Epistemic Experiments: Utilities, Beliefs, and Irrational Play

Paul J. Healy (OSU)

An Embarrassing Timeline

A pet project:

2010: Tenure. First attempt at 2×2 games.

2013: Redo experiment on pencil & paper

2014: Present at Pitt

2015: Add centipede games

2016: Add no-elicitation benchmark

2017: Add sequential-move 2×2 games

2019: "I'm never presenting this again."

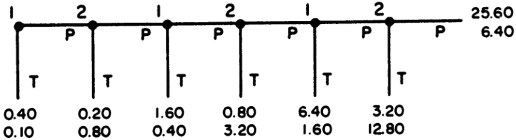
2020: COVID writing retreat, 1st draft

2023: Present at Pitt

The Standard Approach

Standard game theory experiment:

1. Interesting game form



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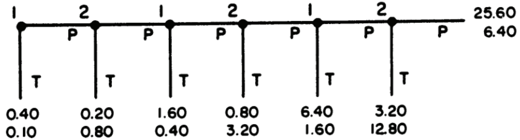
Standard game theory experiment:

1. Interesting game form
2. Baseline theory/assumptions:
    - Selfish prefs, "Rational" behavior (eg, backwards induction)



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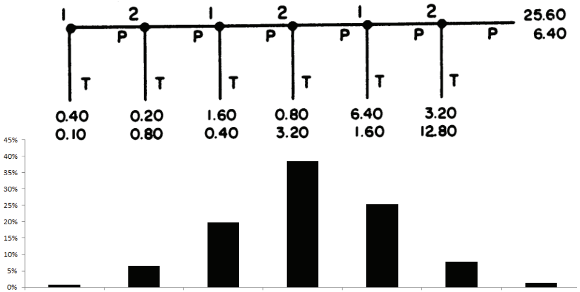

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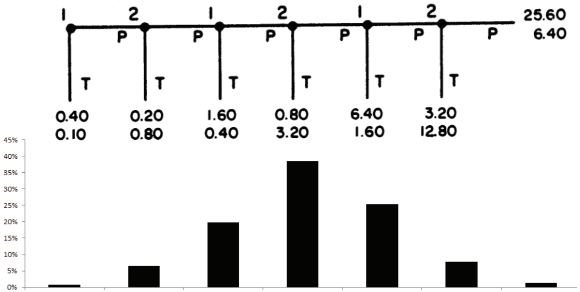
4. Posit alternative theory



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Standard game theory experiment:

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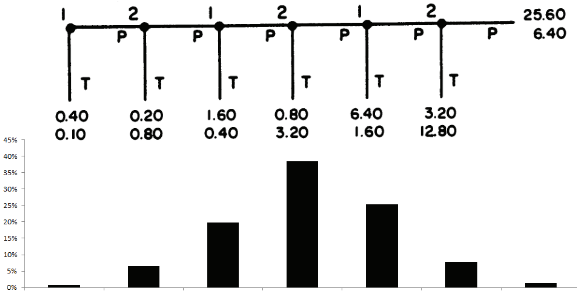
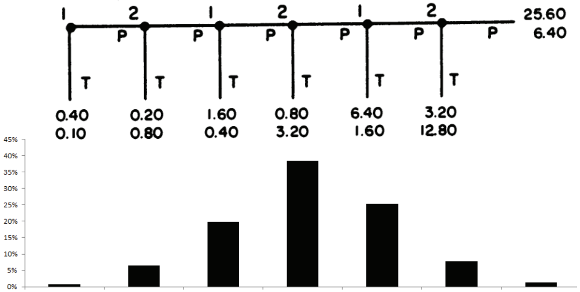
3. Observe deviations
4. Posit alternative theory
5. New treatments to test comparative statics

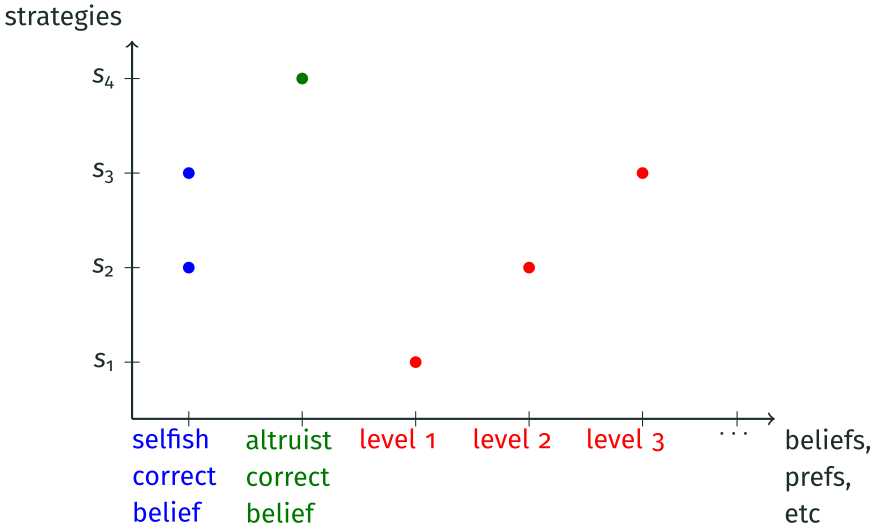


Alternative "Solution Concepts"

1. Nash with Altruism, Inequality Aversion
2. Reputation-building/Gang of Four
3. Level-*k* (wrong beliefs)
4. QRE (noisy equilibrium play)



A Risky Picture



Direct Measurement

Each solution concept makes specific assumptions about utilities, beliefs, rationality, etc.

