Belief-Free Strategies in Repeated Games with Stochastically-Perfect Monitoring: An Experimental Test

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The Prisoners Dilemma

- Two firms in a joint venture

	C	D
C	$30, 30$	$5, 35$
D	$35, 5$	$10, 10$

- **One Shot:** no cooperation
- Cooperation requires:
  1. a future
  2. feedback (monitoring)

Perfect Monitoring

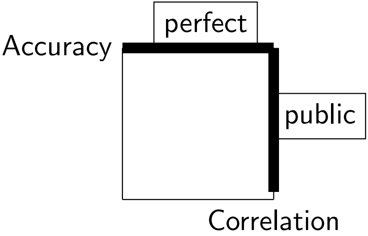
- ∞-repeated PD
- Time discounting: $\delta = 0.9$.
- $u_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_i^t$

	C	D
C	$30, 30$	$5, 35$
D	$35, 5$	$10, 10$

- **Perfect Monitoring:** see other's action (C or D)
  - Easy to execute & coordinate punishments.

Imperfect Monitoring

- **Imperfect Monitoring:** can't see the effort of other party
  - Only see the (random) outcome of the joint venture
- Round t action of player i: $a_i^t \in \{C, D\}$.
- At end of t, i gets signal $z_i^t \in \{cc, cd, dc, dd\}$, depends on (a_i^t, a_j^t).
  - Accuracy: $Pr(z_i^t = (a_i^t, a_j^t))$
  - Correlation $Pr(z_i^t = z_j^t)$
- $u_i^t(a_i^t, z_i^t)$ doesn't depend on a_j^t.



Public Monitoring

- **Public Monitoring:** $z_i^t = z_j^t$
- Example: Cournot duopoly (Green & Porter)
  - Observe period-t price $100 - a_i^t - a_j^t + \varepsilon^t$
  - Can't back out a_j
- Still easy to coordinante punishments
  - Both players can see when it's time to be punishing
- Inefficiency: unnecessary punishments due to wrong signals

Private Monitoring

- **Private Monitoring:** $z_i^t \neq z_j^t$
- This paper: conditionally independent
  - $z_i^t \in \{c, d\}$, depends on $a_j^t \in \{C, D\}$
  - $Pr(c_i|C_j) = Pr(d_i|D_j) = 0.75$
- Can't coordinate on punishment phases, etc.
- Observing $z_i = d_i$ could mean
  1. j played C but signal was wrong
  2. j played D as part of an equilibrium punishment phase
  3. j played D because he deviated from equilibrium!
- j knows his action is hidden, so he can 'get away' with deviations!
- Beliefs needed to calculate are complicated:
  - j's action, the signals j has seen, j's belief about i's action, j's belief about i's signal...
- Solving equilibria is intractable

Belief-Free Equilibrium

- Idea: Mixed-strategy equilibrium (e.g., Ely & Valimaki 2002)
- Suppose perfect monitoring for now
- Same $Pr(C_i)$ every period?
  - No: D is a dominant strategy, so $Pr(C_i) = 0$
- Allow mixing to depend on last period action a_j
  - $Pr(C_i|C_j) > Pr(C_i|D_j)$
  - This incentivizes j to cooperate
  - 'Power' of incentive: $Pr(C_i|C_j) - Pr(C_i|D_j)$
- Can even allow it to depend on own last-period action
  - $Pr(C_i|C_iC_j) > Pr(C_i|C_iD_j)$ and $Pr(C_i|D_iC_j) > Pr(C_i|D_iD_j)$
  - Can have different 'power' of incentives
- Mixing $\Rightarrow$ indifferent
  - But, indifferent in the whole repeated game going forward
- No need to track beliefs b/c they're irrelevant.

Our Question

- Belief-free equilibria are easy to solve
- Seems like just a theorist's trick. Not descriptive.
- But wait... maybe it's plausible!
- Maybe it's a long-run steady state
- Recent evidence:
  - Breitmoser (2015) meta-analysis
  - Romera & Rosokha (WP) explicit mixing
  - Both with perfect monitoring...

