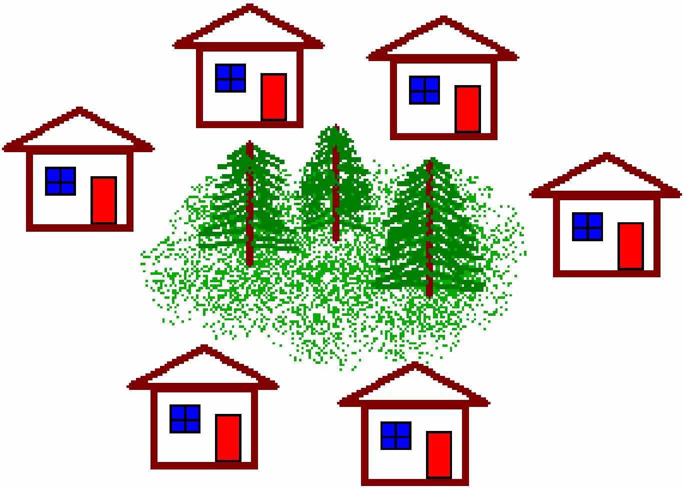
Equilibrium Participation in Public Good Economies

Paul J. Healy

Ohio State

PET'07 Nashville

Participation in Mechanisms



- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01: $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms
Previous Work

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms

Previous Work

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms

Previous Work

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms

Previous Work

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms

Previous Work

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms

Previous Work

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\implies$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms

Previous Work

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01: $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms

Previous Work

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01: $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

- The need for a mechanism
  - Voluntary contributions typically inefficient
  - Misrevelation of preferences
- Mechanisms as part of a larger game:
- Participation decision before the mechanism
  - Saijo & Yamato '99: Participation drops in n
  - Dixit & Olson '00: MSNE Pr(efficient) drops in n
  - Shinohara '07: Prop.C.S. $\Longrightarrow$ Efficient coalitions are strict NE
- Participation decision after the mechanism
  - Renegotiation (Maskin & Moore '99, e.g.)
  - Jackson & Palfrey '01:
    $g(\theta) = a$, but then players move to $h(a, \theta)$
  - This paper...

Participation in Mechanisms

The Current Paper

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

Participation in Mechanisms

The Current Paper

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

Participation in Mechanisms

The Current Paper

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

Participation in Mechanisms

The Current Paper

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

Participation in Mechanisms

The Current Paper

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

Participation in Mechanisms

The Current Paper

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
  - Full enforcement: chosen allocation is realized
  - No enforcement: voluntary contribution game
  - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
  - Property of allocations, not mechanisms
  - Provides a constraint on mechanisms

Participation in Mechanisms

The Current Paper

- Economy with 1 private, 1 public good
- (Unspecified) mechanism selects an allocation
- Allocation defines the participation game
- Possible models of the participation decision:
    - Full enforcement: chosen allocation is realized
    - No enforcement: voluntary contribution game
    - This paper: either contribute as requested or not at all
- An allocation satisfies **equilibrium participation (EP)** if it is a NE for all agents to participate
    - Property of allocations, not mechanisms
    - Provides a constraint on mechanisms

An Example

- 2 agents, 1 pvt good $x = (x_1, x_2)$, 1 pub good y
- Constant marginal cost $c'(y) = 6$
- Endowments of pvt good only: $\omega_1 = \omega_2 = 8$
- For any (x, y), define $t_i = \omega_i - x_i$
- $u_i(x_i, y) = \begin{cases} x_i + 10y & \text{if} \quad y \leq 1/2 \\ x_i + 4y + 3 & \text{if} \quad y \in [1/2, 1] \\ x_i + 1y + 6 & \text{if} \quad y \geq 1 \end{cases}$
- Individual optimum: $y^* = 1/2$
- PO: $y^o = 1$ and $\sum_i (\omega_i - x_i) = 6$

An Example

- 2 agents, 1 pvt good $x = (x_1, x_2)$, 1 pub good y
- Constant marginal cost $c'(y) = 6$
- Endowments of pvt good only: $\omega_1 = \omega_2 = 8$
- For any (x, y), define $t_i = \omega_i - x_i$
- $u_i(x_i, y) = \begin{cases} x_i + 10y & \text{if} \quad y \leq 1/2 \\ x_i + 4y + 3 & \text{if} \quad y \in [1/2, 1] \\ x_i + 1y + 6 & \text{if} \quad y \geq 1 \end{cases}$
- Individual optimum: $y^* = 1/2$
- PO: $y^o = 1$ and $\sum_i (\omega_i - x_i) = 6$

An Example

- 2 agents, 1 pvt good $x = (x_1, x_2)$, 1 pub good y
- Constant marginal cost $c'(y) = 6$
- Endowments of pvt good only: $\omega_1 = \omega_2 = 8$
- For any (x, y), define $t_i = \omega_i - x_i$
- $u_i(x_i, y) = \begin{cases} x_i + 10y & \text{if} \quad y \leq 1/2 \\ x_i + 4y + 3 & \text{if} \quad y \in [1/2, 1] \\ x_i + 1y + 6 & \text{if} \quad y \geq 1 \end{cases}$
- Individual optimum: $y^* = 1/2$
- PO: $y^o = 1$ and $\sum_i (\omega_i - x_i) = 6$

