Incentive Compatible Experiments: An Overview

P.J. Healy
Cast of characters:

• Yaron Azrieli (OSU)
• Chris Chambers (Georgetown)
• Nicolas Lambert (MIT)
• John Kagel (OSU)
• Kirby Nielsen (Caltech)
• Marina Agranov (Caltech)
• Alex Brown (Texas A&M)
• Greg Leo (Vanderbilt)
• Sam(antha) Stelnicki (OSU student)
Part 1: General experiments

Part 2: Belief elicitation
Goal of any experiment: elicit (coarse) information about $\geq$
Goal of any experiment: elicit (coarse) information about \( x \)

Requirement: Incentive compatibility
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Requirement: Incentive compatibility

Classic mechanism design problem, except:
1. Don’t have any particular SCF in mind
   • Any IC payment is fine
2. Allow random mechanisms
3. *Strict* incentive compatibility
Incentives in Experiments

“Incentives in Experiments”
Azrieli, Chambers & Healy
J. Political Economy (2018)

• Experiment: sequence of choices from menus
• Goal: observe their “true” choices (preferences)
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    - Armantier & Treich (2013)
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  • Examples where it’s not IC with RDU
    • Holt (1986), Karni & Safra (1987), Segal (1988), others
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  - Hadn’t been proven either way
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- Which do researchers use?
  - Survey from 2011:
    - Pay all: 56%
    - RPS: 25%
    - Pay some: 13%
    - Other: 6%
Incentives in Experiments

Framework for Analyzing IC:

- Choice objects: $x, y, z \in X$
- (Strict complete) preference: $\succeq \in O$
- Decision problems: $D = (D_1, \ldots, D_k)$, each $D_i \subseteq X$
- “True” choices: $\mu_i(\succeq) \in D_i$
  - $\mu_i(\succeq) \succeq x \ \forall x \in D_i$
- Stated choices (messages): $m_i \in D_i \quad m = (m_1, \ldots, m_k)$
- Payment mechanism: $\phi(m) \in \mathcal{P}(X)$
  - Payment objects: $\mathcal{P}(X)$
- Experiment: $(D, \phi)$
Definition
An experiment $(D, \phi)$ is **incentive compatible** if, for every $\succeq$ and every $m \neq \mu(\succeq)$,

\[ \phi(\mu(\succeq)) \text{ is strictly preferred to } \phi(m). \]
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- RPS: acts
  - \(\Omega = \{\omega_1, \omega_2\}\)
  - \(\phi(m)(\omega_1) = \{\text{Left shoe}\}\)
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\(\succeq\) says **nothing** about how these objects are ranked!
Incentives in Experiments

- Preference Extension: $\succeq$ on $X$, $\succeq^*$ on $\mathcal{P}(X)$.
- Example: $\succeq$ over money, $\succeq^*$ EU over lotteries

**Definition**
An experiment $(D, \phi)$ is **incentive compatible** if, for every $\succeq$ and every $m \neq \mu(\succeq)$,

$$\phi(\mu(\succeq)) \succ^* \phi(m).$$

**Theorem**
*If no restrictions are placed on $\succeq^*$ then an experiment is IC if and only if there is one decision problem and $\phi(m_1) = m_1$.***

**Corollary**
*If $k > 1$ we must talk about $\succeq^*$ and how it relates to $\succeq$.***
Incentives in Experiments

When is the Pay-All mechanism incentive compatible?

• Need an assumption about \( \preceq^* \) over bundles

• No Complementarities at the Top (NCaT):

  • “The bundle of your favorites is your favorite bundle”
  • Apple \( \preceq \) Left shoe & Banana \( \preceq \) Right shoe \( \Rightarrow \) \{Apple, Banana\} is \( \preceq^* \)-maximal
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Assume $D = (D_1, \ldots, D_k)$ is non-redundant ($\cap_i D_i = \emptyset$).
If $\succeq^*$ satisfies NCaT (and nothing else is assumed) then Pay-All is the **only** IC mechanism.
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*Redundant case just adds flexibility on “intransitive” messages.*
Incentives in Experiments

When is the RPS mechanism incentive compatible?