What we do:

1. Look for evidence of this mixing
2. New design feature that tests a stark prediction of belief-free equilibria

Solving Belief-Free Equilibria

	C	D
C	$30,30$	$5,35$
D	$35,5$	$10,10$

- First: perfect monitoring case (for understanding)
- Strategy: $Pr(C_i|a_i a_j)$ (1-period memory)
- $V_i^{a_j}$: continuation value when opponent will play a_j this period

Solving Belief-Free Equilibria

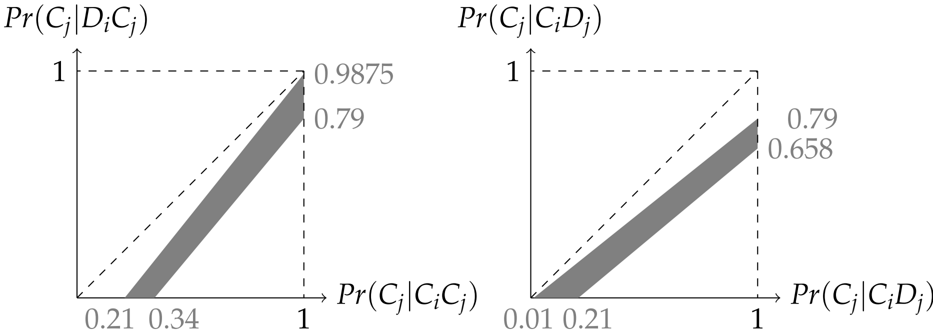
	C	D
C	$30, 30$	$5, 35$
D	$35, 5$	$10, 10$

$$V_i^{C_j} = \underbrace{(1-\delta)30 + \delta\left[Pr(C_j|C_iC_j)V_i^{C_j} + (1 - Pr(C_j|C_iC_j))V_i^{D_j}\right]}_{\text{Payoff under } C_i}$$
$$= \underbrace{(1-\delta)35 + \delta\left[Pr(C_j|D_iC_j)V_i^{C_j} + (1 - Pr(C_j|D_iC_j))V_i^{D_j}\right]}_{\text{Payoff under } D_i}$$

$$V_i^{D_j} = (1-\delta)5 + \delta\left[Pr(C_j|C_iD_j)V_i^{C_j} + (1 - Pr(C_j|C_iD_j))V_i^{D_j}\right]$$
$$= (1-\delta)10 + \delta\left[Pr(C_j|D_iD_j)V_i^{C_j} + (1 - Pr(C_j|D_iD_j))V_i^{D_j}\right]$$

Solving Belief-Free Equilibria ($\delta = 0.95$)

$Pr(C_j|C_iC_j)$ vs. $Pr(C_j|D_iC_j)$ and $Pr(C_j|C_iD_j)$ vs. $Pr(D_j|C_iD_j)$:



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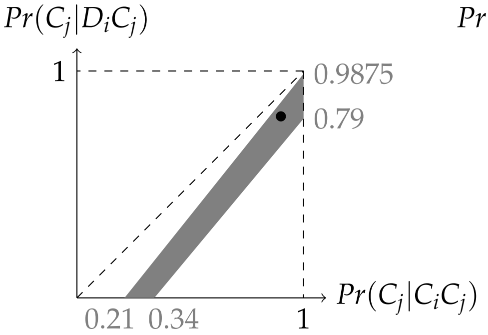

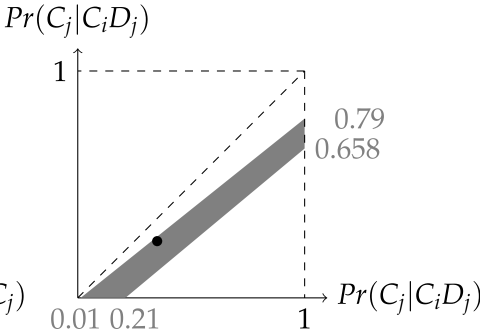

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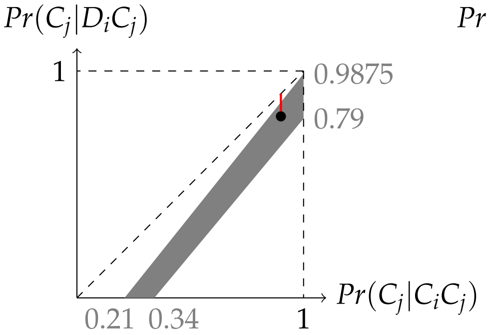