An Example

- 2 agents, 1 pvt good $x = (x_1, x_2)$, 1 pub good y
- Constant marginal cost $c'(y) = 6$
- Endowments of pvt good only: $\omega_1 = \omega_2 = 8$
- For any (x, y), define $t_i = \omega_i - x_i$
- $u_i(x_i, y) = \begin{cases} x_i + 10y & \text{if} \quad y \leq 1/2 \\ x_i + 4y + 3 & \text{if} \quad y \in [1/2, 1] \\ x_i + 1y + 6 & \text{if} \quad y \geq 1 \end{cases}$
- Individual optimum: $y^* = 1/2$
- PO: $y^o = 1$ and $\sum_i (\omega_i - x_i) = 6$

An Example

- 2 agents, 1 pvt good $x = (x_1, x_2)$, 1 pub good y
- Constant marginal cost $c'(y) = 6$
- Endowments of pvt good only: $\omega_1 = \omega_2 = 8$
- For any (x, y), define $t_i = \omega_i - x_i$
- $u_i(x_i, y) = \begin{cases} x_i + 10y & \text{if} \quad y \leq 1/2 \\ x_i + 4y + 3 & \text{if} \quad y \in [1/2, 1] \\ x_i + 1y + 6 & \text{if} \quad y \geq 1 \end{cases}$
- Individual optimum: $y^* = 1/2$
- PO: $y^o = 1$ and $\sum_i (\omega_i - x_i) = 6$

An Example

- 2 agents, 1 pvt good $x = (x_1, x_2)$, 1 pub good y
- Constant marginal cost $c'(y) = 6$
- Endowments of pvt good only: $\omega_1 = \omega_2 = 8$
- For any (x, y), define $t_i = \omega_i - x_i$
- $u_i(x_i, y) = \begin{cases} x_i + 10y & \text{if} \quad y \leq 1/2 \\ x_i + 4y + 3 & \text{if} \quad y \in [1/2, 1] \\ x_i + 1y + 6 & \text{if} \quad y \geq 1 \end{cases}$
- Individual optimum: $y^* = 1/2$
- PO: $y^o = 1$ and $\sum_i (\omega_i - x_i) = 6$

An Example

- 2 agents, 1 pvt good $x = (x_1, x_2)$, 1 pub good y
- Constant marginal cost $c'(y) = 6$
- Endowments of pvt good only: $\omega_1 = \omega_2 = 8$
- For any (x, y), define $t_i = \omega_i - x_i$
- $u_i(x_i, y) = \begin{cases} x_i + 10y & \text{if} \quad y \leq 1/2 \\ x_i + 4y + 3 & \text{if} \quad y \in [1/2, 1] \\ x_i + 1y + 6 & \text{if} \quad y \geq 1 \end{cases}$
- Individual optimum: $y^* = 1/2$
- PO: $y^o = 1$ and $\sum_i (\omega_i - x_i) = 6$

An Example

- Suppose a PO & IR mechanism selects $(y, t_1, t_2) = (1, 3, 3)$

1. If agents can't opt out, allocation stands
2. If agents can freely adjust t_i, get $y^* = 1/2$ in NE
3. Either in (pay t_i) or out (pay nothing):

$t_1 \setminus t_2$	3	0
3	12, 12	10, 13
0	13, 10	8, 8

PSNE: only 1 contributor $\Longrightarrow y = 1/2$

An Example

- Suppose a PO & IR mechanism selects $(y, t_1, t_2) = (1, 3, 3)$

1. If agents can't opt out, allocation stands
2. If agents can freely adjust t_i, get $y^* = 1/2$ in NE
3. Either in (pay t_i) or out (pay nothing):

$t_1 \setminus t_2$	3	0
3	12, 12	10, 13
0	13, 10	8, 8

PSNE: only 1 contributor $\Longrightarrow y = 1/2$

An Example

- Suppose a PO & IR mechanism selects $(y, t_1, t_2) = (1, 3, 3)$

1. If agents can't opt out, allocation stands
2. If agents can freely adjust t_i, get $y^* = 1/2$ in NE
3. Either in (pay t_i) or out (pay nothing):

$t_1 \setminus t_2$	3	0
3	12, 12	10, 13
0	13, 10	8, 8

PSNE: only 1 contributor $\Longrightarrow y = 1/2$

An Example

- Suppose a PO & IR mechanism selects $(y, t_1, t_2) = (1, 3, 3)$

1. If agents can't opt out, allocation stands
2. If agents can freely adjust t_i, get $y^* = 1/2$ in NE
3. Either in (pay t_i) or out (pay nothing):

$t_1 \backslash t_2$	3	0
3	12, 12	10, 13
0	13, 10	8, 8

PSNE: only 1 contributor $\Longrightarrow y = 1/2$

An Example

- A different PO mechanism selects $(y, t_1, t_2) = (1, 6, 0)$

1. If agents can't opt out, allocation stands
2. If agents can freely adjust t_i, get $y^* = 1/2$
3. Either in (pay t_i) or out (pay nothing):

$t_1 \setminus t_2$	0	0
6	9, 15	9, 15
0	8, 8	8, 8

Unique NE outcome: full contribution, $y = 1$

An Example

- A different PO mechanism selects $(y, t_1, t_2) = (1, 6, 0)$

1. If agents can't opt out, allocation stands
2. If agents can freely adjust t_i, get $y^* = 1/2$
3. Either in (pay t_i) or out (pay nothing):

