Epistemic Experiments: Utilities, Beliefs, and Irrational Play

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Motivation

Question:
How do people play games??

E.g.: Do people play equilibrium? If not, why not?

Current methodology:

1. Observe strategy choices
2. Identify likely phenomena
3. Alter the standard model to generate new solution concepts
4. Test/horserace solution concepts

Rather than assuming these alterations, we can *measure* them.

How? Copious amounts of elicitation!

The Problem

Elicitation bumps us into two insurmountable obstacles:

1. Contamination
    - Elicitation changes game play, and vice-versa.
2. Consequentialism
    - People care about more in a game than just its outcomes.

More on this later...

Epistemic Game Theory

How to pick what we should elicit?

Behavioral game theory: many informed guesses (see above)

Epistemic game theory: provides a structured framework for answers.

- Very clear about what players know and don't know.

The Epistemic Framework

In the lab, experimenter chooses a *game form*: $(I, (S_i)_{i \in I}, \pi)$.

- $I = \{1, 2\}$ players
- S_i strategy set
- $\pi : S \to X$ outcome function
  - Typical outcome: $x = (\$10, \$5)$.

Each player i arrives to the lab with a private *state*: $\omega_i = (u_i, s_i, \tau_i)$.

- $u_i : X \to \mathbb{R}$ utility for *outcomes*
- s_i chosen *pure* strategy
- $\tau_i = (p_i^1, p_i^2, \ldots)$ hierarchy of beliefs
  - $p_i^1(u_j, s_j)$ (marginals: $p_i^{1u}(u_j)$ and $p_i^{1s}(s_j)$)
  - $p_i^2(u_j, s_j, p_j^1)$ (marginal: $p_i^{2p}(p_j^1)$)
  - $p_i^3(u_j, s_j, p_j^1, p_j^2)$,
  - $\ldots \Rightarrow p_i(u_j, s_j, \tau_j) = p_i(\omega_j)$

Rationality

Rationality

Definition: Player i is **rational** in state $\omega_i = (u_i, s_i, \tau_i)$
if s_i maximizes $\sum_{s_j} p_i^{1s}(s_j) u_i(s_i, s_j)$ (← expected utility given u_i, p_i^{1s})

Player i **believes** j **is rational** at ω_i
if $p_i(\omega_j)$ puts probability 1 on $\{\omega_j : j \text{ is rational}\}$

("Belief" = probability one)

Theorem: Rationality & Common Belief in Rationality ⇔ Rationalizability

Theorem: Mutual belief in $[\sigma$, rationality, & utility$] \Rightarrow \sigma$ is Nash equil.

What I Will Elicit

"Epistemic experiments":

In each game, elicit:

1. u_i over *outcomes*
2. $p_i^{1u}(u_j)$ ("best guess of u_j")

At each decision node, elicit from *both* players:

4. s_i (complete plan)
5. $p_i^{1s}(s_j)$
6. $p_i^{2p}(p_j^{1s})$ ("best guess of p_j^{1s}")
7. $p_i(\{j \text{ is rational}\})$ ("weighted value theory").

Contamination?

Does elicitation contaminate game play? PROBABLY!
Does game play contaminate elicitation?? PROBABLY!

- Embrace it! This is a *fully contaminated* experiment!

Empirically, I think it actually doesn't matter:

- Strategy choices in popular games (e.g. PD) match previous studies

Elicitation Mechanisms

Eliciting cardinal utility index in a game
What is $u_i(\$15,\$5)$?

	Option A	**vs.**	**Option B**
Q0:	$(\$15,\$5)$	vs.	0% chance of $(\$20,\$20)$
Q1:	$(\$15,\$5)$	vs.	1% chance of $(\$20,\$20)$
		⋮	
Q62:	$(\$15,\$5)$	vs.	62% chance of $(\$20,\$20)$
Q63:	$(\$15,\$5)$	vs.	63% chance of $(\$20,\$20)$
Q64:	$(\$15,\$5)$	vs.	64% chance of $(\$20,\$20)$
		⋮	
Q100:	$(\$15,\$5)$	vs.	100% chance of $(\$20,\$20)$

$$u(\$15,\$5) = 0.63\underbrace{u(\$20,\$20)}_{\to 100} + 0.37\underbrace{u(\$0,\$0)}_{\to 0} = 63.$$

Notes and Caveats

Utility

- Elicit $u_i(\$15, \$5)$, e.g.
- u_i captures non-selfish preferences.
- u_i captures risk aversion.

Problem: Game theory assumes a utility over *strategies* $U_i(s_i, s_j)$

$$\textit{Game}: \ (I, (S_i, U_i)_i)$$

Solution: assume consequentialism:

$$U_i(s_i, s_j) = u_i(\pi(s_i, s_j))$$

Is consequentialism reasonable??

Consequentialism

Violating consequentialism:

	L	R
T	\$5, \$5	\$5, \$5
B	\$100, \$5	\$5, \$5

$U_1(T,L) \neq U_1(B,R)$, but $\pi(T,L) = \pi(B,R)$.

Thus, $U_i(s_i,s_j) \neq u_i(\pi(s_i,s_j))$.

Claim: Cannot elicit $U_i(s_i,s_j)$. Must assume consequentialism.

Messy Solution: Elicit $u_i(\pi(s_i,s_j))$ in the *context* of the game.