• Need an assumption about \( \succeq^* \) over acts

• The RPS mechanism has the "truth dominates lies" property

• Monotonicity: \( \succeq^* \) respects statewise dominance
Incentives in Experiments

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- **Monotonicity:** $\succeq^*$ respects statewise dominance (w.r.t. $\succeq$)

\[ f(\omega) \succeq g(\omega) \forall \omega \implies f \succeq^* g \]
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*Redundant case adds flexibility on “surely-identified” sets.
**Can also add states that pay a fixed prize.
Summary

Pay All: No Complementarities

RPS: Monotonicity w.r.t. statewise dominance
Incentives in Experiments

“Incentives in Experiments with Objective Lotteries”
Azrieli, Chambers & Healy
Experimental Economics (2020)

- RPS with lotteries instead of acts
  - Assume an objective $p \in \Delta(\Omega)$
- More restrictive setting $\Rightarrow$ more IC mechanisms??
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**Theorem**
Assume Monotonicity w.r.t. FOSD (and nothing else).
1. Non-redundant: Same as before (only RPS)
2. Redundant: Added flexibility on “surely-identified” sets; not useful
When Can We Use RPS?

Things we should worry about with Monotonicity/RPS:

Things I don’t think we need to worry much about:
Suppose $X$ are multi-agent payments. $\mathcal{P}(X)$ are lotteries over $X$. Ex-ante fairness $\Rightarrow$ monotonicity violation

Example: Machina’s mom
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What about the RDU examples where RPS wasn’t IC??

Suppose $X$ are lotteries, $\mathcal{P}(X)$ are compound lotteries. Monotonicity + reduction $\Rightarrow \succeq$ satisfies independence (EUT)!
On Monotonicity

What about the RDU examples where RPS wasn’t IC??

Suppose $X$ are lotteries, $\mathcal{P}(X)$ are compound lotteries. Monotonicity + reduction $\Rightarrow$ satisfies independence (EUT)!

Reduction + Non-EU $\Rightarrow$ Monotonicity $\Rightarrow$ RPS may not be IC

The counter-examples all assume Reduction + Non-EU

Halevy (2007): those who reduce are EU maximizers! ✓
When Can We Use RPS?

Things **we should worry about** with Monotonicity/RPS:

- Ex-ante fairness

Things *I don’t think we need to worry much about*:

- Non-expected utility + reduction
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Suppose $X$ are lotteries, $P(X)$ are compound lotteries. Monotonicity + reduction $\implies \succeq$ satisfies independence (EUT)!

Suppose $X$ are acts, $P(X)$ are lotteries over acts (AA). Monotonicity + order-reversal $\implies \succeq$ is ambiguity-neutral!
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Ability to “hedge” away ambiguity...

Should we add ambiguity hedging to the “worry” list??
On Hedging

“A Direct Test of Hedging”
Healy & Stelnicki
Work in Progress

\[ D_1 = \{ \$2.00 \text{ if Red from K, } \$2.10 \text{ if Red from U} \} \]
\[ D_2 = \{ \$2.00 \text{ if Blue from K, } \$2.10 \text{ if Blue from U} \} \]
On Hedging

Picking $UU$:

**Ambiguous 50-50 Lottery For Sure**

$KK \succ^* UU$

$UU \succ^* KK$
On Hedging

• Do people “see” the hedging opportunity?

- Baillon Halevy & Li (forthcoming): Yes.

Our design:

• “I think the probability of me winning a bonus payment is between $\%$ and $\%$.” (incentivized)

• Hedgers: Pick UU, say “between 50% and 50%.”

• True even if the jars aren’t 50-50
On Hedging

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15% $UU$ contains:

- Ambiguity Loving & Monotonicity
- Ambiguity Neutral & $\sim50$-50 beliefs & Monotonicity
- Ambiguity Averse & Hedging

$UK > KU \Rightarrow$ red more likely $\Rightarrow$ Ask One should differ
Belief ranges of the 15% who choose *UU* in Ask Both:

\[ \sim 15\% \text{ are consistent with hedging. Or, } \sim 2\% \text{ overall.} \]
On Hedging

Belief ranges of the 19% who choose KK.

\[
\frac{1}{2}(\frac{1}{8}) + \frac{1}{2}(\frac{7}{8}) = \frac{1}{2}
\]

21% say [50, 50]. 17% say [1/8, 7/8].
On Hedging

Back to UU:

Could be some non-reducers here, but Order Reversal fails
When Can We Use RPS?

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Our conjecture: Preference for randomization (violates Monotonicity)
Randomization

“Stable Randomization”
Agranov, Healy & Nielsen
Working Paper

Bet A: You receive $5 if the number drawn is from 1-16, and $25 if it is from 17-20.