$t_1 \setminus t_2$	0	0
6	9, 15	9, 15
0	8, 8	8, 8

Unique NE outcome: full contribution, $y = 1$

An Example

- A different PO mechanism selects $(y, t_1, t_2) = (1, 6, 0)$

1. If agents can't opt out, allocation stands
2. If agents can freely adjust t_i, get $y^* = 1/2$
3. Either in (pay t_i) or out (pay nothing):

$t_1 \setminus t_2$	0	0
6	9, 15	9, 15
0	8, 8	8, 8

Unique NE outcome: full contribution, $y = 1$

An Example

- A different PO mechanism selects $(y, t_1, t_2) = (1, 6, 0)$

1. If agents can't opt out, allocation stands
2. If agents can freely adjust t_i, get $y^* = 1/2$
3. Either in (pay t_i) or out (pay nothing):

$t_1 \setminus t_2$	0	0
6	9, 15	9, 15
0	8, 8	8, 8

Unique NE outcome: full contribution, $y = 1$

Notation

- Allocations: $(x, y) \in \mathbb{R}_{+}^{n+1}$
- $t_i = \omega_i - x_i$, $T = \sum_i t_i$, $T_{-i} = T - t_i$
- PG production function: $F(T)$
- Economy: $e = (\{\succeq_i, \omega_i\}_{i=1}^{n}, F) \in \mathcal{E}$
- $\mathcal{Z}(e)$ = feasible allocations
- SCC: maps e into subsets of $\mathcal{Z}(e)$
  Example: $\mathcal{IR}(e) = \{(x, y) : (x_i, y) \succeq_i (\omega_i, 0) \ \forall i\}$
- Mechanism: $\Gamma = (S, g)$ where $g : S \to \mathcal{Z}(e)$

Notation

- Allocations: $(x, y) \in \mathbb{R}_{+}^{n+1}$
- $t_i = \omega_i - x_i, \ T = \sum_i t_i, \ T_{-i} = T - t_i$
- PG production function: $F(T)$
- Economy: $e = (\{\succeq_i, \omega_i\}_{i=1}^{n}, F) \in \mathcal{E}$
- $\mathcal{Z}(e) =$ feasible allocations
- SCC: maps e into subsets of $\mathcal{Z}(e)$
  Example: $\mathcal{IR}(e) = \{(x, y) : (x_i, y) \succeq_i (\omega_i, 0) \ \ \forall i\}$
- Mechanism: $\Gamma = (S, g)$ where $g : S \rightarrow \mathcal{Z}(e)$

Notation

- Allocations: $(x, y) \in \mathbb{R}_+^{n+1}$
- $t_i = \omega_i - x_i, \; T = \sum_i t_i, \; T_{-i} = T - t_i$
- PG production function: $F(T)$
- Economy: $e = \left(\{\succeq_i, \omega_i\}_{i=1}^n, F\right) \in \mathcal{E}$
- $\mathcal{Z}(e)$ = feasible allocations
- SCC: maps e into subsets of $\mathcal{Z}(e)$
  Example: $\mathcal{IR}(e) = \{(x, y) : (x_i, y) \succeq_i (\omega_i, 0) \;\; \forall i\}$
- Mechanism: $\Gamma = (S, g)$ where $g : S \rightarrow \mathcal{Z}(e)$

Notation

- Allocations: $(x, y) \in \mathbb{R}_{+}^{n+1}$
- $t_i = \omega_i - x_i, \ T = \sum_i t_i, \ T_{-i} = T - t_i$
- PG production function: $F(T)$
- Economy: $e = \left(\{\succeq_i, \omega_i\}_{i=1}^{n}, F\right) \in \mathcal{E}$
- $\mathcal{Z}(e)$ = feasible allocations
- SCC: maps e into subsets of $\mathcal{Z}(e)$
  Example: $\mathcal{IR}(e) = \{(x, y) : (x_i, y) \succeq_i (\omega_i, 0) \ \forall i\}$
- Mechanism: $\Gamma = (S, g)$ where $g : S \to \mathcal{Z}(e)$

Notation

- Allocations: $(x, y) \in \mathbb{R}_{+}^{n+1}$
- $t_i = \omega_i - x_i$, $T = \sum_i t_i$, $T_{-i} = T - t_i$
- PG production function: $F(T)$
- Economy: $e = (\{\succeq_i, \omega_i\}_{i=1}^{n}, F) \in \mathcal{E}$
- $\mathcal{Z}(e)$ = feasible allocations
- SCC: maps e into subsets of $\mathcal{Z}(e)$
  Example: $\mathcal{IR}(e) = \{(x, y) : (x_i, y) \succeq_i (\omega_i, 0)\ \forall i\}$
- Mechanism: $\Gamma = (S, g)$ where $g : S \to \mathcal{Z}(e)$