Why not measure these things directly???

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OK... but then what exactly should we measure?

We need a framework that encompasses *all* such theories

...a level playing field in which no theory is the null hypothesis

The Epistemic Game Theory Framework

The Observable Experiment: (I, S, X, π)

1. Players: $i \in I = \{1, 2\}$
2. Strategies: $s_i \in S_i$ *Ex*: when to Take
3. Outcomes: $(x_1, x_2) \in X$ *Ex*: $(\$6.40, \$1.60)$
4. Outcome function: $\pi(s_1, s_2) \in X$

i's Private Information: $\omega_i = (u_i, s_i, \vec{p}_i)$

1. Utility: $u_i(x_1, x_2)$
    - Non-selfish, but consequentialist
2. Chosen Strategy: $s_i \in S_i$
    - Mixing is only in beliefs (Aumann)
3. Beliefs
    - First-order: $p_i^1(u_{-i}, s_{-i})$
    - Second-order: $p_i^2(p_{-i}^1, u_{-i}, s_{-i})$
    - Hierarchy: $\vec{p}_i = (p_i^1, p_i^2, p_i^3, \ldots)$

Example: Nash Equilibrium

Players are in a (selfish) Nash equilibrium at $\omega = (\omega_1, \ldots, \omega_n)$ if:

1. Utility: $u_i(x_i, x_{-i}) = x_i$ (“selfish”)
2. Beliefs: correct beliefs about u_{-i}, s_{-i}.
3. Strategy: $s_i \in \arg\max_{s_i'} \left[\sum_{(u_{-i},s_{-i})} p_i^1(u_{-i}, s_{-i}) u_i(\pi(s_i', s_{-i}))\right]$

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    - Player i is **rational** at $\omega_i = (u_i, s_i, \vec{p}_i)$ if this is true
    - Let R_i be those (p_i^1, u_i, s_i) where i is rational
    - i's belief that $-i$ is rational is $p_i^2(R_{-i})$
    - Can define common knowledge of rationality, etc.

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    - Can define common knowledge of rationality, etc.
    - Aumann (1995): Nash equil. does **not** require c.k. of rationality
    - Rationality & c.k. of rationality $\Rightarrow$ IESDS

Example: Altruistic Nash Equilibrium

Players are in an Altruistic Nash equilibrium at $\omega = (\omega_1, \ldots, \omega_n)$ if:

1. Utility: $u_i(x_i, x_{-i}) = x_i + \alpha x_{-i}$
2. Beliefs: correct beliefs about u_{-i}, s_{-i}.
3. Strategy: $s_i \in \arg\max_{s_i'} \left[\sum_{(u_{-i}, s_{-i})} p_i^1(u_{-i}, s_{-i}) u_i(\pi(s_i', s_{-i}))\right]$
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Example: Level-k

Level-1:

1. Utility: selfish
2. Beliefs: u_{-i} selfish, s_{-i} uniformly distributed
3. Strategy: s_i is rational, given utility & beliefs

Level-$k > 1$:

1. Utility: selfish
2. Beliefs: u_{-i} selfish, s_{-i} is Level-$k-1$ strategy
3. Strategy: s_i is rational, given utility & beliefs

So, What Should We Measure?

1. Utility: $u_i(\pi(s_1, s_2))$

A Game Form

	L	R
U	\$2,\$2	\$0,\$3
D	\$3,\$0	\$1,\$1

$\swarrow \qquad \downarrow \qquad \searrow$

A Game

	L	R
U	$2, 2$	$0, 3$
D	$3, 0$	$1, 1$

A Game

	L	R
U	$4, 4$	$0, 0$
D	$0, 0$	$2, 2$

A Game

	L	R
U	$0, 0$	$0, 0$
D	$0, 0$	$0, 0$

How To Measure Cardinal Utility

	L	R
U	\$2,\$2	\$0,\$3
D	\$3,\$0	\$1,\$1

A Game Form

- Elicit $u_i(x_1, x_2)$ for each cell
  - or, for each terminal node
- How?
  - Let $\overline{x} = (\$20, \$20)$, $\underline{x} = (\$0, \$0)$
  - "I'm indifferent between $(\$3, \$0)$ and getting $\overline{x}$ w/ prob. p^*"

$$
\begin{aligned}
u_i(\$3, \$0) &= p^* \underbrace{u_i(\overline{x})}_{=1} + (1 - p^*) \underbrace{u_i(\underline{x})}_{=0} \\
&= p^*
\end{aligned}
$$