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Randomization

- “PM” Questions: dominance
- “RS” Questions: risky-safe

Percentage of people who mix:

Mixing highly correlated across decisions and games. “Mixing types”
When Can We Use RPS?

Things *we should worry about* with Monotonicity/RPS:

- Ex-ante fairness
- Repeated choices (same or similar)

Things *I don’t think we need to worry much about*:

- Non-expected utility + reduction
- Ambiguity hedging
## Separated Decisions

"Separated Decisions"

Brown & Healy

*EER* (2018)

<table>
<thead>
<tr>
<th>Row #</th>
<th><strong>Option A</strong></th>
<th></th>
<th><strong>Option B</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Balls 1-10 pay $10 (50% chance of $10)</td>
<td>Balls 11-20 pay $5 (50% chance of $5)</td>
<td>or</td>
<td>Ball 1 pays $15 (5% chance of $15)</td>
</tr>
<tr>
<td>2</td>
<td>Balls 1-10 pay $10 (50% chance of $10)</td>
<td>Balls 11-20 pay $5 (50% chance of $5)</td>
<td>or</td>
<td>Balls 1-2 pay $15 (10% chance of $15)</td>
</tr>
<tr>
<td>3</td>
<td>Balls 1-10 pay $10 (50% chance of $10)</td>
<td>Balls 11-20 pay $5 (50% chance of $5)</td>
<td>or</td>
<td>Balls 1-3 pay $15 (15% chance of $15)</td>
</tr>
<tr>
<td>4</td>
<td>Balls 1-10 pay $10 (50% chance of $10)</td>
<td>Balls 11-20 pay $5 (50% chance of $5)</td>
<td>or</td>
<td>Balls 1-4 pay $15 (20% chance of $15)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>19</td>
<td>(50% chance of $10)</td>
<td>(50% chance of $5)</td>
<td>or</td>
<td>(90% chance of $15)</td>
</tr>
<tr>
<td>20</td>
<td>Balls 1-10 pay $10 (50% chance of $10)</td>
<td>Balls 11-20 pay $5 (50% chance of $5)</td>
<td>or</td>
<td>Balls 1-19 pay $15 (95% chance of $15)</td>
</tr>
<tr>
<td>20</td>
<td>Balls 1-10 pay $10 (50% chance of $10)</td>
<td>Balls 11-20 pay $5 (50% chance of $5)</td>
<td>or</td>
<td>All Balls pay $15 (100% chance of $15)</td>
</tr>
</tbody>
</table>
Separated Decisions

Direct test of Monotonicity:

- List-RPS: See all rows, RPS payment
- List-R14: See all rows, only paid for row 14
Direct test of Monotonicity:

- List-RPS: See all rows, RPS payment
- List-R14: See all rows, only paid for row 14

<table>
<thead>
<tr>
<th></th>
<th>% Risky on Row 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>List-RPS</td>
<td>52%</td>
</tr>
<tr>
<td>List-14</td>
<td>70%</td>
</tr>
</tbody>
</table>

List formatting violates monotonicity.
Separated Decisions

Direct test of Monotonicity:

• Separated-RPS:
  See all rows on separate screens in random order, RPS payment

• Separated-R14:
  See all rows on separate screens in random order, pay row 14
Separated Decisions

Direct test of Monotonicity:

• Separated-RPS:
  See all rows on separate screens in random order, RPS payment

• Separated-R14:
  See all rows on separate screens in random order, pay row 14

<table>
<thead>
<tr>
<th></th>
<th>% Risky on Row 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep-RPS</td>
<td>59%</td>
</tr>
<tr>
<td>Sep-R14</td>
<td>56%</td>
</tr>
</tbody>
</table>

Separated formatting restores monotonicity.
Multiple switching: 5% → 33%, but usually very minor
Recommendation: Separate your decisions!
When Can We Use RPS?

Things we should worry about with Monotonicity/RPS:

• Ex-ante fairness
• Repeated choices (same or similar)
• Showing choices all together

Things I don’t think we need to worry much about:

• Non-expected utility + reduction
• Ambiguity hedging
Things we should worry about with Monotonicity/RPS:

- Ex-ante fairness
- Repeated choices (same or similar)
- Showing choices all together

Things I don’t think we need to worry much about:

- Non-expected utility + reduction
- Ambiguity hedging

That’s it!
“Constrained Preference Elicitation”
Azrieli, Chambers & Healy
*Theoretical Economics* (2021)

Structure theorems on what we can learn about $\succeq$ from any experiment.
“Minimal Experiments”
Healy & Leo
Work in Progress

Given: Something you want to learn about $\geq$.