Notation

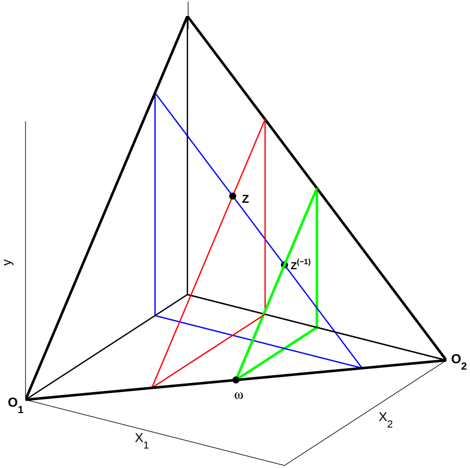
- Allocations: $(x, y) \in \mathbb{R}_{+}^{n+1}$
- $t_i = \omega_i - x_i,\ T = \sum_i t_i,\ T_{-i} = T - t_i$
- PG production function: $F(T)$
- Economy: $e = \left(\{\succeq_i, \omega_i\}_{i=1}^{n}, F\right) \in \mathcal{E}$
- $\mathcal{Z}(e)$ = feasible allocations
- SCC: maps e into subsets of $\mathcal{Z}(e)$
  Example: $\mathcal{IR}(e) = \{(x, y) : (x_i, y) \succeq_i (\omega_i, 0)\ \forall i\}$
- Mechanism: $\Gamma = (S, g)$ where $g : S \rightarrow \mathcal{Z}(e)$

Notation

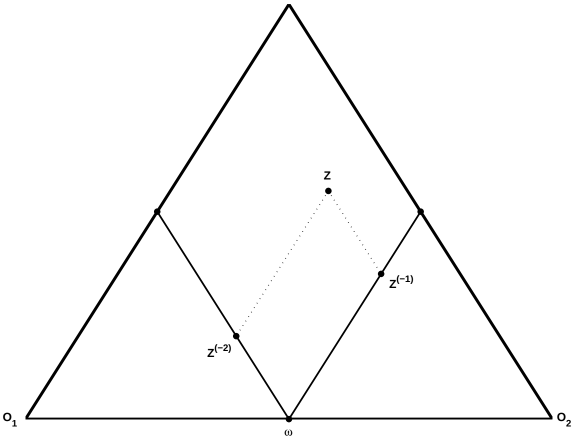
- Allocations: $(x, y) \in \mathbb{R}_{+}^{n+1}$
- $t_i = \omega_i - x_i$, $T = \sum_i t_i$, $T_{-i} = T - t_i$
- PG production function: $F(T)$
- Economy: $e = \left(\{\succeq_i, \omega_i\}_{i=1}^{n}, F\right) \in \mathcal{E}$
- $\mathcal{Z}(e)$ = feasible allocations
- SCC: maps e into subsets of $\mathcal{Z}(e)$
  Example: $\mathcal{IR}(e) = \{(x, y) : (x_i, y) \succeq_i (\omega_i, 0) \ \forall i\}$
- Mechanism: $\Gamma = (S, g)$ where $g : S \rightarrow \mathcal{Z}(e)$

Kolm Triangle

- Let $\omega = (1/2, 1/2)$ and $F(T) = T$

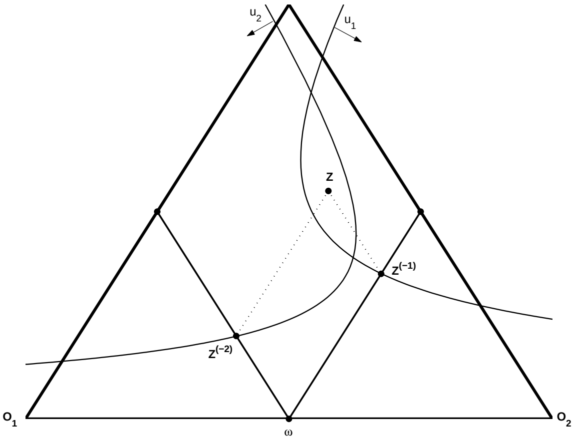


Kolm Triangle: Drop-Out Points



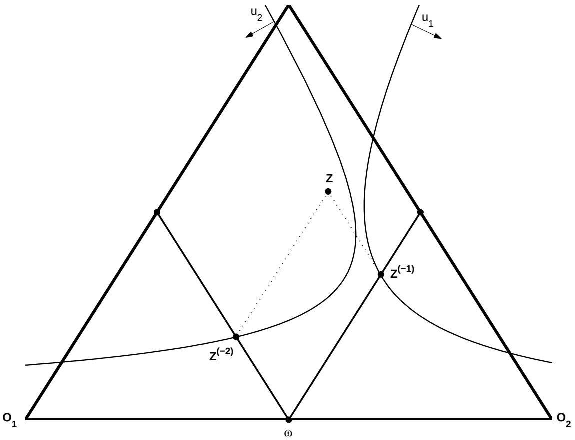
Kolm Triangle: Equilibrium Participation

- Equilibrium Participation (EP): $(x, y) \succeq_i (\omega_i, y^{(-i)})\ \forall i$