Redefining Rationality

Definition: Player i is **rational** in state $\omega_i = (u_i, s_i, \tau_i)$ if s_i maximizes $\sum_{s_j} p_i^{1s}(s_j) u_i(\pi(s_i, s_j))$

Thus, "rational" means

1. EU-maximizing, $\underline{\text{and}}$
2. consequentialism

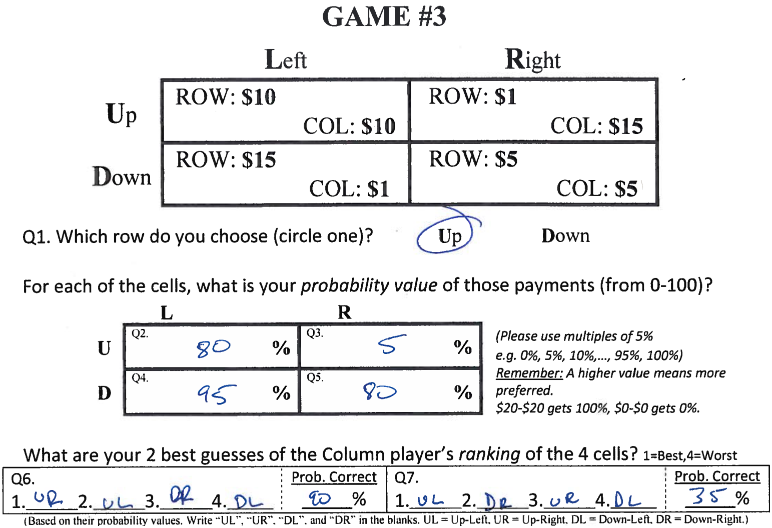
"Irrational" ⇒ "Non-EU" $\underline{\text{or}}$ "Non-consequentialist"

Design Summary

3 experiments

1. Five 2×2 game forms $n_1 = 150$
    - One-shot play w/ elicitation. Paper & pencil.
2. Same five game forms, but now sequential-move. $n_2 = 64$
    - One-shot play w/ elicitation. Paper & pencil.
3. Centipede game forms (4 payoff treatments, btwn-subject) $n_3 = 226$
    - Play 4 times w/ feedback. Elicitation in last 2. Study last.

2 × 2 Game Forms



GAME #3

	Left	Right
Up	ROW: $10 COL: $10	ROW: $1 COL: $15
Down	ROW: $15 COL: $1	ROW: $5 COL: $5

Q1. Which row do you choose (circle one)? [✓ Up] Down

For each of the cells, what is your *probability value* of those payments (from 0-100)?

	L	R
U	Q2. 80 %	Q3. 5 %
D	Q4. 95 %	Q5. 80 %

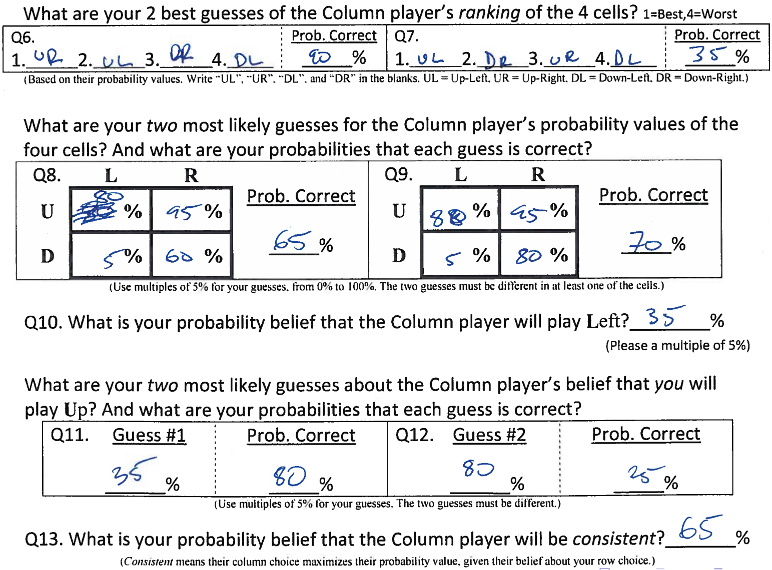
(Please use multiples of 5%
e.g. 0%, 5%, 10%,..., 95%, 100%)
Remember: A higher value means more preferred.
$20-$20 gets 100%, $0-$0 gets 0%.

What are your 2 best guesses of the Column player's *ranking* of the 4 cells? 1=Best,4=Worst

Q6. 1. UR 2. UL 3. DR 4. DL	Prob. Correct 90 %	Q7. 1. UL 2. DR 3. UR 4. DL	Prob. Correct 35 %

(Based on their probability values. Write "UL", "UR", "DL", and "DR" in the blanks. UL = Up-Left, UR = Up-Right, DL = Down-Left, DR = Down-Right.)

2×2 Game Forms



What are your 2 best guesses of the Column player's *ranking* of the 4 cells? 1=Best,4=Worst

Q6.	Prob. Correct	Q7.	Prob. Correct
1. UR 2. UL 3. DR 4. DL	90 %	1. UL 2. DR 3. UR 4. DL	35 %

(Based on their probability values. Write "UL", "UR", "DL", and "DR" in the blanks. UL = Up-Left, UR = Up-Right, DL = Down-Left, DR = Down-Right.)

What are your *two* most likely guesses for the Column player's probability values of the four cells? And what are your probabilities that each guess is correct?