- Example: is $p(E)$ in $[0, \frac{1}{3})$, $[\frac{1}{3}, \frac{2}{3})$, or $[\frac{2}{3}, 1]$?

Step 1: Which experiments would elicit that?

Step 2: Which experiment is the “simplest”?

- $D_1 = \{\text{$10 if } E, \text{ $10 if } E^C, \text{ $10 w/ 66\%}\}$
Part 1: General experiments

Part 2: Belief elicitation
Belief Elicitation Mechanisms:

- Quadratic scoring rule (QSR; Brier 1950)
  - Logarithmic, spherical...
  - QSR corrected for risk aversion (Harrison et al. 2014)
- Binarized scoring rules (BSR; Savage 1971, Hossain & Okui 2013)
- BDM for probabilities (Marschak 1963, Grether 1981)
  - Clock BDM (Karni 2009)
- Multiple Price List (MPL; Holt & Smith 2016)
What Do The Data Say?

- Offerman & Sonnemans (2004): QSR ∼ None
- Armantier & Treich (2013): QSR ≻ None
- Huck & Weizsacker (2002): QSR ≻ BDM
- Hollars et al. (2010): BDM ≻ QSR
- Hao & Houser (2012): BDM-Clock ≻ BDM
- Hossain & Okui (2013): BSR ≻ QSR
- Harrison et al. (2014): BSR ∼ QSR-Corr ≻ QSR
- Holt & Smith (2016); MPL ≻ BDM

Best performers: BSR and MPL
Our Motivations

- Offerman & Sonnemans (2004): QSR $\sim$ None
- Armantier & Treich (2013): QSR $\succ$ None
- Huck & Weizsacker (2002): QSR $\succ$ BDM
- Hollars et al. (2010): BDM $\succ$ QSR
- Hao & Houser (2012): BDM-Clock $\succ$ BDM
- Hossain & Okui (2013): BSR $\succ$ QSR
- Harrison et al. (2014): BSR $\sim$ QSR-Corr $\succ$ QSR
- Holt & Smith (2016); MPL $\succ$ BDM

**Motivation:** Compare MPL to BSR in theory and in the lab
Suppose $X \in \{0, 1\}$.  
Want to elicit $p = Pr(X = 1)$.  
Subject announces $q$, gets paid:

$$S(q, X) = 1 - (X - q)^2$$

IC requires risk neutrality.

Solution: pay in probabilities
Proof of Incentive Compatibility:

- **Announce $p$:**
  - If $X=1$, the payoff is $S(p, 1)$.
  - If $X=0$, the payoff is $1-S(p, 1)$.
  - If $X=1$, the payoff is $S(p, 0)$.
  - If $X=0$, the payoff is $1-S(p, 0)$.

- **Announce $q$:**
  - If $X=1$, the payoff is $S(q, 1)$.
  - If $X=0$, the payoff is $1-S(q, 1)$.
  - If $X=1$, the payoff is $S(q, 0)$.
  - If $X=0$, the payoff is $1-S(q, 0)$.

This requires “Subjective-Objective Reduction”

- Weakening of ROCL: Applies only to two-prize lotteries
### Multiple Price Lists (MPL)

<table>
<thead>
<tr>
<th>Row#</th>
<th><strong>Option A</strong></th>
<th>OR</th>
<th><strong>Option B</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } 1%$</td>
</tr>
<tr>
<td>2</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } 2%$</td>
</tr>
<tr>
<td>$q$</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } q%$</td>
</tr>
<tr>
<td>$q + 1$</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } q + 1%$</td>
</tr>
<tr>
<td>$q + 2$</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } q + 2%$</td>
</tr>
<tr>
<td>$q + 3$</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } q + 3%$</td>
</tr>
<tr>
<td>$q + 4$</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } q + 4%$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>99</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } 99%$</td>
</tr>
<tr>
<td>100</td>
<td>$8 \text{ if } X = 1$</td>
<td>or</td>
<td>$8 \text{ w/ prob } 100%$</td>
</tr>
</tbody>
</table>