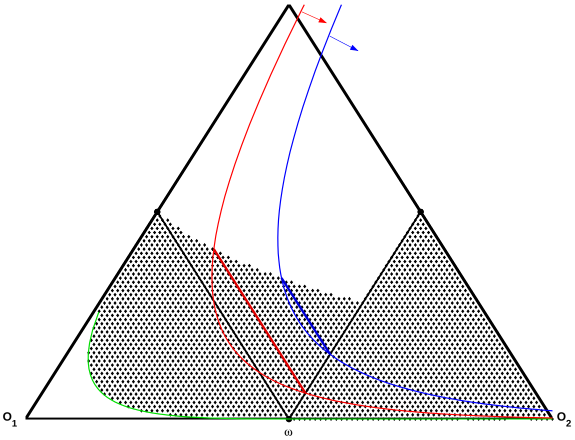
Kolm Triangle: EP Fails

- Equilibrium Participation: $(x, y) \succeq_i (\omega_i, y^{(-i)}) \; \forall i$



Kolm Triangle: The EP Set

- $\mathcal{EP}_i(e) = \{(x, y) : (x, y) \text{ satisfies EP for } i\}$ and $\mathcal{EP}(e) = \bigcap_i \mathcal{EP}_i(e)$



EP: General Definition

Definition

Given any allocation $(x, y) \in \mathcal{Z}(e)$, for each i define

$$x^{(-i)} = \omega_i$$

$$y^{(-i)} = \begin{cases} F(T_{-i}) & \text{if} \quad t_i \geq 0,\ T_{-i} \geq 0,\ \&\ y \geq F(T_{-i}) \\ 0 & \text{if} \quad T_{-i} < 0 \\ y & \text{otherwise} \end{cases}$$

The allocation (x, y) satisfies **equilibrium participation (EP$_i$) for** i if $(x, y) \succeq_i (x^{(-i)}, y^{(-i)})$. The allocation (x, y) satisfies **equilibrium participation (EP)** if it satisfies EP$_i$ for every i.

EP vs. NE and IR

- $\mathcal{NE}(e) = \{ (x, y) : (x, y)$ is a PSNE outcome of the VCM $\}$

Theorem

$\mathcal{NE}(e) \subseteq \mathcal{EP}(e) \;\; \forall e$

- $\mathcal{IR}(e) = \{(x, y) : (x, y) \succeq_i (\omega, 0) \;\; \forall i\}$

Theorem

If preferences are monotonic, then $\mathcal{EP}(e) \subseteq \mathcal{IR}(e)$

Proof.

$(x, y) \succeq_i \left(\omega, y^{(-i)}\right) \succeq_i (\omega, 0)$ □

EP vs. Pareto Optimality

Theorem

There are economies in which no PO point is the equilibrium outcome of an induced participation game (thus, $\mathcal{PO}(e) \cap \mathcal{EP}(e) = \emptyset$)

- Fix $n \geq 2$, let $F(T) = T$, $\omega_i = 1/n$, and
$$u_i(x, y) = \begin{cases} \frac{3}{2n}y + x_i & \text{if} \quad y \leq 1 \\ \frac{1}{2n}y + \frac{1}{n} + x_i & \text{if} \quad y \geq 1 \end{cases} \quad \forall i$$
- Note: $MRS < MC$ everywhere
- PO: $(x^o, y^o) : y^o = \sum_i t_i^o = 1$
- If $t_i^o > 0$ then $y^{o(-i)} = 1 - t_i^o < 1$, so

$$u_i(x^o, y^o) = \frac{3}{2n} + \omega_i - t_i^o$$
$$u_i\left(\omega, y^{o(-i)}\right) = \frac{3}{2n}(1 - t_i^o) + \omega_i$$

EP vs. Pareto Optimality

Theorem

There are economies in which no PO point is the equilibrium outcome of an induced participation game (thus, $\mathcal{PO}(e) \cap \mathcal{EP}(e) = \emptyset$)

- Fix $n \geq 2$, let $F(T) = T$, $\omega_i = 1/n$, and
  $$u_i(x, y) = \begin{cases} \frac{3}{2n} y + x_i & \text{if} \quad y \leq 1 \\ \frac{1}{2n} y + \frac{1}{n} + x_i & \text{if} \quad y \geq 1 \end{cases} \quad \forall i$$
- Note: $MRS < MC$ everywhere
- PO: $(x^o, y^o) : y^o = \sum_i t_i^o = 1$
- If $t_i^o > 0$ then $y^{o(-i)} = 1 - t_i^o < 1$, so

$$u_i(x^o, y^o) = \frac{3}{2n} + \omega_i - t_i^o$$
$$u_i\left(\omega, y^{o(-i)}\right) = \frac{3}{2n}(1 - t_i^o) + \omega_i$$

EP vs. Pareto Optimality

Theorem

There are economies in which no PO point is the equilibrium outcome of an induced participation game (thus, $\mathcal{PO}(e) \cap \mathcal{EP}(e) = \emptyset$)

- Fix $n \geq 2$, let $F(T) = T$, $\omega_i = 1/n$, and

$$u_i(x, y) = \begin{cases} \frac{3}{2n} y + x_i & \text{if} \quad y \leq 1 \\ \frac{1}{2n} y + \frac{1}{n} + x_i & \text{if} \quad y \geq 1 \end{cases} \quad \forall i$$

- Note: $MRS < MC$ everywhere
- PO: $(x^o, y^o) : y^o = \sum_i t_i^o = 1$
- If $t_i^o > 0$ then $y^{o(-i)} = 1 - t_i^o < 1$, so

$$u_i(x^o, y^o) = \frac{3}{2n} + \omega_i - t_i^o$$

$$u_i\left(\omega, y^{o(-i)}\right) = \frac{3}{2n}(1 - t_i^o) + \omega_i$$

EP vs. Pareto Optimality

Theorem

There are economies in which no PO point is the equilibrium outcome of an induced participation game (thus, $\mathcal{PO}(e) \cap \mathcal{EP}(e) = \emptyset$)