Q8.	L	R	Prob. Correct
U	80 %	95 %	65 %
D	5 %	60 %	

Q9.	L	R	Prob. Correct
U	88 %	95 %	70 %
D	5 %	80 %	

(Use multiples of 5% for your guesses, from 0% to 100%. The two guesses must be different in at least one of the cells.)

Q10. What is your probability belief that the Column player will play Left? 35 %

(Please a multiple of 5%)

What are your *two* most likely guesses about the Column player's belief that *you* will play Up? And what are your probabilities that each guess is correct?

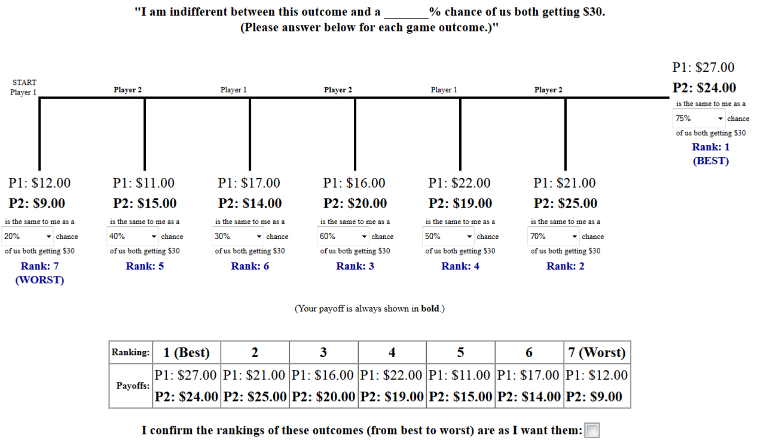
Q11. Guess #1	Prob. Correct	Q12. Guess #2	Prob. Correct
35 %	80 %	80 %	25 %

(Use multiples of 5% for your guesses. The two guesses must be different.)

Q13. What is your probability belief that the Column player will be *consistent*? 65 %

(*Consistent* means their column choice maximizes their probability value, given their belief about your row choice.)

Centipede Game Forms



"I am indifferent between this outcome and a _______% chance of us both getting $30.
(Please answer below for each game outcome.)"

P1: $27.00
P2: $24.00
is the same to me as a
75% chance
of us both getting $30
Rank: 1
(BEST)

P1: $12.00
P2: $9.00
is the same to me as a
20% chance
of us both getting $30
Rank: 7
(WORST)

P1: $11.00
P2: $15.00
is the same to me as a
40% chance
of us both getting $30
Rank: 5

P1: $17.00
P2: $14.00
is the same to me as a
30% chance
of us both getting $30
Rank: 6

P1: $16.00
P2: $20.00
is the same to me as a
60% chance
of us both getting $30
Rank: 3

P1: $22.00
P2: $19.00
is the same to me as a
50% chance
of us both getting $30
Rank: 4

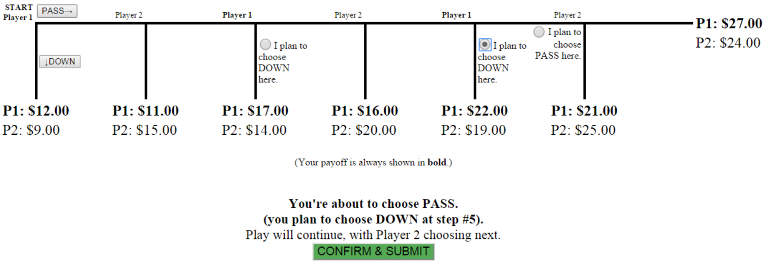
P1: $21.00
P2: $25.00
is the same to me as a
70% chance
of us both getting $30
Rank: 2

(Your payoff is always shown in **bold**.)

Ranking:	**1 (Best)**	**2**	**3**	**4**	**5**	**6**	**7 (Worst)**
Payoffs:	P1: $27.00 **P2: $24.00**	P1: $21.00 **P2: $25.00**	P1: $16.00 **P2: $20.00**	P1: $22.00 **P2: $19.00**	P1: $11.00 **P2: $15.00**	P1: $17.00 **P2: $14.00**	P1: $12.00 **P2: $9.00**

I confirm the rankings of these outcomes (from best to worst) are as I want them: ☐

Centipede Game Forms



○ I plan to choose DOWN here.

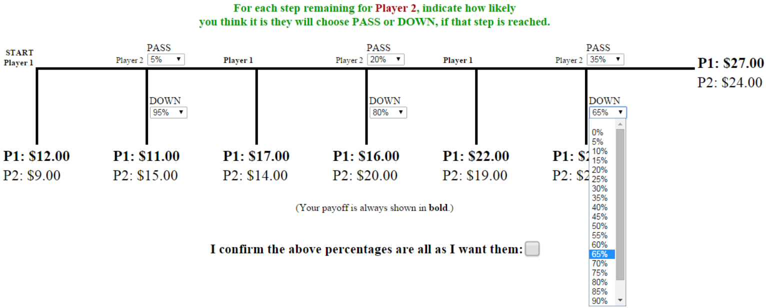
● I plan to choose DOWN here.

○ I plan to choose PASS here.

(Your payoff is always shown in **bold**.)

You're about to choose PASS.
(you plan to choose DOWN at step #5).
Play will continue, with Player 2 choosing next.