Multiple Price List Elicitation

Row#	**Option A**	OR	**Option B**
1	(\$3,\$0)	or	(\$20,\$20) w/ prob 1%
2	(\$3,\$0)	or	(\$20,\$20) w/ prob 2%
$\vdots$	$\vdots$	$\vdots$	$\vdots$
q	(\$3,\$0)	or	(\$20,\$20) w/ prob q%
$q+1$	(\$3,\$0)	or	(\$20,\$20) w/ prob $q+1$%
$q+2$	(\$3,\$0)	or	(\$20,\$20) w/ prob $q+2$%
$q+3$	(\$3,\$0)	or	(\$20,\$20) w/ prob $q+3$%
$\vdots$	$\vdots$	$\vdots$	$\vdots$
99	(\$3,\$0)	or	(\$20,\$20) w/ prob 99%
100	(\$3,\$0)	or	(\$20,\$20) w/ prob 100%

Choose Option A or Option B (single switch point q)

One row randomly selected for payment

Multiple Price List Elicitation

Row#	Option A	OR	Option B
1	(\$3,\$0)	or	(\$20,\$20) w/ prob 1%
2	(\$3,\$0)	or	(\$20,\$20) w/ prob 2%
$\vdots$	$\vdots$	$\vdots$	$\vdots$
q	(\$3,\$0)	or	(\$20,\$20) w/ prob q%
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$\vdots$	$\vdots$	$\vdots$	$\vdots$
99	(\$3,\$0)	or	(\$20,\$20) w/ prob 99%
100	(\$3,\$0)	or	(\$20,\$20) w/ prob 100%

If you lie, you get the less-preferred option on some rows

I.C. as long as subject respects **statewise dominance** in rows

Issue 1: Consequentialism

- Elicit $u_i(\$15, \$5)$, e.g.
- u_i captures non-selfish preferences.
- u_i captures risk aversion.

Problem: Game theory: utility over *strategies*: $U_i(s_i, s_j)$

We elicit: utility over *outcomes*: $u_i(\pi(s_i, s_j))$

Solution: Assume consequentialism:

$$U_i(s_i, s_j) = u_i(\pi(s_i, s_j))$$

Is consequentialism reasonable?? Is it even testable??

Issue 1: Consequentialism

Example violating consequentialism:

	Nice	Mean
Foolish	\$5, \$5	\$5, \$5
Wise	\$100, \$5	\$5, \$5

$\pi(\text{Foolish}, \text{Nice}) = \pi(\text{Wise}, \text{Mean})$, but, intuitively $U_1(\text{Foolish}, \text{Nice}) \neq U_1(\text{Wise}, \text{Mean})$.

But how could you possibly observe that??

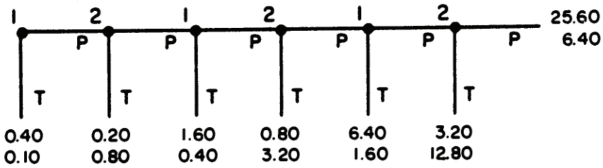
Claim: Cannot elicit $U_i(s_i, s_j)$. *Must* assume consequentialism.

Messy Solution: Elicit $u_i(\pi(s_i, s_j))$ in the *context* of the game.

So, What Should We Measure?

2. Strategies: s_i

- Easy. Just play the game.
- Complete contingent plan
  - "When will you Take?"
- Can re-elicit this at each node
  - Even when not active



So, What Should We Measure?

3. Beliefs: $p_i^1(u_{-i}, s_{-i})$, $p_i^2(p^1_{-i}, u_{-i}, s_{-i})$, ...

Measure before the game:

1. Best guess of $u_{-i}(x_1, x_2)$ at each terminal node

Measure at every node:

1. Probability of each s_{-i} (call that $p_i^1(s_{-i})$)
2. Best guess of $p^1_{-i}(s_i)$
3. Probability $-i$ is *rational*

My Wish List:

1. Entire distribution over u_{-i}
2. Correlation between u_{-i} and s_{-i}
3. Correlation between p^1_{-i} and s_{-i}

Issue 2: Contamination

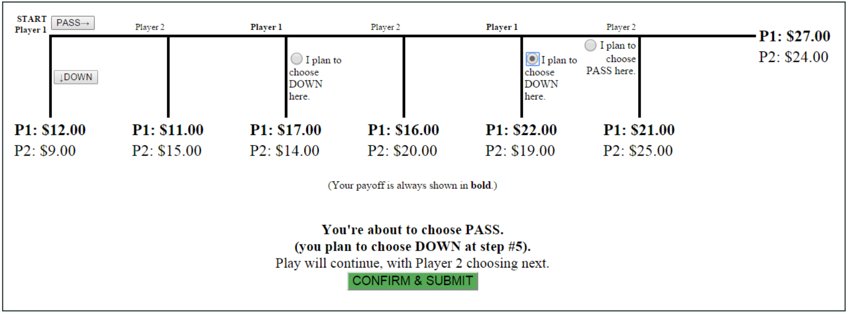
Does elicitation contaminate game play? PROBABLY!
Does game play contaminate elicitation?? PROBABLY!