- Fix $n \geq 2$, let $F(T) = T$, $\omega_i = 1/n$, and
$$u_i(x, y) = \begin{cases} \frac{3}{2n} y + x_i & \text{if} \quad y \leq 1 \\ \frac{1}{2n} y + \frac{1}{n} + x_i & \text{if} \quad y \geq 1 \end{cases} \quad \forall i$$
- Note: $MRS < MC$ everywhere
- PO: $(x^o, y^o) : y^o = \sum_i t_i^o = 1$
- If $t_i^o > 0$ then $y^{o(-i)} = 1 - t_i^o < 1$, so

$$u_i(x^o, y^o) = \frac{3}{2n} + \omega_i - t_i^o$$
$$u_i\left(\omega, y^{o(-i)}\right) = \frac{3}{2n}(1 - t_i^o) + \omega_i$$

Quasi-Linear Economies

- $u_i(x, y) = v_i(y) - t_i$
- Feasibility: $t_i \geq c(y) - c(y^{(-i)}) = \int_{y^{(-i)}}^{y} c'(y)\,dy$
- EP: $t_i \leq v_i(y) - v_i(y^{(-i)}) = \int_{y^{(-i)}}^{y} v_i'(y)\,dy$
- Thus:

$$\int_{y^{(-i)}}^{y} c'(y)\,dy \leq t_i \leq \int_{y^{(-i)}}^{y} v_i'(y)\,dy$$

Quasi-Linear Economies

- $u_i(x, y) = v_i(y) - t_i$
- Feasibility: $t_i \geq c(y) - c(y^{(-i)}) = \int_{y^{(-i)}}^{y} c'(y)\, dy$
- EP: $t_i \leq v_i(y) - v_i(y^{(-i)}) = \int_{y^{(-i)}}^{y} v_i'(y)\, dy$
- Thus:

$$\int_{y^{(-i)}}^{y} c'(y)\, dy \leq t_i \leq \int_{y^{(-i)}}^{y} v_i'(y)\, dy$$

Quasi-Linear Economies

- $u_i(x, y) = v_i(y) - t_i$
- Feasibility: $t_i \geq c(y) - c(y^{(-i)}) = \int_{y^{(-i)}}^{y} c'(y)\, dy$
- EP: $t_i \leq v_i(y) - v_i(y^{(-i)}) = \int_{y^{(-i)}}^{y} v_i'(y)\, dy$
- Thus:

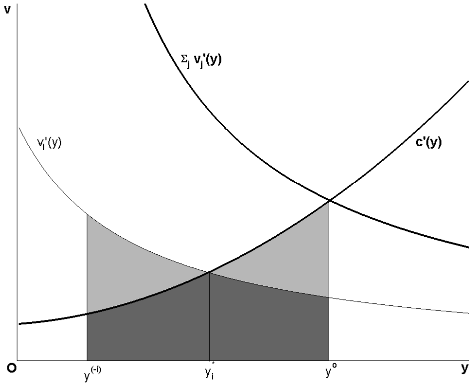
$$\int_{y^{(-i)}}^{y} c'(y)\, dy \leq t_i \leq \int_{y^{(-i)}}^{y} v_i'(y)\, dy$$

Quasi-Linear Economies

- $u_i(x, y) = v_i(y) - t_i$
- Feasibility: $t_i \geq c(y) - c(y^{(-i)}) = \int_{y^{(-i)}}^{y} c'(y)\, dy$
- EP: $t_i \leq v_i(y) - v_i(y^{(-i)}) = \int_{y^{(-i)}}^{y} v_i'(y)\, dy$
- Thus:

$$\int_{y^{(-i)}}^{y} c'(y)\, dy \leq t_i \leq \int_{y^{(-i)}}^{y} v_i'(y)\, dy$$

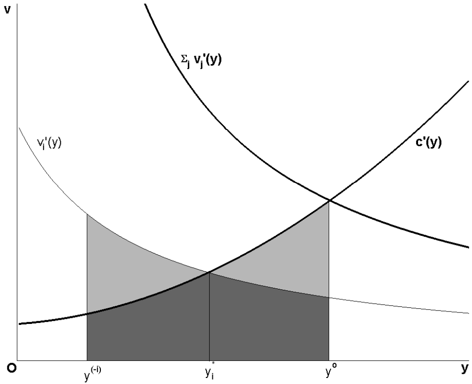
- $\int_{y^{(-i)}}^{y} c'(y)\,dy \leq \int_{y^{(-i)}}^{y} v_i'(y)\,dy \Longrightarrow$ a big transfer is needed for PO



- Especially true for large *n*

Quasi-Linear Economies

- $\int_{y^{(-i)}}^{y} c'(y)\, dy \leq \int_{y^{(-i)}}^{y} v_i'(y)\, dy \implies$ a big transfer is needed for PO