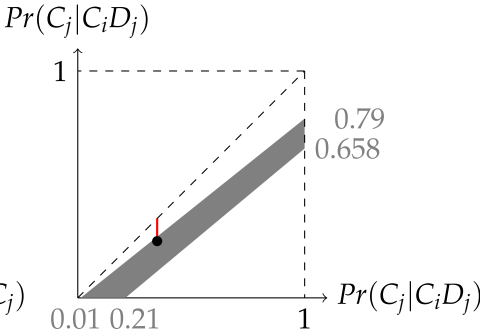

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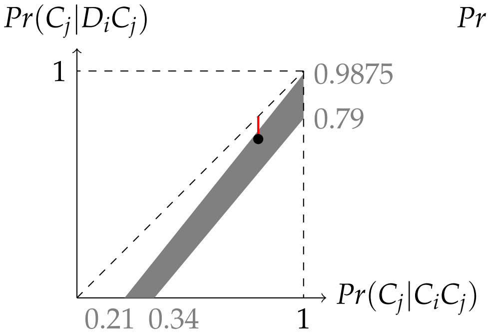

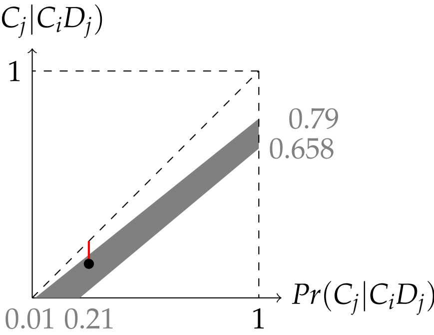

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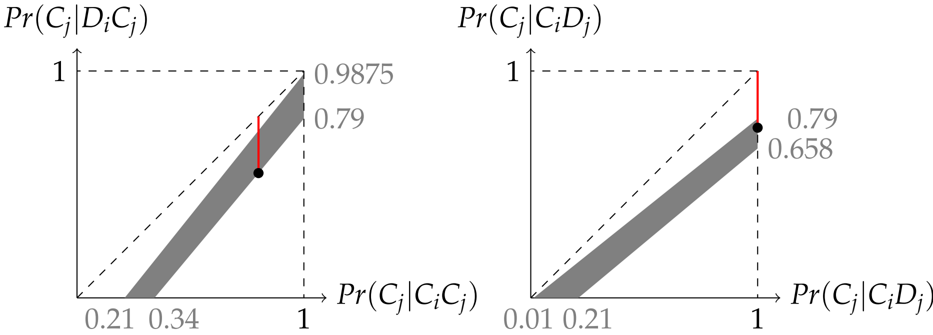

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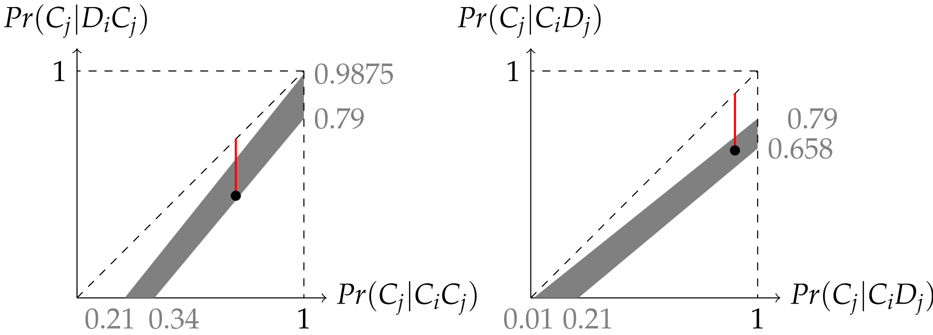

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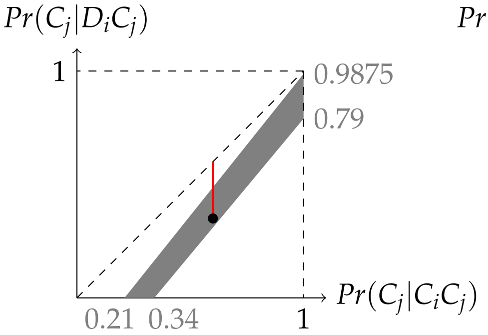