- I embrace it! This is a *fully contaminated* experiment!
  - Necessary evil for the methodology
  - Intuitively: provides an upper bound on rationality

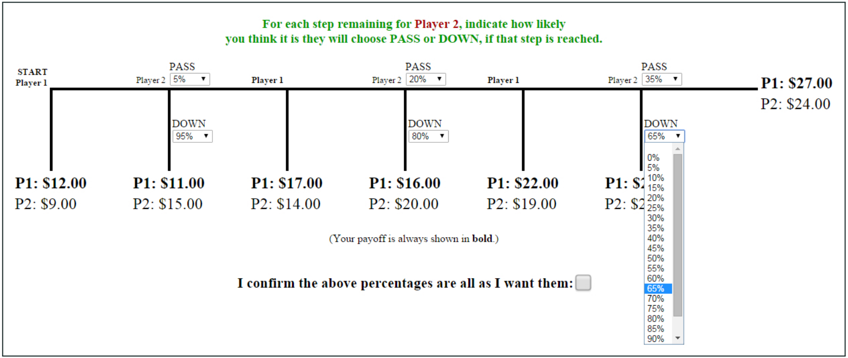
Empirically, I think it probably doesn't matter:

- In five 2×2 games, play w/out elicitation was the same in 4 of 5
- Behavior pretty similar to previous papers

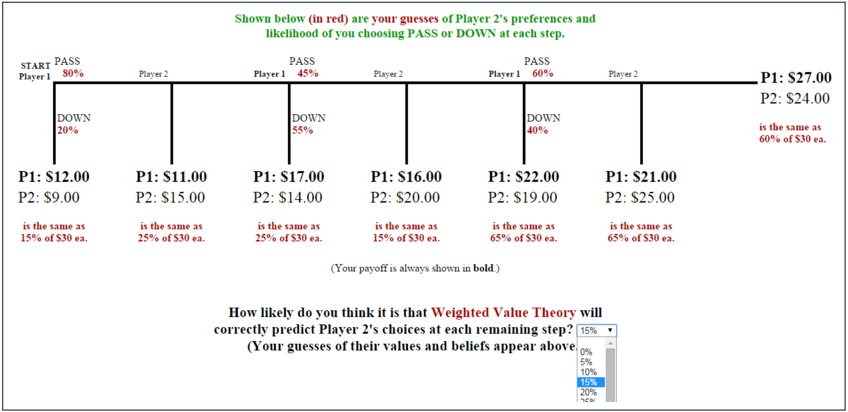
Screenshot: Eliciting Strategies



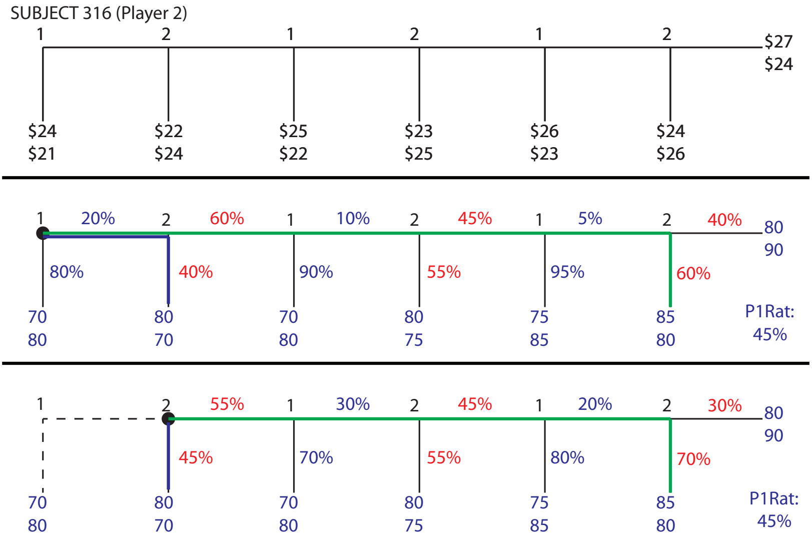
Screenshot: 1st-Order Beliefs



Screenshot: Belief of Rationality



Example Observation



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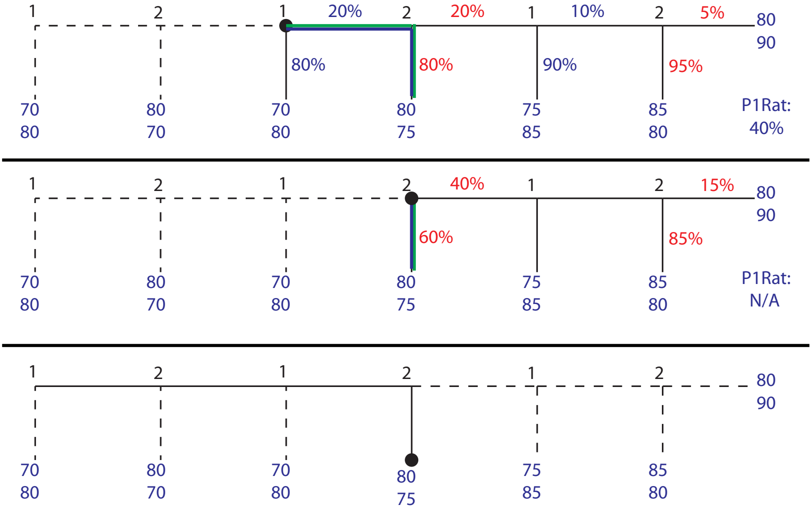
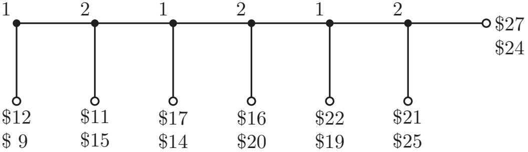
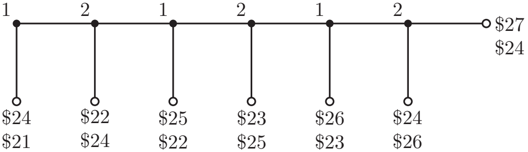