Centipede Game Forms


For each step remaining for Player 2, indicate how likely you think it is they will choose PASS or DOWN, if that step is reached.

Player 2: PASS 5%, DOWN 95%

Player 2: PASS 20%, DOWN 80%

Player 2: PASS 35%, DOWN 65%

(Your payoff is always shown in **bold**.)

I confirm the above percentages are all as I want them: ☐

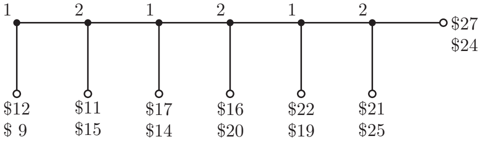
Results, Part 1

Results, Part 1: The Importance of Utilities

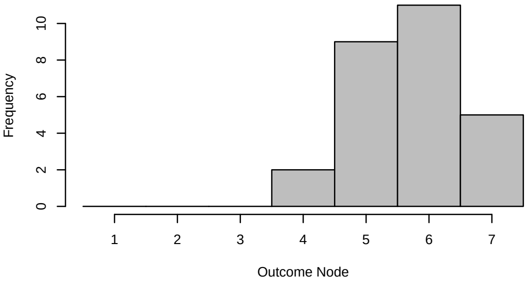
The Centipede Game Form

The Centipede Game Form

Treatment #1: Risk $1 to gain $5

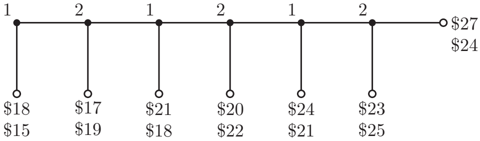


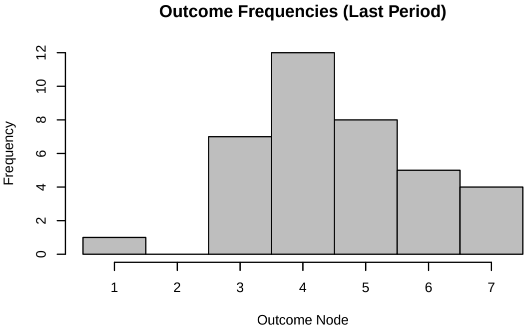
Outcome Frequencies (Last Period)



The Centipede Game Form

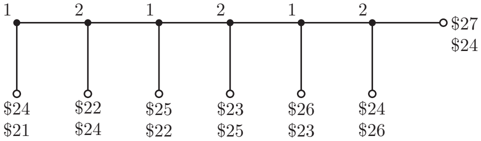
Treatment #2: Risk $1 to gain $3



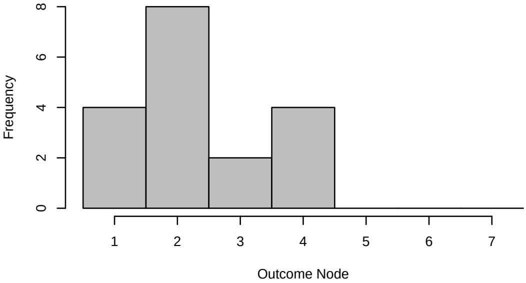


The Centipede Game Form

Treatment #3: Risk $2 to gain $1



Outcome Frequencies (Last Period)



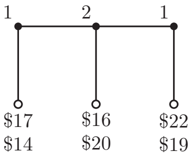
Why Is This Happening?

Why do payoffs have such a drastic impact on outcomes?

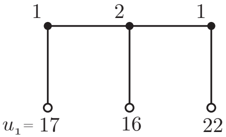
Turn to elicitation data for answers...

Bottom line: Preferences matter a LOT

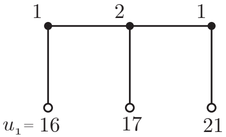
Risk \$1 to Gain \$5



Game Form



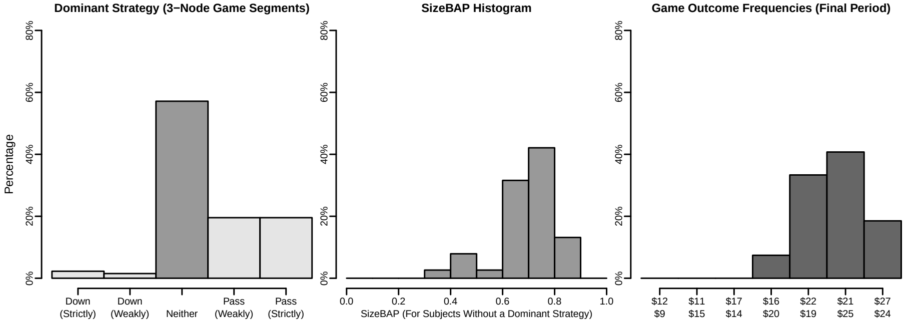
‘Selfish’



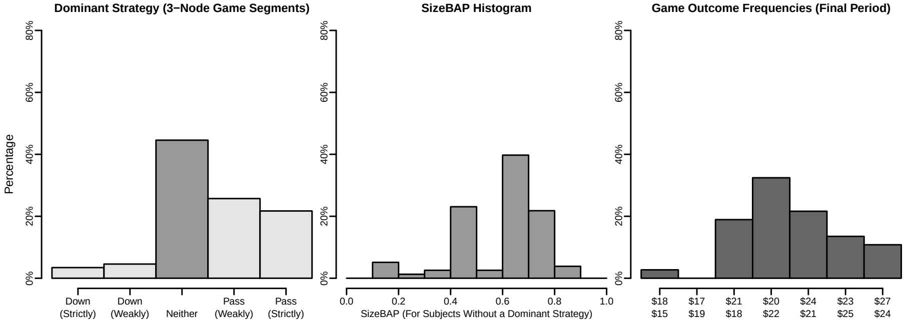
‘Altruistic’