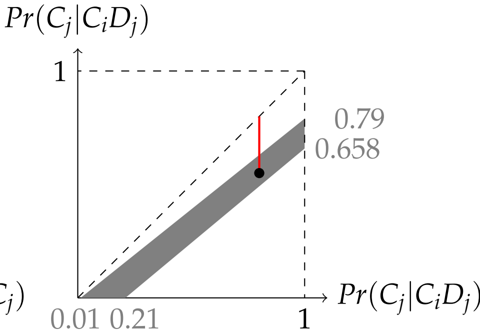

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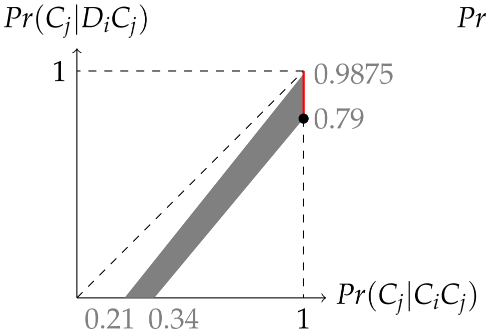

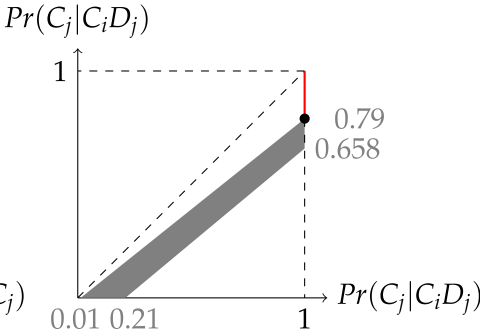

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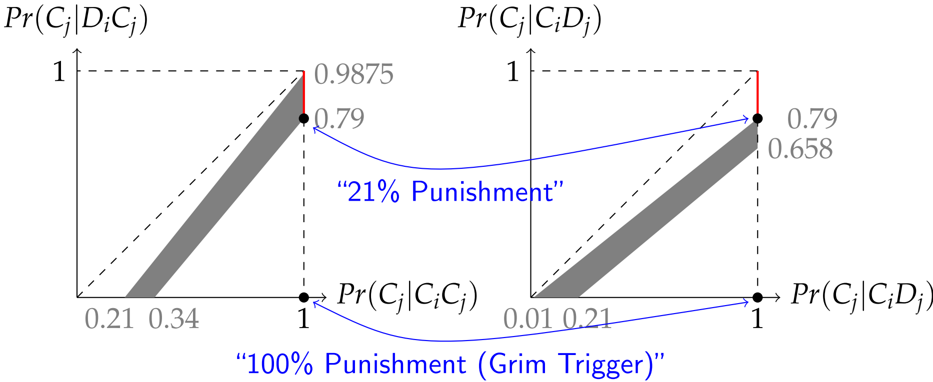

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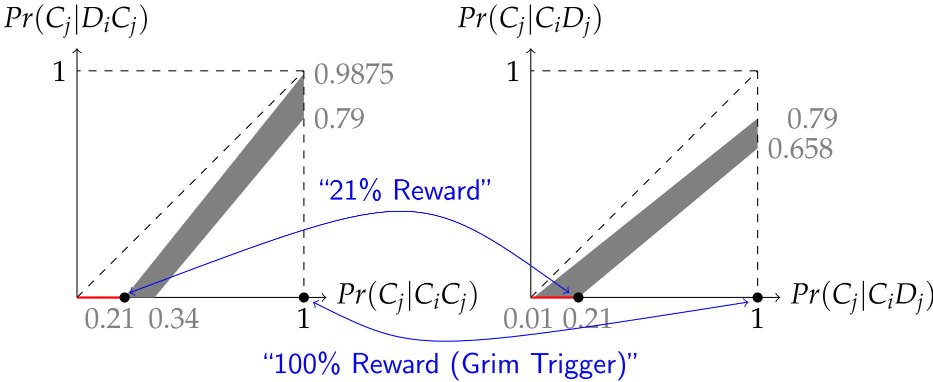