Centipede Treatments

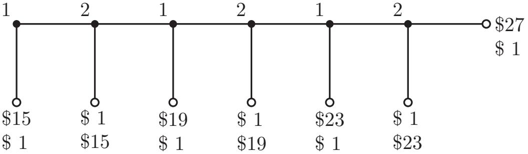
CENT-LO:
“Risk $1 to gain $5”



CENT-HI:
“Risk $2 to gain $1”



CENT-ALL:
“Risk ALL to gain $4”



Design Details

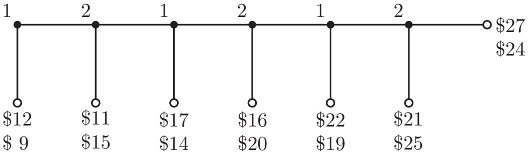
- OSU subject pool
- Custom software, ORSEE recruiting
- Between-subjects treatment (LO vs HI vs ALL)
- Play 4 periods. Elicitation only in last 2
  - Random rematching with feedback
- Only one game *or* elicitation is paid
- $19 average
- # subjects:

CENT-LO	CENT-HI	CENT-ALL
54	36	62

Results

The Story

CENT-LO:
"Risk \$1 to gain \$5"



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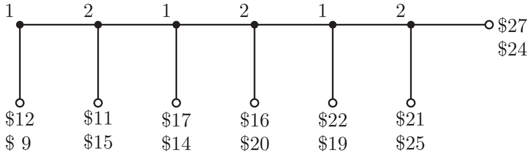

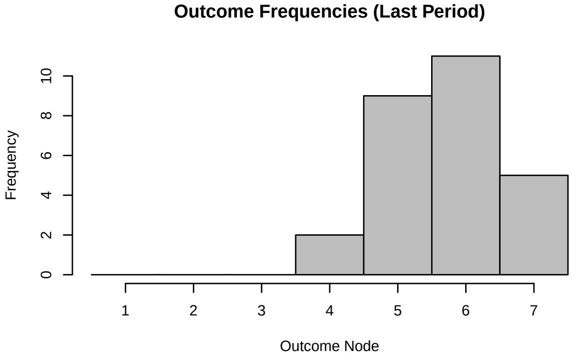

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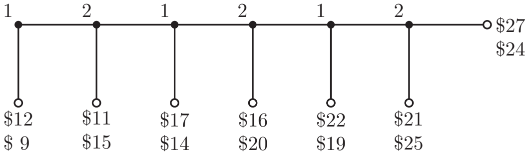
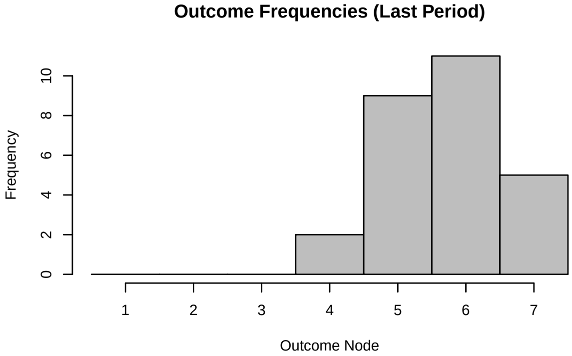
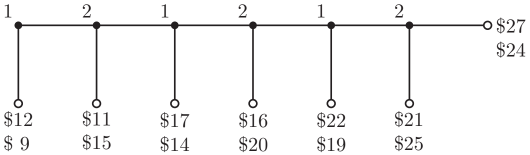




1. Node 7 freq
2. Story???

The Story

CENT-LO:
"Risk $1 to gain $5"

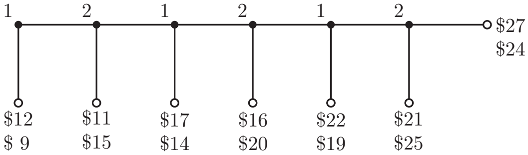


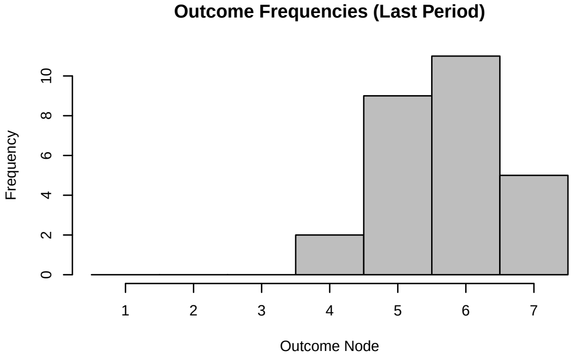
What do we learn from elicitation?

