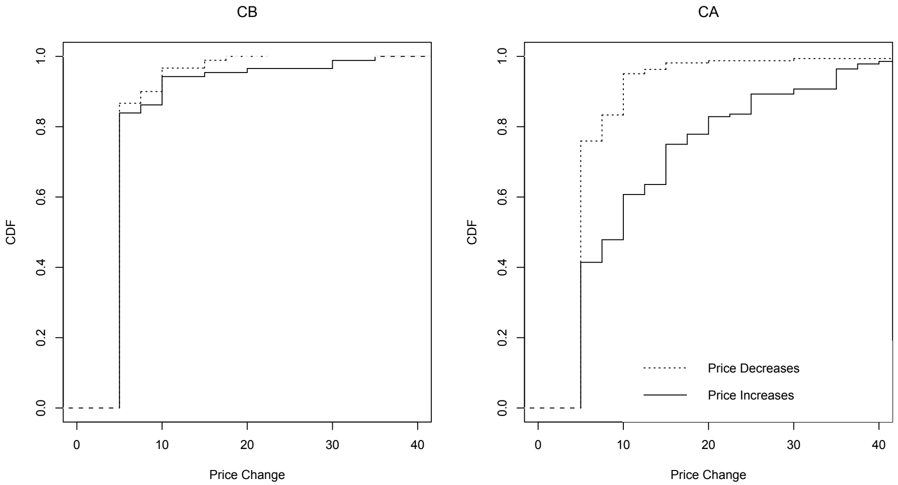
45-degree line: $p_i = p_j$

Size of Increases vs Decreases



Increases: dashed. Decreases: solid

Size of Increases vs Decreases



Increases: dashed. Decreases: solid

Total Variation

- Total Variation: sum of all price changes

Treat.	10%-ile	25%	50%	75%	90%
Total Variation (TV)					
DA	0.00	35.00	127.50	196.25	372.50
DB	15.00	40.00	100.00	215.00	327.50
CA	0.00	25.00	77.50	208.75	297.50
CB	0.00	5.00	35.00	71.25	115.00

- Low in CB: consistent with Friedman & Oprea (2012)
- Suggests strong ‘collusion’ in CB
- Cycling in DA and CA as predicted (better evidence later)
- DB has high variance, no cycling

Market Price Distribution

- Take the average price across time

Treat.	10%ile	25%	50%	75%	90%
			Sales Prices		
DA	13.75	15.00	30.00	40.00	55.00
DB	15.00	15.00	25.00	45.00	50.00
CA	15.00	15.00	25.00	45.00	50.00
CB	20.00	30.00	45.00	50.00	55.00

- CB avg price dist'n FOSD's other treatments
- All: 90% at or below monopoly price
- Cycling doesn't affect *average* price (DA & CA vs. DB)

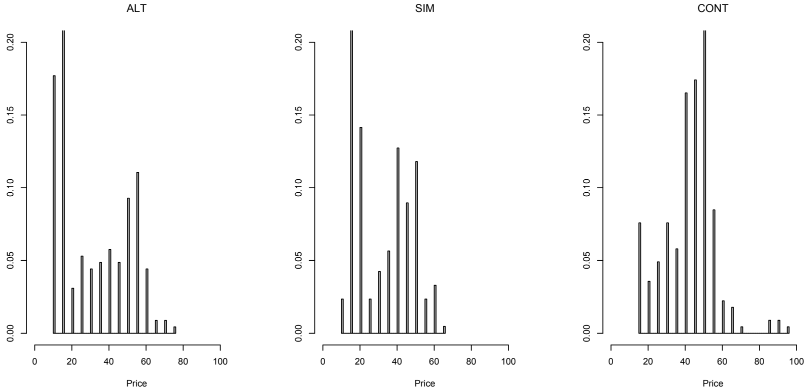
Average Profit Distribution

- Take the average profits across time per firm

Treat.	10%ile	25%	50%	75%	90%
Profits					
DA	118.75	209.38	405.00	713.12	1080.00
DB	112.50	183.12	365.00	620.62	987.50
CA	99.50	162.75	366.00	661.25	975.50
CB	219.70	357.88	622.75	853.75	1320.90

- CB collusion gives highest profits
- Cycling profits not much better than DB

Stable Prices

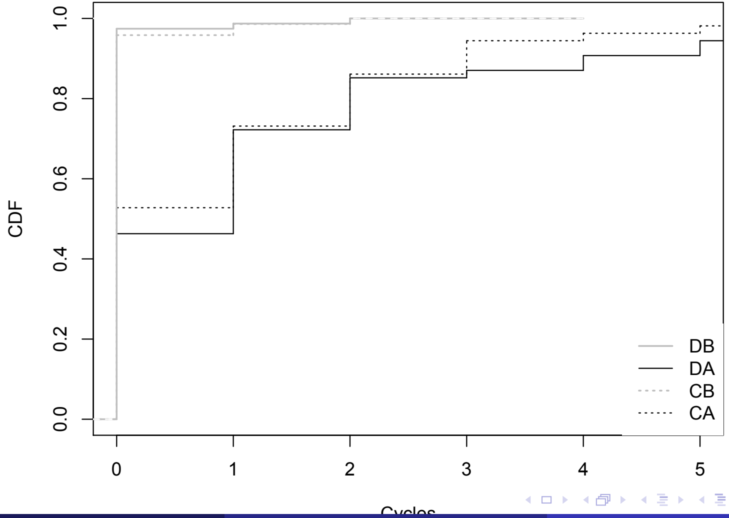


Histograms of price levels observed during stable price regimes lasting at least 25 seconds.