Solving Belief-Free Equilibria

	C	D
C	$30, 30$	$5, 35$
D	$35, 5$	$10, 10$

- Now: private monitoring case
- $Pr(c|C) = \rho = 0.75$. $Pr(c|D) = 1 - \rho = 0.25$.
- Let $\pi_j^{dC} = Pr(C_j|d_j C_j)$, etc
- *Actual* cooperation probabilities:
  - $Pr(C_j|C_i C_j) = \rho \pi_j^{cC} + (1-\rho)\pi_j^{dC}$
  - $Pr(C_j|D_i C_j) = (1-\rho)\pi_j^{cC} + \rho \pi_j^{dC}$
  - $\vdots$

Solving Belief-Free Equilibria

	C	D
C	$30, 30$	$5, 35$
D	$35, 5$	$10, 10$

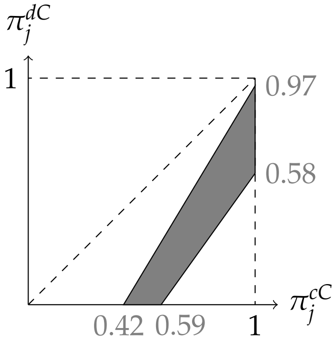
$$V_i^{C_j} = (1-\delta)30 + \delta[\underbrace{Pr(C_j|C_iC_j)}_{\rho\pi_j^{cC}+(1-\rho)\pi_j^{dC}} V_i^{C_j} + \underbrace{(1-Pr(C_j|C_iC_j))}_{\rho(1-\pi_j^{cC})+(1-\rho)(1-\pi_j^{dC})} V_i^{D_j}]$$

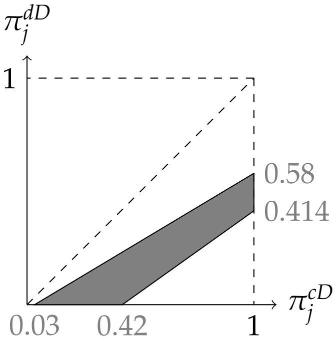
$$= (1-\delta)35 + \delta[\underbrace{Pr(C_j|D_iC_j)}_{(1-\rho)\pi_j^{cC}+\rho\pi_j^{dC}} V_i^{C_j} + \underbrace{(1-Pr(C_j|D_iC_j))}_{(1-\rho)(1-\pi_j^{cC})+\rho(1-\pi_j^{dC})} V_i^{D_j}]$$

$$V_i^{D_j} = \cdots$$

Solving Belief-Free Equilibria

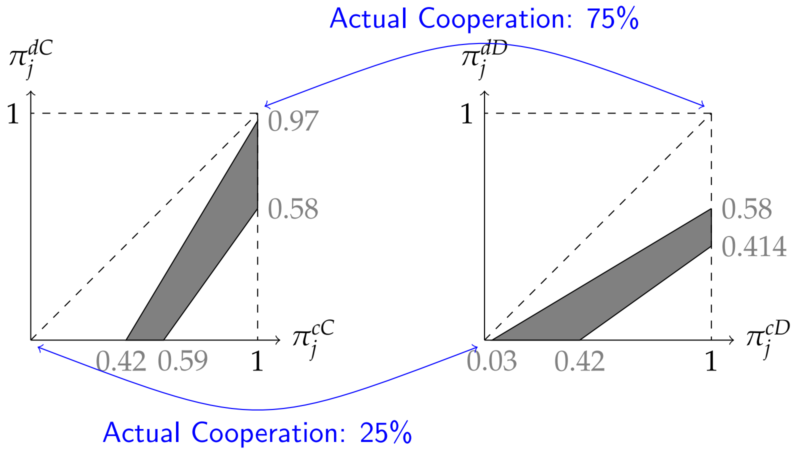
π_j^{cC} vs. π_j^{dC} and π_j^{cD} vs. π_j^{dD}:





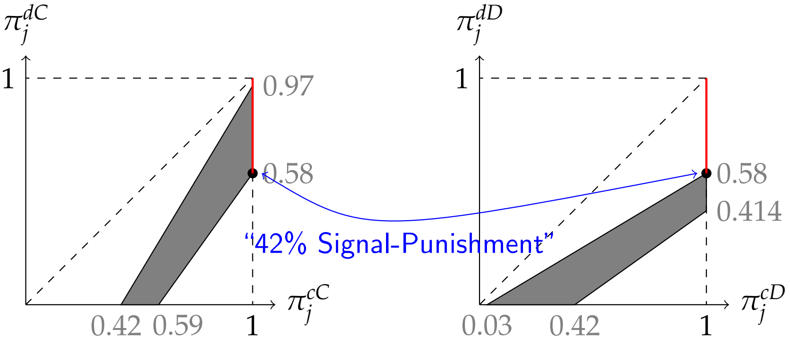
Solving Belief-Free Equilibria

You can't get 100% actual cooperation...