Choose Option A or Option B (single switch point $q$)
One row randomly selected for payment
<table>
<thead>
<tr>
<th>Row#</th>
<th>Option A</th>
<th>OR</th>
<th>Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob 1%$</td>
</tr>
<tr>
<td>2</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob 2%$</td>
</tr>
<tr>
<td>$q$</td>
<td>$8 if X = 1$</td>
<td>or</td>
<td>$8 w/ prob q%$</td>
</tr>
<tr>
<td>$q+1$</td>
<td>$8 if X = 1$</td>
<td>or</td>
<td>$8 w/ prob q + 1%$</td>
</tr>
<tr>
<td>$q+2$</td>
<td>$8 if X = 1$</td>
<td>or</td>
<td>$8 w/ prob q + 2%$</td>
</tr>
<tr>
<td>$q+3$</td>
<td>$8 if X = 1$</td>
<td>or</td>
<td>$8 w/ prob q + 3%$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>99</td>
<td>$8 if X = 1$</td>
<td>or</td>
<td>$8 w/ prob 99%$</td>
</tr>
<tr>
<td>100</td>
<td>$8 if X = 1$</td>
<td>or</td>
<td>$8 w/ prob 100%$</td>
</tr>
</tbody>
</table>

“Multiple Price List” (MPL) version of BDM for probabilities
Holt & Smith (2016)
### Multiple Price Lists (MPL)

<table>
<thead>
<tr>
<th>Row#</th>
<th>Option A</th>
<th>OR</th>
<th>Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob 1%</td>
</tr>
<tr>
<td>2</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob 2%</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td></td>
<td>...</td>
</tr>
<tr>
<td>$q$</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob q%$</td>
</tr>
<tr>
<td>$q+1$</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob q + 1%$</td>
</tr>
<tr>
<td>$q+2$</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob q + 2%$</td>
</tr>
<tr>
<td>$q+3$</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob q + 3%$</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td></td>
<td>...</td>
</tr>
<tr>
<td>99</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob 99%$</td>
</tr>
<tr>
<td>100</td>
<td>$8 if $X = 1$</td>
<td>or</td>
<td>$8 w/ prob 100%$</td>
</tr>
</tbody>
</table>

If you lie, you get the less-preferred option on some rows I.C. as long as subject respects **statewise dominance** in rows...
Proposition:

All BSRs are I.C.

\[ \iff \]

Subjective-Objective Reduction

\[ \iff \]

Statewise Dominance

\[ \iff \]

Any MPL is I.C.
Our Experiment

- Compare **BSR** to **MPL**
- Put subjects in teams of two, working together via chat
- Scan chat transcripts for (1) true beliefs, (2) manipulation
- Variety of questions (objective, subjective)
  - Focus here on objective questions
Q3: What do you think is the probability (from 0% to 100%) that a RED marble will be drawn? **60%**

Your answer to Q3 determines what you choose in each row below. One row will be chosen at random for payment.

<table>
<thead>
<tr>
<th>Pick</th>
<th>Option A</th>
<th>OR</th>
<th>Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 57:</td>
<td>$8 if RED is drawn</td>
<td>OR</td>
<td>$8 with probability 57%</td>
</tr>
<tr>
<td>Row 58:</td>
<td>$8 if RED is drawn</td>
<td>OR</td>
<td>$8 with probability 58%</td>
</tr>
<tr>
<td>Row 59:</td>
<td>$8 if RED is drawn</td>
<td>OR</td>
<td>$8 with probability 59%</td>
</tr>
<tr>
<td>Row 60:</td>
<td>$8 if RED is drawn</td>
<td>OR</td>
<td>$8 with probability 60%</td>
</tr>
<tr>
<td>Row 61:</td>
<td>$8 if RED is drawn</td>
<td>OR</td>
<td>$8 with probability 61%</td>
</tr>
<tr>
<td>Row 62:</td>
<td>$8 if RED is drawn</td>
<td>OR</td>
<td>$8 with probability 62%</td>
</tr>
<tr>
<td>Row 63:</td>
<td>$8 if RED is drawn</td>
<td>OR</td>
<td>$8 with probability 63%</td>
</tr>
</tbody>
</table>

Remember: you maximize your overall probability of getting $8 when you report truthfully.