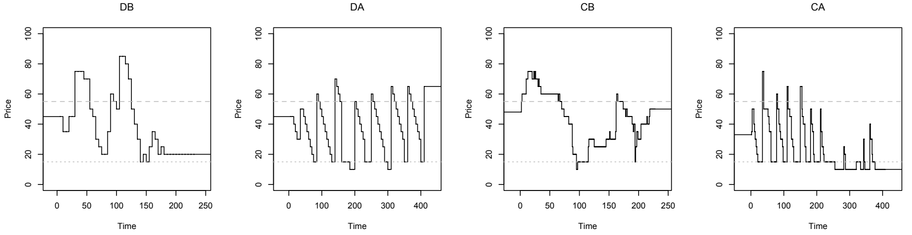
Identifying Cycles

- 18 new subjects: ‘identifiers’
- Shown 1/2 the data
- Count cycles in each period
- Paid based on count agreement
- Robustness checks: ∼80% consistent

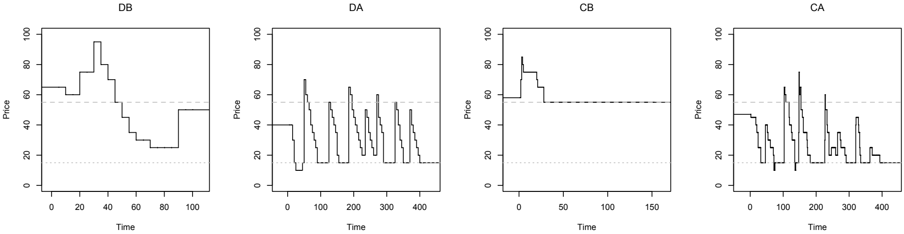
Cycle Counts



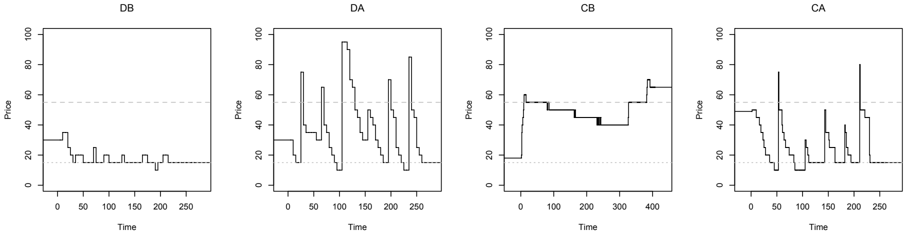
Periods With Most Cycles



Periods With 2nd Most Cycles



Periods With 3rd Most Cycles



The Markov Assumption: Discrete Time

	Treatment **DA**		Treatment **DB**	
Coefficient	Estimate	p-value	Estimate	p-value
Const.	26.39**	0.003	27.88**	0.001
p_j^{t-1}-mid	-17.50**	0.004	-22.23**	0.003
p_j^{t-1}-hi	37.21**	< 0.001	6.62	0.496
p_j^{t-1}	-0.31	0.392	-0.89	0.058
$p_j^{t-1} \times (p_j^{t-1}$-mid)	1.15**	0.005	1.55**	0.003
$p_j^{t-1} \times (p_j^{t-1}$-hi)	0.22	0.469	1.07*	0.034
p_j^{t-2}-mid	-5.23	0.206	0.45	0.882
p_j^{t-2}-hi	6.69	0.204	11.28	0.155
p_j^{t-2}	-0.29	0.321	0.30	0.138
$p_j^{t-2} \times (p_j^{t-2}$-mid)	0.38	0.191	-0.10	0.624
$p_j^{t-2} \times (p_j^{t-2}$-hi)	0.22	0.424	-0.27	0.249
Cluster: matches	$N = 21,348,\ R^2 = 0.53$		$N = 15,418,\ R^2 = 0.41$	

The Markov Assumption: Continuous Time

	Treatment **CA**		Treatment **CB**	
Coefficient	Estimate	p-value	Estimate	p-value
Const.	18.36**	< 0.001	28.79**	< 0.001
p_j^{t-1}-mid	-15.32**	< 0.001	-25.06**	< 0.001
p_j^{t-1}-hi	26.18**	0.002	-0.28	0.968
p_j^{t-1}	-0.33*	0.017	-0.90**	< 0.001
$p_j^{t-1} \times (p_j^{t-1}$-mid)	1.11**	< 0.001	1.73**	< 0.001
$p_j^{t-1} \times (p_j^{t-1}$-hi)	0.38*	0.042	1.32**	< 0.001
p_j^{t-2}-mid	3.36*	0.033	-1.37	0.496
p_j^{t-2}-hi	10.67*	0.026	-5.13	0.363
p_j^{t-2}	0.28*	0.011	0.07	0.546
$p_j^{t-2} \times (p_j^{t-2}$-mid)	-0.19	0.070	0.04	0.718
$p_j^{t-2} \times (p_j^{t-2}$-hi)	-0.28*	0.013	0.13	0.374
Cluster: matches	$N = 98,532,\ R^2 = 0.48$		$N = 72,000,\ R^2 = 0.71$	

Estimating Repeated Game Strategies I

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Estimating Repeated Game Strategies II

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Estimating Repeated Game Strategies III

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Summary

- Strong support for responding to $t-1$
- Not *quite* Markovian
  - $t-1$ may not be payoff relevant
- State space $= \times_{i=1}^{n} A^{t-1}$ (blows up in n)
- Reminiscent of learning dynamics literature
- Still more work to be done on analysis
- Open question: what if $t-2$ is payoff relevant?

Thank you!