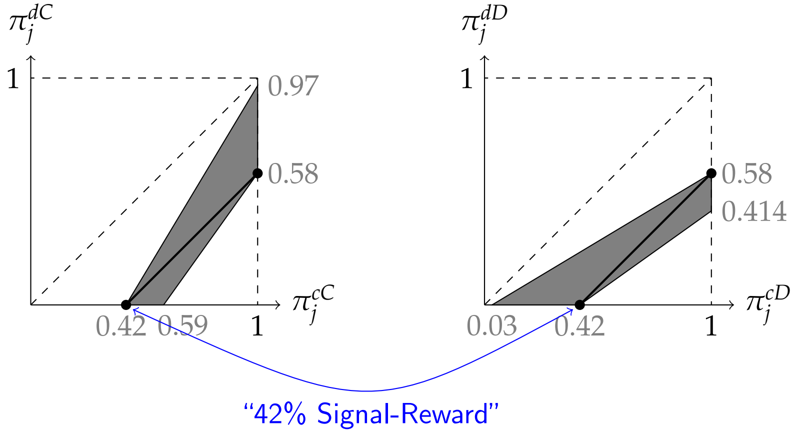
Solving Belief-Free Equilibria

π_j^{cC} vs. π_j^{dC} and π_j^{cD} vs. π_j^{dD}:



Solving Belief-Free Equilibria

Playing the same regardless of C_j: π_j^c and π_j^d



Outcomes

Best equilibrium: $\pi_j^c = 1$, $\pi_j^d = 0.58$:

	CC	DC	CD	DD
CC	80%	9%	9%	1%
CD	61%	7%	28%	3%
DC	61%	28%	7%	3%
DD	47%	22%	22%	10%

Stochastically-Perfect Monitoring

- In belief-free equilibrium, you are indifferent
- Beliefs don't matter!
- Even if you found out new information, you wouldn't change
- We test that prediction
- New Twist: **Stochastically perfect monitoring**
  - In each period, after a_i^t is chosen, a 'golden signal' is revealed with probability $\beta = 0.1$
  - 'Golden signal': $(a_j^1, z_j^1; \ldots; a_j^{t-1}, z_j^{t-1})$
  - Players *don't* know when opponents got golden signals
    - Still private monitoring.
  - After seeing golden signal, player can revise a_i^t.
  - Notation: original action $= a_i^{t-}$. Revised action $= a_i^{t+}$.

Belief-Free Equilibria Revisited

	C	D
C	$30, 30$	$5, 35$
D	$35, 5$	$10, 10$

- Belief-free equilibria still exist
- They *must* ignore the golden signal
- Same equilibrium set as before

Other Strategies

- "Stochastic Grim Trigger": Cooperate until a golden signal reveals D_j
- SGT is equilibrium for δ and β large, and noise small.
- SGT is equilibrium in our experiment
- Beliefs matter (off-path), but aren't too hard

Other strategies?? (Still private monitoring; beliefs intractable.)

Experimental Design

Design:

- Subject plays multiple matches, each against random opponent.
- Stage game payoff:

	c	d
C	\$30	\$5
D	\$35	\$10

- Payment: last realized round of 1 randomly-chosen match

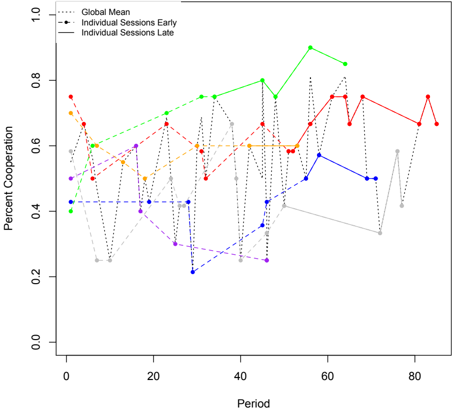
Discount Factor: $\delta = 0.9$
Signal Accuracy: $\rho = 0.75$
Golden Signal Probability: $\beta = 0.10$

Results

Interested in convergence behavior...