- Let p be probability Player 2 plays Pass
- Selfish Player 1: Pass if $p \in [1/6, 1]$.
    - $\mathit{SizeBAP} = 5/6$.
- Altruist Player 1: *Dominant Strategy* to pass ($p \geq 0$)
    - $\mathit{SizeBAP} = 1$.
    - Not a centipede game!
- Selfish Player 1: Pass if $1/6$ of players are Altruists

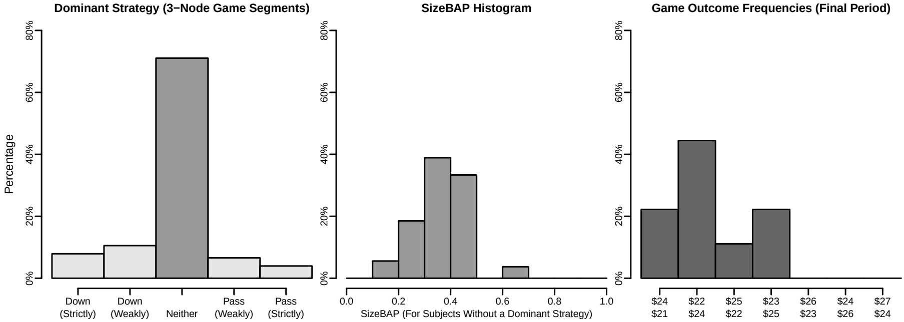
Treatment #1: Risk $1 to Gain $5



Treatment #2: Risk $1 to Gain $3



Treatment #3: Risk $2 to Gain $1



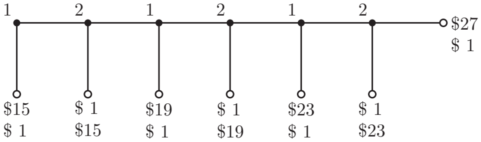
Problem

None of these are complete-information centipede games!

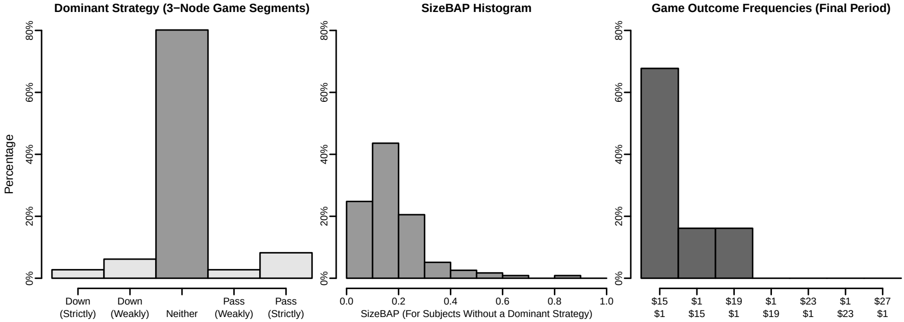
Not really testing backwards induction.

The Centipede Game Form

Treatment #4: Risk (Almost) Everything to Gain $4



Treatment #4: Risk Everything to Gain $4



Results, Part 1

Results, Part 1: The Importance of Utilities

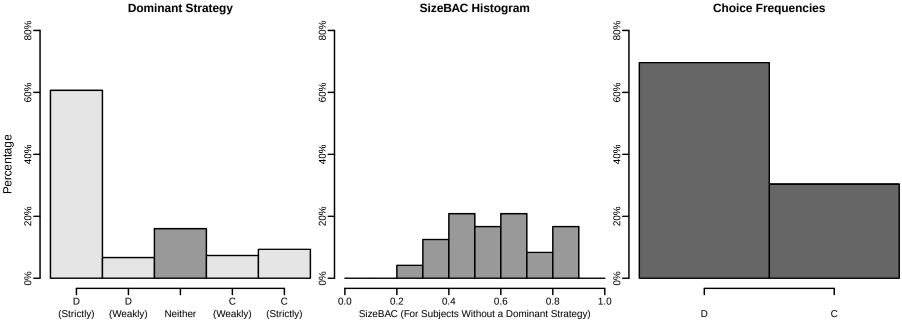
The Prisoners' Dilemma Game Form

The Prisoners' Dilemma: Action Choices

	35%	65%
26%	\$10, 10	\$1, 15
74%	\$15, 1	\$5, 5

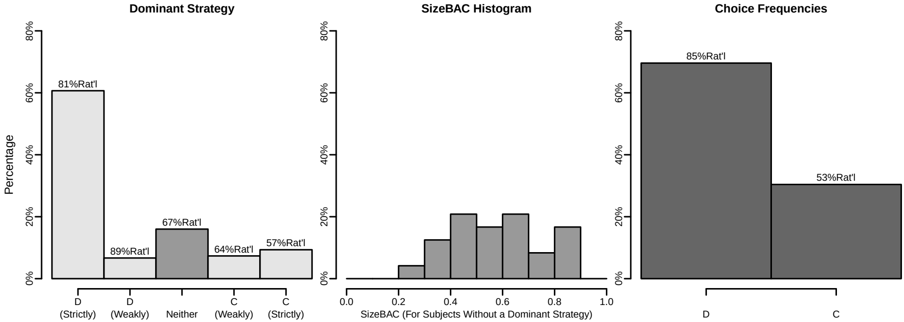
Why do 30% of people cooperate?