Confirm and lock in your choices:

[Lock In Your Choices]

**Note:** subjects saw the same phrase in all three treatments.
The Mechanism Interfaces: BSR

Q3: What do you think is the probability (from 0% to 100%) that a RED marble will be drawn? 60%

Your answer to Q3 determines your payment probabilities below:

<table>
<thead>
<tr>
<th>If RED is drawn</th>
<th>you get $8 with probability 72%</th>
</tr>
</thead>
<tbody>
<tr>
<td>If BLUE is drawn</td>
<td>you get $8 with probability 62%</td>
</tr>
</tbody>
</table>

If the true probability is 60% then your payment probabilities for each possible report are:

<table>
<thead>
<tr>
<th>If You Report</th>
<th>Overall Probability</th>
<th>You get $8 with probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td></td>
<td>67.88%</td>
</tr>
<tr>
<td>26%</td>
<td></td>
<td>67.92%</td>
</tr>
<tr>
<td>27%</td>
<td></td>
<td>67.95%</td>
</tr>
<tr>
<td>28%</td>
<td></td>
<td>67.98%</td>
</tr>
<tr>
<td>29%</td>
<td></td>
<td>68.00%</td>
</tr>
<tr>
<td>30%</td>
<td></td>
<td>68.03%</td>
</tr>
<tr>
<td>31%</td>
<td></td>
<td>68.05%</td>
</tr>
<tr>
<td>32%</td>
<td></td>
<td>68.08%</td>
</tr>
<tr>
<td>33%</td>
<td></td>
<td>68.10%</td>
</tr>
<tr>
<td>34%</td>
<td></td>
<td>68.13%</td>
</tr>
<tr>
<td>35%</td>
<td></td>
<td>68.15%</td>
</tr>
<tr>
<td>36%</td>
<td></td>
<td>68.18%</td>
</tr>
<tr>
<td>37%</td>
<td></td>
<td>68.20%</td>
</tr>
<tr>
<td>38%</td>
<td></td>
<td>68.23%</td>
</tr>
<tr>
<td>39%</td>
<td></td>
<td>68.25%</td>
</tr>
<tr>
<td>40%</td>
<td></td>
<td>68.28%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Remember: you maximize your overall probability of getting $8 when you report truthfully.

Confirm and lock in your choices: [Lock In Your Choices]

Note: subjects saw the same phrase in all three treatments.
Q3: What do you think is the probability (from 0% to 100%) that a RED marble will be drawn? 60%

Time remaining: 199  PARTNER: current choice:  locked in
Pause timer:  Skip 30s

Remember: you maximize your overall probability of getting $8 when you report truthfully.

Confirm and lock in your choices:

Note: subjects saw the same phrase in all three treatments
Teams Interface

- Use chat window to communicate
- Must lock in the same number to proceed
- If time runs out, one choice is randomly used
Misreporting Rate: Objective Probabilities

Rate of Misreporting

<table>
<thead>
<tr>
<th></th>
<th>MPL</th>
<th>BSR</th>
<th>NoInfo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INDIVIDUALS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marble Counting (60%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coin Flip (50%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Update w/ 1 Draw</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Update w/ 2 Draws</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TEAMS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marble Counting (60%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coin Flip (50%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Update w/ 1 Draw</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Update w/ 2 Draws</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chat Encoding

Two Types of Evidence of IC Failures:

**Deviate**  Discuss deviating from their belief
  • May not specify *why* they’re deviating

**Manipulate**  Discuss manipulation of payoffs
  • May not end up deviating from their belief

**Warning:** So far, only encoded by me
Two Types of Evidence of IC Failures:

**Devi ate** Discuss deviating from their belief
  - May not specify *why* they’re deviating

**Manipulate** Discuss manipulation of payoffs
  - May not end up deviating from their belief

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>MPL</th>
<th>BSR</th>
<th>NoInfo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviate</td>
<td>2/33</td>
<td>2/34</td>
<td>0/27</td>
</tr>
<tr>
<td>Manipulate</td>
<td>1/33</td>
<td>5/34</td>
<td>0/27</td>
</tr>
</tbody>
</table>
Marble Counting Chat

<table>
<thead>
<tr>
<th>ID#181</th>
<th>MPL</th>
<th>ID#187</th>
</tr>
</thead>
<tbody>
<tr>
<td>i have 12 for red and 8 for blue</td>
<td>12, 20, and 75%? yes</td>
<td>75 sounds good with me</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td>75%</td>
</tr>
</tbody>
</table>
**Coin Flip Chat**