- we only analyze matches that start in the last half of the session.

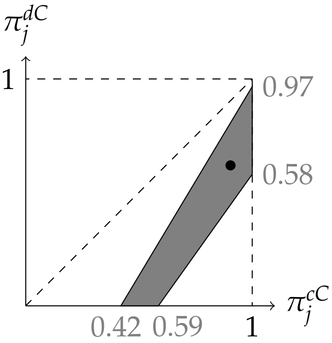
First-Period Cooperation Rates

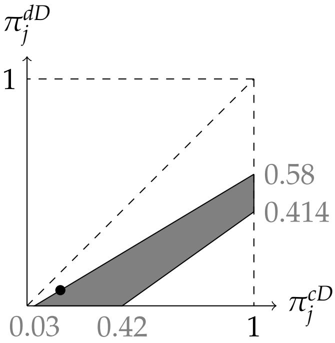


1.Compare:1-shot PD **2.**Compare:SGT **3.**Heterogeneity **4.**No learning

Actual Strategies

Actual strategies played, in aggregate:





On What Do Strategies Depend?

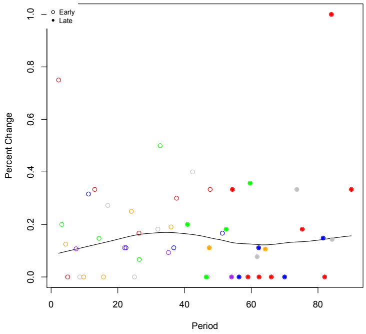
Frequencies of cooperation based on previous period outcome:

(a_i^{t-1}, z_i^{t-1})	$Pr(C)$
(C,c)	0.904
(C,d)	0.619
(D,c)	0.147
(D,d)	0.069

- Looks like own action plays bigger role than partner's signal.
  - Suggests heterogeneity.
  - Cooperative type vs. Defect(ive) type
- Cooperative types give higher-powered incentives
  - $\pi^{cC} - \pi^{dC} = 0.285$
  - $\pi^{cD} - \pi^{dD} = 0.078$
  - Can't be equilibrium with these payoffs

Switch Rates

Do they switch actions after seeing a golden signal?



Overall pretty low switching rate!

Romero & Rosokha: 18% switch (perfect monitoring, costly switch)

Switch Rates w/ Mixing

Only look at periods with golden signals:

a_i^{t-1}	z_i^{t-1}	$Pr(a_i^{t-} = C)$	$Pr(a_i^{t+} = C)$	p-value
C	c	0.863	0.745	0.212
C	d	0.578	0.311	*0.020*
D	c	0.231	0.192	0.810
D	d	0.051	0.081	0.566

Mostly not a big switch in (mixed) strategies. Exception: (C, d)

Next: When they do switch (which isn't much), what in the golden signals causes the switch?

When Do People Switch?

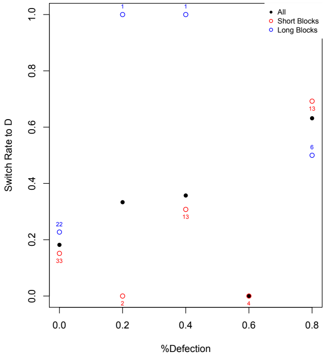
Last period's signal vs. actual action:

a_i^{t-}	z_i^{t-1}	a_j^{t-1}	$Pr(\text{switch})$
C	c	C	0.146
C	c	D	**0.625**
C	d	C	0.250
C	d	D	**0.522**
D	c	C	0.125
D	c	D	0.043
D	d	C	0.000
D	d	D	0.047

Cooperators react to a_j^{t-1}, even when not a surprise. Defectors don't react.

Stochastic Grim Trigger?

Look at those with $a_i^{t-} = C$.
How much defection in the golden signal (as %) causes a switch to D?



Somewhat responsive, but not grim trigger. Note 20% at 0%!!