- Especially true for large n

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
  - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
  - Add new agent $n+1$, set $t_{n+1} = 0$
  - EP still satisfied
  - Repeat
- Fundamental discontinuity
  - Constant $y > 0$ for any finite n
  - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
  - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
  - Add new agent $n + 1$, set $t_{n+1} = 0$
  - EP still satisfied
  - Repeat
- Fundamental discontinuity
  - Constant $y > 0$ for any finite n
  - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
  - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
  - Add new agent $n + 1$, set $t_{n+1} = 0$
  - EP still satisfied
  - Repeat
- Fundamental discontinuity
  - Constant $y > 0$ for any finite n
  - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
  - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
  - Add new agent $n + 1$, set $t_{n+1} = 0$
  - EP still satisfied
  - Repeat
- Fundamental discontinuity
  - Constant $y > 0$ for any finite n
  - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
  - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
  - Add new agent $n + 1$, set $t_{n+1} = 0$
  - EP still satisfied
  - Repeat
- Fundamental discontinuity
  - Constant $y > 0$ for any finite n
  - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
  - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
  - Add new agent $n + 1$, set $t_{n+1} = 0$
  - EP still satisfied
  - Repeat
- Fundamental discontinuity
  - Constant $y > 0$ for any finite n
  - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
    - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
    - Add new agent $n + 1$, set $t_{n+1} = 0$
    - EP still satisfied
    - Repeat
- Fundamental discontinuity
    - Constant $y > 0$ for any finite n
    - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
  - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
  - Add new agent $n + 1$, set $t_{n+1} = 0$
  - EP still satisfied
  - Repeat
- Fundamental discontinuity
  - Constant $y > 0$ for any finite n
  - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Hypothesis: $\mathcal{EP}(e)$ 'shrinks' as economy 'grows'
- Counter-example:
  - Pick (x, y) with $y > 0$ and $t_2 = \cdots = t_n = 0$ satisfying EP
  - Add new agent $n+1$, set $t_{n+1} = 0$
  - EP still satisfied
  - Repeat
- Fundamental discontinuity
  - Constant $y > 0$ for any finite n
  - Continuum economy: 1 contributor is negligible, $y = 0$

Large Economies

- Can 1 agent provide equivalent PGs in large vs. small societies?
- Milleron '72: Splitting economies:
- Types $i \in \{1, \ldots, n\}$
- Replicates $r \in \{1, \ldots, R\}$
- Agent $= (i, r)$
- $\omega_{i,r} = \omega_i / R$, so $\sum_i \sum_r \omega_{i,r} = \sum_i \omega_i$
- $(x_{i,r}, y) \succeq_{i,r} (x'_{i,r}, y') \Leftrightarrow (R\, x_{i,r}, y) \succeq_i (R\, x'_{i,r}, y')$
- Agents care about x_i / ω_i

Large Economies

- Can 1 agent provide equivalent PGs in large vs. small societies?
- Milleron '72: Splitting economies:
- Types $i \in \{1, \ldots, n\}$
- Replicates $r \in \{1, \ldots, R\}$
- Agent $= (i, r)$
- $\omega_{i,r} = \omega_i / R$, so $\sum_i \sum_r \omega_{i,r} = \sum_i \omega_i$
- $(x_{i,r}, y) \succeq_{i,r} (x'_{i,r}, y') \Leftrightarrow (R\,x_{i,r}, y) \succeq_i (R\,x'_{i,r}, y')$
- Agents care about x_i / ω_i

Large Economies

- Can 1 agent provide equivalent PGs in large vs. small societies?
- Milleron '72: Splitting economies:
- Types $i \in \{1, \ldots, n\}$
- Replicates $r \in \{1, \ldots, R\}$
- Agent $= (i, r)$
- $\omega_{i,r} = \omega_i / R$, so $\sum_i \sum_r \omega_{i,r} = \sum_i \omega_i$
- $(x_{i,r}, y) \succeq_{i,r} (x'_{i,r}, y') \Leftrightarrow (R\, x_{i,r}, y) \succeq_i (R\, x'_{i,r}, y')$
- Agents care about x_i / ω_i

Large Economies

- Can 1 agent provide equivalent PGs in large vs. small societies?
- Milleron '72: Splitting economies:
- Types $i \in \{1, ..., n\}$
- Replicates $r \in \{1, ..., R\}$
- Agent $= (i, r)$
- $\omega_{i,r} = \omega_i / R$, so $\sum_i \sum_r \omega_{i,r} = \sum_i \omega_i$
- $(x_{i,r}, y) \succeq_{i,r} (x'_{i,r}, y') \Leftrightarrow (R x_{i,r}, y) \succeq_i (R x'_{i,r}, y')$
- Agents care about x_i / ω_i

Large Economies

- Can 1 agent provide equivalent PGs in large vs. small societies?
- Milleron '72: Splitting economies:
- Types $i \in \{1, \ldots, n\}$
- Replicates $r \in \{1, \ldots, R\}$
- Agent $= (i, r)$
- $\omega_{i,r} = \omega_i / R$, so $\sum_i \sum_r \omega_{i,r} = \sum_i \omega_i$
- $(x_{i,r}, y) \succeq_{i,r} (x'_{i,r}, y') \Leftrightarrow (R x_{i,r}, y) \succeq_i (R x'_{i,r}, y')$
- Agents care about x_i / ω_i