<table>
<thead>
<tr>
<th>ID#257</th>
<th>BSR</th>
<th>ID#260</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>50 ?</td>
</tr>
<tr>
<td>id say 60</td>
<td></td>
<td>Why</td>
</tr>
<tr>
<td>cause heads is always more likely</td>
<td></td>
<td>Thats just false</td>
</tr>
<tr>
<td>55 is a compromise</td>
<td></td>
<td>Which is also wrong but whatever</td>
</tr>
<tr>
<td>55%</td>
<td></td>
<td>55%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ID#357</th>
<th>BSR</th>
<th>ID#365</th>
</tr>
</thead>
<tbody>
<tr>
<td>(no chat)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75%</td>
<td></td>
<td>75%</td>
</tr>
</tbody>
</table>
\[ \frac{12}{20} = 60\% \]

<table>
<thead>
<tr>
<th>ID#352</th>
<th>MPL</th>
<th>ID#353</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 red marbles, 20 total, so 60%</td>
<td></td>
<td>60%</td>
</tr>
<tr>
<td>Yea but I am thinking should we really put the correct number for probability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I mean yeah i think</td>
<td></td>
<td>alright</td>
</tr>
<tr>
<td>Although its random, its the best “odds” then</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60%</td>
<td></td>
<td>60%</td>
</tr>
</tbody>
</table>
i noticed that the higher you make their percentage, the higher our probability percentage gets

yeah that’s true but the closer to 50, the more equal the probs

i say we go for a big one

<table>
<thead>
<tr>
<th>ID#407</th>
<th>BSR</th>
<th>ID#414</th>
</tr>
</thead>
<tbody>
<tr>
<td>hi</td>
<td></td>
<td>hi</td>
</tr>
<tr>
<td>i noticed that the higher you make their percentage, the higher our probability percentage gets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>yeah that’s true but the closer to 50, the more equal the probs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i say we go for a big one</td>
<td></td>
<td></td>
</tr>
<tr>
<td>85</td>
<td></td>
<td>85</td>
</tr>
</tbody>
</table>
• Chats conclude they’re **not** successfully manipulating
  • Maybe slightly more *attempts* in BSR?

• NoInfo performs well when easy, worst when hard

• Implication: Mechanism details can be distracting **or** useful
  • Easy problems: details get in the way, ↑ mistakes
  • Harder problems: details maybe help focus, ↓ mistakes
Summary

• Theory:
  1. MPL has superior IC properties
  2. Some scoring rules are equiv. to an MPL, but not BQSR

• Empirics:
  1. MPL and BSR perform similarly
  2. NoInfo works well when easy, not when hard
  3. Very little evidence of manipulation
     • Subjects are confused/overwhelmed, not manipulating
"Coarse Elicitation"
Healy & Leo
Work in Progress

Expected Payoff (True Belief = 50%)
“Coarse Elicitation”
Healy & Leo
Work in Progress
"Midpoint Property"

**Theorem:** The only* differentiable scoring rule that satisfies the midpoint property for *any* grid is the quadratic scoring rule.

*Up to a rescaling.

Simple alternative: Coarse MPL
“Elicitability”
Azrieli, Chambers, Healy & Lambert
Work in Progress

- **Goal:** elicit subjective $p(E)$ for some event $E \subseteq \Omega$
- **Problem:** states $\omega \in \Omega$ are not observable! Only signals $y \in Y$.

Examples:
- Climate change
- Beliefs in repeated PD w/ private monitoring
- Vaccine effectiveness

**Question:** can we still learn beliefs over $\Omega$ using only $Y$?
**Vaccine Example (of course)**

**State:** efficacy. \( \omega \in \Omega = \{0, 1/2, 1\} \)

**Agent:** medical researcher. Has belief \( p \in \Delta(\Omega) \)

**Principal:** management. Wants to learn about \( p \)

**Signal:** outcome of 1 trial. \( y \in Y = \{S, H\} \)

**Info Structure:** \( \Pi(y|\omega) \)

\[
\begin{array}{c|cc}
\Pi & S & H \\
\hline
\Omega & 0 & 1/2 & 1 \\
\hline
0 & 1 & 0 \\
0.5 & 0.5 & 0.5 \\
1 & 0 & 1 \\
\end{array}
\]