On What Do Strategies Depend?

Regression. Dependant variable: $Pr(a_i^2 = C)$. PERIOD 2 ONLY.

	Coeff. (p-value)
Constant	0.161*** (0.0002)
$z_i^{t-1} = c$	0.042 (0.563)
$a_i^{t-1} = C$	0.539*** (< 0.0001)
Interaction	0.205* (0.034)
	$n = 328$

1. Own action always matters (not 'action-free').
2. Signal only matters if you played C.
3. "Cooperative types" respond to signal? "Defect types" don't?

On What Do Strategies Depend?

$Pr(a_i^3 = C)$. $t = 3$ ONLY, excluding golden signal at $t = 2$ and fixing period 2 outcome.

$(a_i^2, z_i^2) =$	(C,c)	(C,d)	(D,c)	(D,d)
Constant	0.667*	0.428*	0.091	0.053
	(0.019)	(0.031)	(0.165)	(0.333)
$z_i^{t-2} = c$	0.133	-0.229	0.083	0.090
	(0.691)	(0.249)	(0.451)	(0.431)
$a_i^{t-2} = C$	0.238	-0.058	0.020	0.114
	(0.417)	(0.792)	(0.875)	(0.372)
$(a_i^{t-2}, z_i^{t-2}) = (C,c)$	-0.054	0.589*	0.806***	-0.007
	(0.888)	(0.013)	(< 0.0001)	(0.982)
	$n = 97$	$n = 74$	$n = 64$	$n = 55$

Not much from $t-2$ matters!

For cooperators: Others' d's are forgiven, and own D's are temporary.

What are Earnings?

Average Earnings:

Overall	\$21.15
$a_i^1 = C$	\$20.54
$a_i^1 = D$	\$21.97

1. Belief-free equil. E(payoff) range: \$12.50–\$27.50.
   DD E(payoff): \$16.25. CC E(payoff): \$23.75.

2. Mixing means equal payoff for C vs. D.
   p-value 0.11... roughly in population-level equilibrium?

3. *Slight* advantage to defectors

Experimental Literature

- **Perfect Monitoring:** (Dal Bo & Frechette 2016 Survey)
  - Cooperation rates depend on δ, temptation & sucker payoffs
  - Strategies: Always Defect & T4T most common, then Grim.
- **Mixed Strategies w/ Perfect Monitoring:**
  - Breitmoser (2015) meta-analysis:
  - Romero & Rosokha (WP) direct elicitation:
    - $Pr(C_j|C_iC_j) = 0.95 > Pr(C_j|D_iC_j) = 0.25$
    - $Pr(C_j|C_iD_j) = 0.60 > Pr(C_j|D_iD_j) = 0.10$

Experimental Literature

- **Imperfect: Public Monitoring:** (Dreber et al 2008; Aoyagi & Frechette 2009; Dal Bo & Frechette 2011; Fudenberg et al 2012; Rojas 2012; Embrey et al 2013; Aoyagi et al 2014)
  - Cooperation occurs, especially when cooperative SPNE exists
  - Effect of signal noise is ambiguous
  - Leniency and forgiveness

Experimental Literature

- **Imperfect: Private Monitoring:**
  - Feinberg & Snyder 2002: PD w/ added dominated action. Revealing noise *ex post* increases cooperation.
  - Matsushima & Toyama 2011: High & low accuracy signals. Cooperation higher than one-shot PD w/ highly accurate signals, but not as good as best SPNE. Theory: response to signal higher with low accuracy. Data: Response is higher with high accuracy.
  - Aoyagi, Bhaskar & Frechette 2014: Perfect vs. Public vs. Private. Cooperation w/ private slightly lower than perfect or public. Lenient & forgiving strategies.

Summary

- **Shockingly little switching** $(< 20\%)$
  - It's only the cooperators who react to golden signals...
  - and they react even when revealed action isn't surprising
- Cooperators also react (some) to private signal, but not defectors
- Heterogeneity of types $\Rightarrow$ session differences
- No evidence (yet) of long-memory strategies
- Though they don't switch much, the finer predictions of belief-free equilibrium aren't borne out.
- Cooperation rate $\approx$ 50% is good, not great.
- Need to do: Strategy estimations