Large Economies

- Can 1 agent provide equivalent PGs in large vs. small societies?
- Milleron '72: Splitting economies:
- Types $i \in \{1, \ldots, n\}$
- Replicates $r \in \{1, \ldots, R\}$
- Agent $= (i, r)$
- $\omega_{i,r} = \omega_i / R$, so $\sum_i \sum_r \omega_{i,r} = \sum_i \omega_i$
- $(x_{i,r}, y) \succeq_{i,r} (x'_{i,r}, y') \Leftrightarrow (R x_{i,r}, y) \succeq_i (R x'_{i,r}, y')$
- Agents care about x_i / ω_i

Large Economies

- Can 1 agent provide equivalent PGs in large vs. small societies?
- Milleron '72: Splitting economies:
- Types $i \in \{1, \ldots, n\}$
- Replicates $r \in \{1, \ldots, R\}$
- Agent $= (i, r)$
- $\omega_{i,r} = \omega_i / R$, so $\sum_i \sum_r \omega_{i,r} = \sum_i \omega_i$
- $(x_{i,r}, y) \succeq_{i,r} \left(x'_{i,r}, y'\right) \Leftrightarrow (R\, x_{i,r}, y) \succeq_i \left(R\, x'_{i,r}, y'\right)$
- Agents care about x_i / ω_i

Large Economies

- Can 1 agent provide equivalent PGs in large vs. small societies?
- Milleron '72: Splitting economies:
- Types $i \in \{1, \ldots, n\}$
- Replicates $r \in \{1, \ldots, R\}$
- Agent $= (i, r)$
- $\omega_{i,r} = \omega_i / R$, so $\sum_i \sum_r \omega_{i,r} = \sum_i \omega_i$
- $(x_{i,r}, y) \succeq_{i,r} \left(x'_{i,r}, y'\right) \Leftrightarrow (R\, x_{i,r}, y) \succeq_i \left(R\, x'_{i,r}, y'\right)$
- Agents care about x_i / ω_i

Large Economies

Theorem

If preferences are monotonic (strictly in the private good) and continuous and the production function is continuous, then $\{(x, y) : (x, y)$ *satisfies EP*$\}$ *shrinks to* $\{(\omega, 0)\}$ *as an economy is infinitely split.*

Proof.

As economy is split, budget constraints (ω_i / R) shrink so each agent becomes 'small' in production. By continuity, $\left|y^{(-i)} - y\right| \to 0$, but $|t_i / \omega_i| \not\to 0$ for some i. By strict monotonicity in the private good and continuity of preferences i must prefer to opt out for large enough R. □

- Also true for replica economies with crowding in PG

Conclusion

- Small economies:
  - Can try to use large, asymmetric transfers, though unfair
  - Often need enforcement to realize allocations $\neq (\omega, 0)$
- Large (splitting) economies
  - Always need enforcement to realize allocations $\neq (\omega, 0)$
- Casual evidence:
  - Kyoto protocol
  - Condo associations & eviction power

Conclusion

- Small economies:
  - Can try to use large, asymmetric transfers, though unfair
  - Often need enforcement to realize allocations $\neq (\omega, 0)$
- Large (splitting) economies
  - Always need enforcement to realize allocations $\neq (\omega, 0)$
- Casual evidence:
  - Kyoto protocol
  - Condo associations & eviction power

Conclusion

- Small economies:
  - Can try to use large, asymmetric transfers, though unfair
  - Often need enforcement to realize allocations $\neq (\omega, 0)$
- Large (splitting) economies
  - Always need enforcement to realize allocations $\neq (\omega, 0)$
- Casual evidence:
  - Kyoto protocol
  - Condo associations & eviction power

Conclusion

- Small economies:
  - Can try to use large, asymmetric transfers, though unfair
  - Often need enforcement to realize allocations $\neq (\omega, 0)$
- Large (splitting) economies
  - Always need enforcement to realize allocations $\neq (\omega, 0)$
- Casual evidence:
  - Kyoto protocol
  - Condo associations & eviction power

Conclusion

- Small economies:
  - Can try to use large, asymmetric transfers, though unfair
  - Often need enforcement to realize allocations $\neq (\omega, 0)$
- Large (splitting) economies
  - Always need enforcement to realize allocations $\neq (\omega, 0)$
- Casual evidence:
  - Kyoto protocol
  - Condo associations & eviction power

Conclusion

- Small economies:
  - Can try to use large, asymmetric transfers, though unfair
  - Often need enforcement to realize allocations $\neq (\omega, 0)$
- Large (splitting) economies
  - Always need enforcement to realize allocations $\neq (\omega, 0)$
- Casual evidence:
  - Kyoto protocol
  - Condo associations & eviction power

Conclusion

- Small economies:
  - Can try to use large, asymmetric transfers, though unfair
  - Often need enforcement to realize allocations $\neq (\omega, 0)$
- Large (splitting) economies
  - Always need enforcement to realize allocations $\neq (\omega, 0)$
- Casual evidence:
  - Kyoto protocol
  - Condo associations & eviction power

Conclusion

- Small economies:
  - Can try to use large, asymmetric transfers, though unfair
  - Often need enforcement to realize allocations $\neq (\omega, 0)$
- Large (splitting) economies
  - Always need enforcement to realize allocations $\neq (\omega, 0)$
- Casual evidence:
  - Kyoto protocol
  - Condo associations & eviction power