**Induced Belief on \( Y \):** \( p_{\Pi}(S) = \tilde{p} \cdot \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} \)
## Vaccine Example: A Tale of Three Agents

<table>
<thead>
<tr>
<th></th>
<th>Sick (S)</th>
<th>Healthy (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ann’s p</strong></td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

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<tr>
<th></th>
<th>Sick (S)</th>
<th>Healthy (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bob’s p</strong></td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>1/2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1/2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Sick (S)</th>
<th>Healthy (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Charlie’s p</strong></td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>1/3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1/3</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1/3</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Vaccine Example

\[ p_\Pi(S) = \]

\[
p(1) = \begin{cases} 
0.9 & 
0.8 \\
0.7 & 
0.6 \\
0.5 & 
0.4 \\
0.3 & 
0.2 \\
0.1 & 
0 
\end{cases} 
\]

\[ p(0) \quad p(1/2) \]

\[ q \]

\[ p \]
Given $\Pi$, what can we learn about $p$?

**Main Result:**
$\Pi$ generates a partition of $\Delta(\Omega)$ based on $p_{\Pi}$.
$p$ and $q$ can be distinguished iff $p_{\Pi} \neq q_{\Pi}$

**Assumptions:**
1. $\Pi$ is known
2. $p_{\Pi}$ is derived from $p$ and $\Pi$ via *reduction*
3. $p_{\Pi}$ can be elicited (BQSR, MPL, ...)


Vaccine Example: Two Subjects

Now suppose vaccine trial has two patients (iid) $Y = \{0, 1, 2\}$ gives # of Healthy patients

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1/2</td>
<td>0.25</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Three linearly independent columns! $\Pi$ has full rank.

$p_\Pi = \bar{p} \cdot \Pi \implies p_\Pi \cdot \Pi^{-1} = \bar{p}$!

Full rank $\Rightarrow$ We can perfectly back out any belief!
In general, with $k$ observations, you learn the first $k$ moments of $p$

Three states: two moments is enough to learn $p$

$|\Omega| = n$: then $n - 1$ observations gives you $p$
Other Stuff We Know

• Can elicit median of $\omega \leftrightarrow$ can elicit entire $p$
• Can add covariates
  • $\Pi_{\text{man}}$ and $\Pi_{\text{woman}}$, $Y = (Y_{\text{man}} \times Y_{\text{woman}})$
• Infinite states & signals
  • Gaussian linear model: $y = \beta_0 + \beta_1 x + \varepsilon$
    • Full rank! One observation gives entire $p$
  • Non-parametric linear model: $E[y|x] = \beta_0 + \beta_1 x$
    • One obs: $E_p[\beta_0]$, $E_p[\beta_1]$.
    • Two obs: $Var_p[\beta_0]$, $Var_p[\beta_1]$.
    • ... 
  • Probit: $y = \mathbb{1}_{\{\beta_0 + \beta_1 x + \varepsilon > 0\}}$
    • Need infinite data to get $E_p[\beta_0]$, $E_p[\beta_1]$!!
• New ordering of Information Structures
  • “$\Pi_2$ elicits more than $\Pi_1$”
  • Blackwell Dominance $\Rightarrow$ Elicits More
• BQSR and MPL both work fine
• BQSR and MPL both work fine
• Manipulation doesn’t seem to be a huge problem
Summary of Belief Elicitation

- BQSR and MPL both work fine
- Manipulation doesn’t seem to be a huge problem
- You can do coarse elicitation
Summary of Belief Elicitation

- BQSR and MPL both work fine
- Manipulation doesn’t seem to be a huge problem
- You can do coarse elicitation
- Unobservable states limits what we can learn
  - More observations helps
Sorry!!
Do Incentives Matter?

Overarching goal: Strict incentive compatibility of experiments

Why pay?

• Real payments $\uparrow$ risk aversion
  • Holt & Laury (2005): hypothetical stake size doesn’t matter
• Real payments $\uparrow$ selfishness
  • Sefton (1992); Forsythe, Horowitz, Savin, & Sefton (1994); Clot, Grolleau & Ibanez (2018)
• Real payments $\uparrow$ correlation with Big 5
  • Lönnqvist et al. (2011)
• Hypothetical bias is real, hard to predict
  • Haghani et al. (2021); Laury & Holt (2008)
• But there are arguments not to pay...
  • Rubinstein (2001,2013); Harbi et al. (2015); Falk et al. (2016); Ben-Ner et al. (2008)