The Prisoners' Dilemma: Preferences



Can social preferences explain cooperation in the PD?

The Prisoners' Dilemma: Preferences



Preferences can only explain 53% of the cooperation!

- Only 60% when C is dominant!
- Failure of consequentialism? $U_i(C,C) \neq u_i(\$10,\$10)$

Sequential-Move PD

What about sequential-move PD?

	67%	33%
38%	$10, 10	$1, 15
	0%	100%
62%	$15, 1	$5, 5

- Play C after C: 7 of 8 rational (88%)
- Play D after C: 3 of 4 rational (75%)
- Play C after D: N/A
- Play D after D: 18 of 18 rational (100%)
- Irrationality *disappears* when strategic uncertainty is removed
- Strategic uncertainty even causes dominance violations (!?)
- Only 2 preference reversals (out of 30) between elicitation and choice

Results, Part 2

Results, Part 2: Rationality

Iterated Dominance

Elicited utility $\equiv$ Selfish

Iterated Dominance

	25%	75%
100%	$^{\$}10, 5$	$^{\$}15, 15$
0%	$^{\$}5, 10$	$^{\$}1, 1$

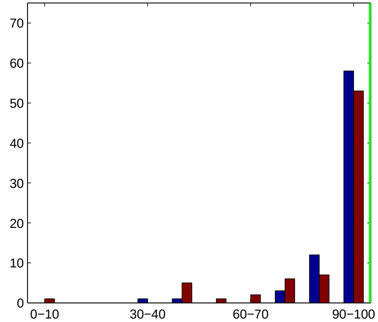
Why do 25% of Column players play Left?

Iterated Dominance

	25%	75%
100%	$^{\$}10, 5$	$^{\$}15, 15$
0%	$^{\$}5, 10$	$^{\$}1, 1$

Row's actual % Up

Col's $p(U)$ & Row's guess

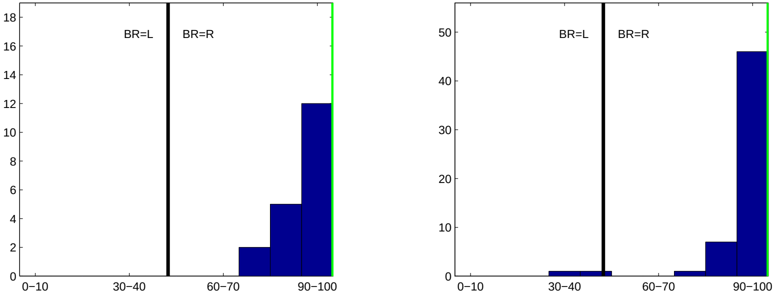


Iterated Dominance

	25%	75%
100%	$^\$10,5$	$^\$15,15$
0%	$^\$5,10$	$^\$1,1$

Col's $p(U)$ | Play L

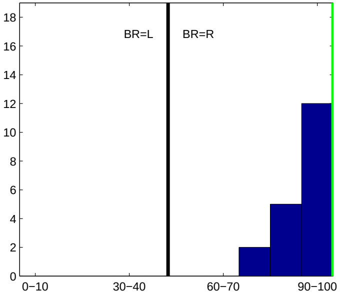
Col's $p(U)$ | Play R



Iterated Dominance

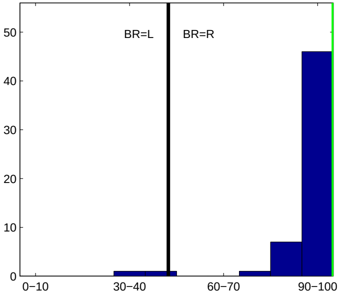
	25%	75%
100%	$10, 5	$15, 15
0%	$5, 10	$1, 1

Col's $p(U)$ | Play L



Rational: **17%** (all 'non-selfish')

Col's $p(U)$ | Play R



Rational: 98%

Iterated Dominance

The sequential-move experiment:

	6%	94%
100%	$10, 5	$15, 15
	–%	–%
0%	$5, 10	$1, 1

- Play L: 1 of 2 are rational
- Play R: 29 of 29 are rational
- Again, irrationality *disappears* when uncertainty is removed

Results, Part 2

Results, Part 2: Rationality

Asymmetric Coordination

Elicited utility $\equiv$ Selfish

Asymmetric Coordination

Asymmetric Coordination.

Data:

	49%	51%
93%	$15,5	$2,1
7%	$1,2	$5,10

Risk-neutral MSNE:

	18%	82%
67%	$15,5	$2,1
33%	$1,2	$5,10

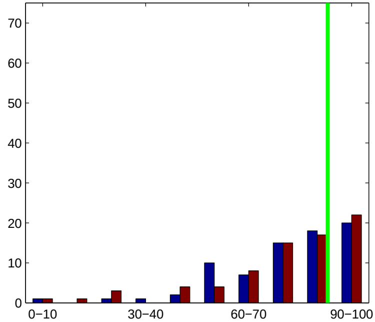
Why are 51% of COL playing Right?