1. There are altruists who prefer Pass even if opponent will Take
    - Many people will give up $1 to give $6
2. Selfish people know that altruists are common
3. *Early nodes:* Selfish people Pass, knowing altruists Pass back
4. *Later nodes:* Selfish people Take. Altruists keep Passing

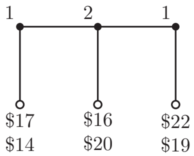
The Story

CENT-LO:
"Risk $1 to gain $5"

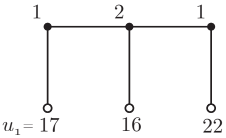




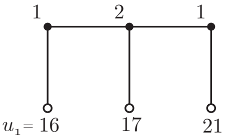
The Unit of Analysis: 3-Node Segments



A 3-Node Segment



Selfish u_1



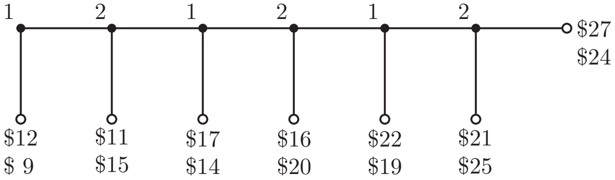
Altruist u_1

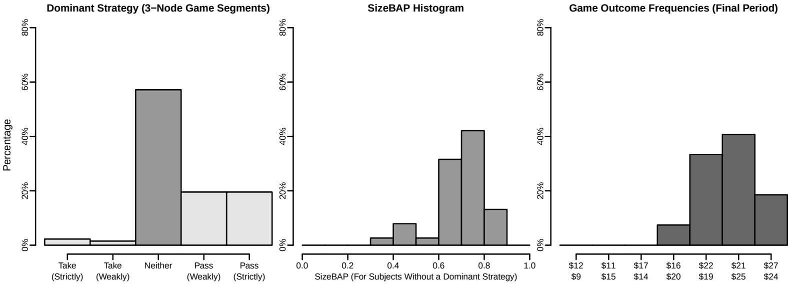
SizeBAP: A measure of the temptation to Pass

- Let $p =$ subjective prob. next mover will Pass
- **Selfish:** Pass is BR if $p \in [1/6, 1]$
  - SizeBAP for this u_1 is 5/6. Very likely to Pass.
- **Altruist:** Pass is BR if $p \in [0, 1]$ (strict Dom.Strat.)
  - SizeBAP for this u_1 is 1. Guaranteed to Pass.
- SizeBAP is a statistic for u_i (and nothing else)

Pooling All 3-Node Segments

CENT-LO Treatment:

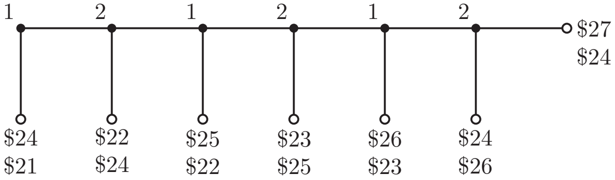


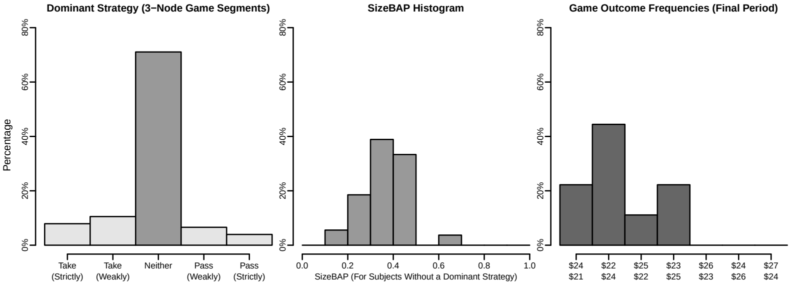


Selfish SizeBAP $\approx$ 0.833

CENT-HI

CENT-HI Treatment:

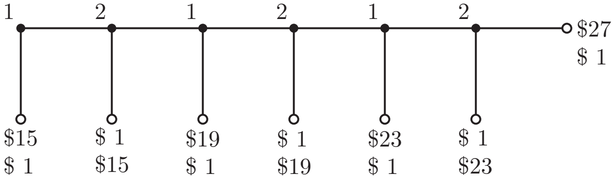


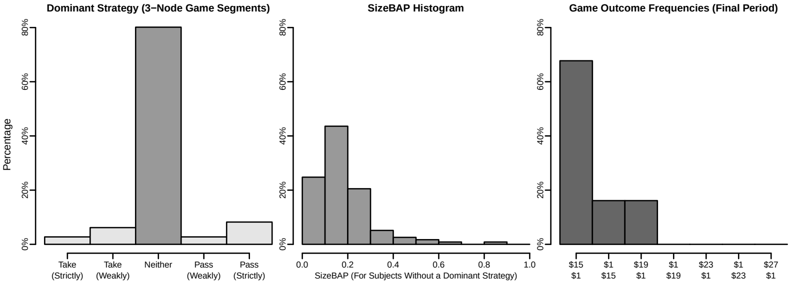


Selfish SizeBAP ≈ 0.333

CENT-ALL

CENT-ALL Treatment:





Selfish SizeBAP $\approx 0.22, 0.18, 0.15$

Verifying the Story: CENT-LO

CENT-LO:

1. Altruists exist
   - Pass is DomStrat in 43.7% of segments
2. Altruists pass
   - 89.7% of the time
   - $43.7\% \times 89.7\% = 39.2\%$ overall chance of Pass from altruists
3. Non-altruists believe Pass is reasonably likely
   - 54.8% have $Pr(Pass) > 39.2\%$ (median = 40%)
   - Self-similarity hides direct belief in altruism
4. Non-altruists BR to that belief
   - 83.8% play BR, given p_i^1 and u_i

Verifying the Story: CENT-HI

CENT-HI:

1. Altruists don't exist
   - Pass is DomStrat in 8.9% of segments
2. Altruists pass but they're very rare
   - Small sample: 6 out of 9
   - $8.9\% \times 66.6\% = 5.9\%$ overall chance of Pass
3. Non-altruists believe Pass is reasonably unlikely
   - Median = 20%
   - Self-similarity hides direct belief in altruism
4. Non-altruists BR to that belief
   - 58.5% play BR, given p_i^1 and u_i
   - Beliefs only elicited for those that Pass, which is a small sample

Verifying the Story: CENT-ALL

CENT-ALL:

1. Altruists don't exist
    - Pass is DomStrat in 8.7% of segments
2. Altruists pass but they're very rare
    - Small sample: 2 out of 12
    - $8.7\% \times 16.67\% = 1.45\%$ overall chance of Pass
3. Non-altruists believe Pass is reasonably unlikely
    - Median = 17.5%
    - Self-similarity hides direct belief in altruism
4. Non-altruists BR to that belief
    - 38.3% play BR, given p_i^1 and u_i
    - Beliefs only elicited for those that Pass, which is a small sample

Belief in Rationality & Backward Induction

- Does common belief in rationality $\Rightarrow$ backwards induction?
- Depends how people react to surprises (Reny 1993)
  - RCSBR: continue to believe in rationality after surprises
  - (Surprises $\Rightarrow$ belief in irrationality) $\Rightarrow$ Surprises!
- Surprise: Pr(Take)=100%, Pr(Rational)=100%, but then Pass

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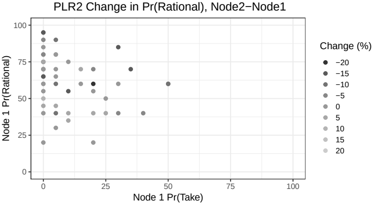
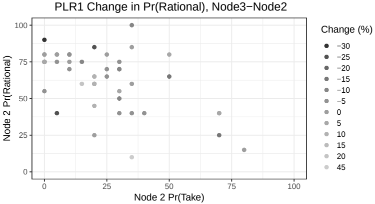
- Surprise: Pr(Take)=100%, Pr(Rational)=100%, but then Pass
- CENT-LO: Pr(Take) never near 100%
  - It's *not* a game of complete information!





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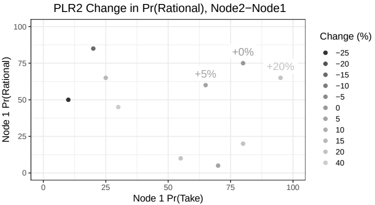
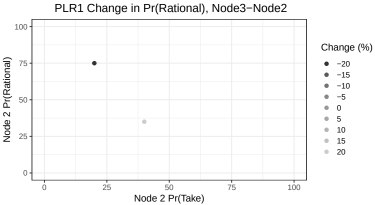
- Surprise: Pr(Take)=100%, Pr(Rational)=100%, but then Pass
- CENT-ALL: Very few surprises since everyone Takes!
  - Unsurprisingly, surprises are rare





Prisoners Dilemma

The Prisoners' Dilemma

	C	D
C	\$10, \$10	\$1, \$15
D	\$15, \$1	\$5, \$5

The Prisoners' Dilemma Game Form

- New treatment: SIM
- Five 2×2 games without feedback, random matching
- Elicitation in every game
- Pencil & paper
- $n = 150$

The Prisoners' Dilemma

	C	D
C	\$10, \$10	\$1, \$15
D	\$15, \$1	\$5, \$5

The Prisoners' Dilemma Game Form

30.4% play C.

Why???