Asymmetric Coordination

	49%	51%
93%	$^{\$}15,5$	$^{\$}2,1$
7%	$^{\$}1,2$	$^{\$}5,10$

Row's actual % Up

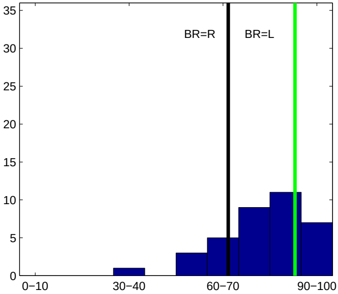
Col's $p(U)$ & Row's guess



Asymmetric Coordination

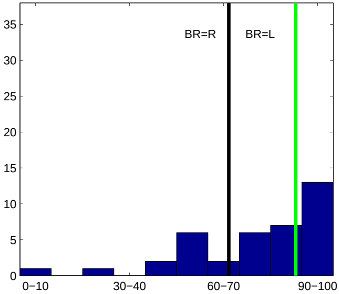
$$\begin{array}{r|c|c|} & 49\% & 51\% \\ \hline 93\% & ^{\$}15,5 & ^{\$}2,1 \\ \hline 7\% & ^{\$}1,2 & ^{\$}5,10 \\ \hline \end{array}$$

Col's $p(U)$ | Play L



% Rational: 86%

Col's $p(U)$ | Play R

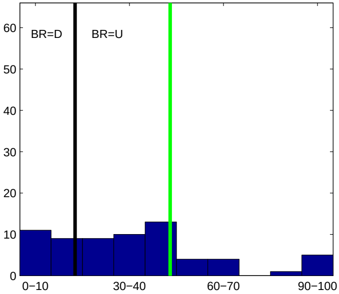


% Rational: 29%

Asymmetric Coordination

	49%	51%
93%	$^{\$}15,5$	$^{\$}2,1$
7%	$^{\$}1,2$	$^{\$}5,10$

Row's $p(L)$ | Play U



% Rational: 70%

Asymmetric Coordination

	49%	51%
93%	$^{\$}15, 5$	$^{\$}2, 1$
7%	$^{\$}1, 2$	$^{\$}5, 10$

Overall, 38% irrational.

- Betting against their beliefs.
- Over-optimism in strategies, not beliefs.
- Non-EU regret aversion?

(Non-EU may be non-consequentialism)

Asymmetric Coordination - Sequential Move

	93%	7%
90%	$^{\$}15, 5$	$^{\$}2, 1$
	0%	100%
10%	$^{\$}1, 2$	$^{\$}5, 10$

- Play L after U: 26 of 26 (100%) Rational
- Play R after U: 0 of 2 (0%) Rational
- Play L after D: N/A
- Play R after D: 3 of 3 (100%) Rational

Removing strategic uncertainty removes irrationality.

Results, Part 2

Results, Part 2: Rationality

Asymmetric Matching Pennies

Elicited utility $\equiv$ Selfish

Asymmetric Matching Pennies

No pure strategy Nash Equil.

Data:

	44%	56%
88%	$15, 5	$5, 10
12%	$5, 10	$10, 5

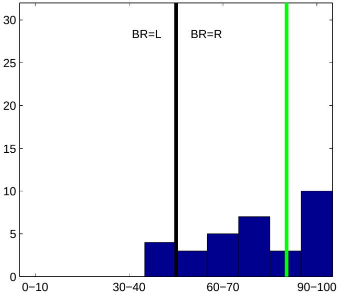
Risk-neutral $ MSNE:

	33%	67%
50%	$15, 5	$5, 10
50%	$5, 10	$10, 5

Asymmetric Matching Pennies

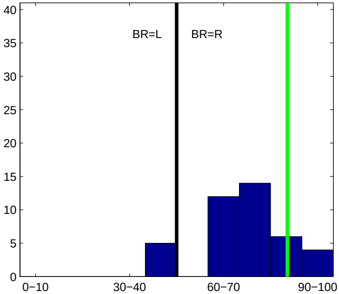
	44%	56%
88%	$^{\$}15,5$	$^{\$}5,10$
12%	$^{\$}5,10$	$^{\$}10,5$

Col's $p(U)$ | Play L



Rational: **35%**

Col's $p(U)$ | Play R



Rational: 90%

Asymmetric Matching Pennies

	13%	87%
97%	$^{\$}15, 5$	$^{\$}5, 10$
	100%	0%
3%	$^{\$}5, 10$	$^{\$}10, 5$

G4: Asym. Matching Pennies

25%Rat'l	96%Rat'l
$^{\$}15, 5$	$^{\$}5, 10$
100%Rat'l	N/A
$^{\$}5, 10$	$^{\$}10, 5$

- Non-consequentialism for those that played L (small %)

Results, Part 2

Results, Part 2: Cross-Game Correlation

Irrationality Correlation

% of irrational players in game i who were also irrational in game j:

		% Irrat in Game j				
		DomSolv	SymCoor	PD	AsymMP	AsymCoor
Game i	% Irrat.	11%	3%	24%	29%	37%
DomSolv	11%	– –	0%	19%	40%	47%
SymCoor	3%	0%	– –	60%	20%	0%
PD	24%	8%	9%	– –	30%	44%
AsymMP	29%	15%	2%	25%	– –	45%
AsymCoor	37%	13%	0%	28%	34%	– –

Results, Part 3

Results, Part 3: Robustness Check

Symmetric Coordination

Symmetric Coordination

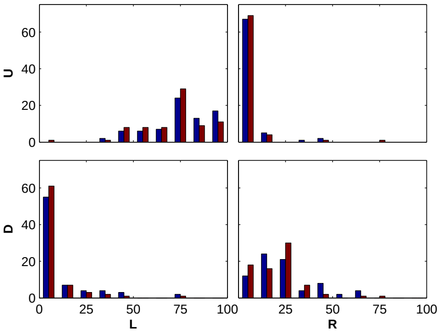
Robustness Check: A Super Easy Game

	97%	3%
97%	$15, 15	$1, 1
3%	$2, 2	$5, 5

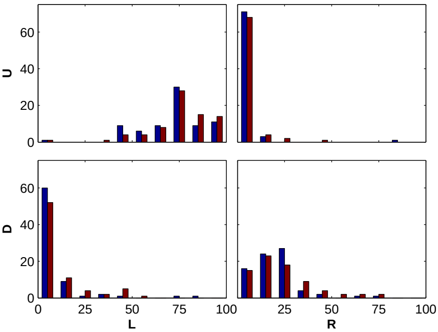
Symmetric Coordination - Utilities

	97%	3%
97%	$^{\$}15,15$	$^{\$}1,1$
3%	$^{\$}2,2$	$^{\$}5,5$

Row's $u_i(\cdot)$ & Col's belief



Col's $u_i(\cdot)$ & Row's belief

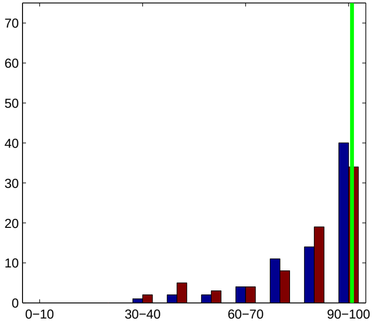


Symmetric Coordination - Beliefs

	97%	3%
97%	$^{\$}15,15$	$^{\$}1,1$
3%	$^{\$}2,2$	$^{\$}5,5$

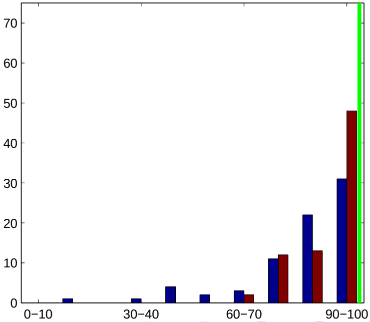
Row's actual % Up

Col's $p(U)$ & Row's guess



Col's actual % Left

Row's $p(L)$ & Col's guess

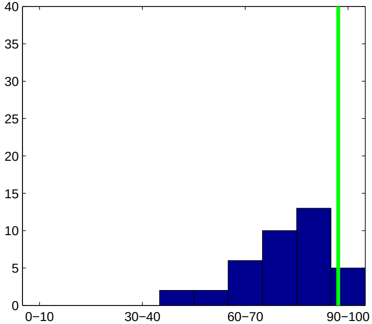


Symmetric Coordination - Rationality

	97%	3%
97%	$15, 15	$1, 1
3%	$2, 2	$5, 5

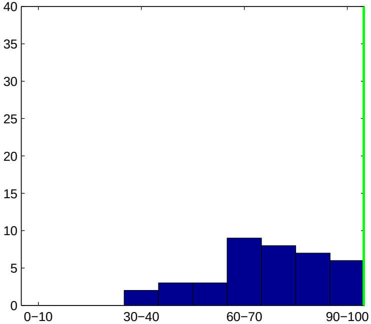
Row's % rational

Col's belief of rationality



Col's % rational

Row's belief of rationality



Summary

- Non-selfish preferences in *some* games
  - Seems to be where we'd expect them
  - Can drive the behavior of selfish types
  - Respect for Bayesian games
  - Why not measure utilities after every experiment?
- Overall rationality: 79%
  - Is that high or low?
  - Rises to 90% for second-movers
  - Strategic uncertainty drives irrationality
  - Irrationality may be non-consequentialism
  - Irrationality may be non-EU
  - Story seems to vary by game :(

- WARNING: reliability of elicitation procedure.
  - See 2010 and 2011 data

The End.

Game Forms & Raw Choice Data

	25%	75%
100%	$^{\$}10, 5$	$^{\$}15, 15$
0%	$^{\$}5, 10$	$^{\$}1, 1$

G1: Dominance Solvable

	99%	1%
96%	$^{\$}15, 15$	$^{\$}1, 1$
4%	$^{\$}2, 2$	$^{\$}5, 5$

G2: Sym. Coordination

	35%	65%
26%	$^{\$}10, 10$	$^{\$}1, 15$
74%	$^{\$}15, 1$	$^{\$}5, 5$

G3: Prisoners' Dilemma

	44%	56%
88%	$^{\$}15, 5$	$^{\$}5, 10$
12%	$^{\$}5, 10$	$^{\$}10, 5$

G4: Asym. Matching Pennies

	49%	51%
93%	$^{\$}15, 5$	$^{\$}2, 1$
7%	$^{\$}1, 2$	$^{\$}5, 10$

G5: Asymmetric Coordination

*11 missing actions (1.5% of data), all in later games.