The Prisoners' Dilemma

	C	D
C	\$10, \$10	\$1, \$15
D	\$15, \$1	\$5, \$5

The Prisoners' Dilemma Game Form

				$BR_i(p_i^{1s}\|u_i) = C$		$BR_i(p_i^{1s}\|u_i) = D$	
Pref. Type	$BR_i(C)$	$BR_i(D)$	% Subj.	$s_i = C$	$s_i = D$	$s_i = C$	$s_i = D$
Selfish	D	D	68.0%	–	–	18	**79**
Cond. Coop.	C	D	19.7%	15	5	3	6
Reverse	D	C	2.7%	1	2	0	1
Uncond. Coop.	C	C	9.5%	8	6	–	–

The Prisoners' Dilemma

	C	D
C	\$10, \$10	\$1, \$15
D	\$15, \$1	\$5, \$5

The Prisoners' Dilemma Game Form

				$BR_i(p_i^{1s}\|u_i) = C$		$BR_i(p_i^{1s}\|u_i) = D$	
Pref. Type	$BR_i(C)$	$BR_i(D)$	% Subj.	$s_i = C$	$s_i = D$	$s_i = C$	$s_i = D$
Selfish	D	D	68.0%	–	–	18	**79**
Cond. Coop.	C	D	19.7%	15	5	3	6
Reverse	D	C	2.7%	1	2	0	1
Uncond. Coop.	C	C	9.5%	8	6	–	–

Only 53% of cooperation (C) is rational

Failure of consequentialism or dominance

Iterated Dominance

A Dominance-Solvable Game Form

	L	R
U	\$10, \$ 5	\$15, \$15
D	\$5, \$10	\$1, \$1

A Dominance Solvable Game Form

- Row players: 100% play U
  - 71 of 75: U is a dominant strategy
  - 4 of 75: U is a best response
- Column players: 25% play L
  - Why???

A Dominance-Solvable Game Form

	L	R
U	\$10, \$ 5	\$15, \$15
D	\$5, \$10	\$1, \$1

A Dominance Solvable Game Form

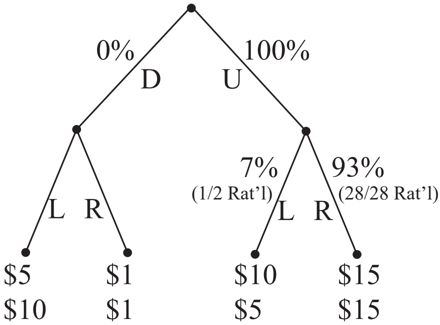
				$BR_i(p_i^{1s}\|u_i) = L$		$BR_i(p_i^{1s}\|u_i) = R$	
Pref. Type	$BR_i(U)$	$BR_i(D)$	% Subj.	$s_i = L$	$s_i = R$	$s_i = L$	$s_i = R$
Selfish	R	L	91.9%	0	0	14	53
DomStrat L	L	L	5.4%	3	1	–	–
DomStrat R	R	R	2.7%	–	–	1	1
Reversed	L	R	0%	0	0	0	0

Violation of consequentialism and/or EU

Conjecture: avoiding (\$1, \$1), despite stated preferences. Strategic uncertainty.

Sequential-Move DomSolv

SEQ treatment: $n = 60$



Irrationality *disappears* when strategic uncertainty is removed

Coordination

Asymmetric Coordination

	L	R
U	\$15, \$ 5	\$2, \$1
D	\$1, \$2	\$5, \$10

An Asymmetric Coordination Game Form

- Row: 93% play U
- Col: 49.3% play L
    - Why??? Beliefs?

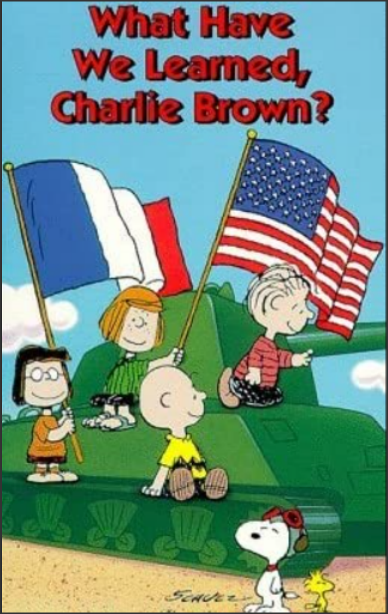
Asymmetric Coordination

	L	R
U	\$15, \$ 5	\$2, \$1
D	\$1, \$2	\$5, \$10

An Asymmetric Coordination Game Form

				$BR_1(p_1^{1S}\|u_1) = U$		$BR_1(p_1^{1S}\|u_1) = D$	
Row's Type	$BR_1(L)$	$BR_1(R)$	% Subj.	$s_1 = U$	$s_1 = D$	$s_1 = U$	$s_1 = D$
Selfish	U	D	95.8%	43	2	20	3
DomStrat U	U	U	4.2%	3	0	–	–

				$BR_2(p_2^{1S}\|u_2) = L$		$BR_2(p_2^{1S}\|u_2) = R$	
Col's Type	$BR_2(U)$	$BR_2(D)$	% Subj.	$s_2 = L$	$s_2 = R$	$s_2 = L$	$s_2 = R$
Selfish	L	R	93.0%	26	27	5	8
DomStrat L	L	L	7.0%	5	0	–	–


What Have
We Learned,
Charlie Brown?

Lessons

1. Most experiments are Bayesian games, not complete info
2. The story changes from one game to the next
3. Centipede game forms:
    - Altruists pass $\Rightarrow$ selfish pass
    - Backwards induction seems to work fine here
4. Prisoners' dilemma:
    - Non-consequential preference for cooperating
5. Beliefs are generally pretty accurate
6. Don't write a solo-authored paper post-tenure