Public Goods, Attention Deficit Disorder†
and Equilibrium in the Internet Economy

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and

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Abstract

We consider a pure public goods economy with a continuum of agents. In general, the optimal public good levels are infinitely larger than the private good consumption levels in such an economy. This does not seem to be a reasonable, or even a comprehensible limit of a large economy. We propose instead to model the internet web pages and other intellectual content as our canonical public goods. We argue that in such an environment each agent would consume a finite amounts of a finite number of public goods. The key factor driving this is that agents have limited attention spans and this prevents them from consuming all, or even a large number of public goods. We show that the infinitely large aggregate public goods contributions in such an economy end up being absorbed in an infinite diversity of public goods rather than by infinite levels of any finite set of public goods. We conclude by showing that while approximately Pareto optimal allocations exist, price systems that satisfy the standard welfare theorems are difficult to define. Since price systems don't signal profit opportunities in equilibrium, there may indeed be opportunities for economic profit in the internet economy

1. Introduction

Aumann (1964) was the first to present a model of a private goods economy with an continuum of agents. He argues that it is only in such an environment the the competitive assumption that agents are price takers is fully justified. The attraction of his approach is it provides a very clean mathematical structure to gain insights about large finite economies. For example, he was able to show that Debreu and Scarf's (1963) result that the core converges to the competitive allocations as the economy gets large holds exactly in the sense that the core and equilibrium allocations are equivalent in a continuum economy.

Although the Aumann's mathematical approach is very productive, the only justification for thinking about continuum economies is that they are an economically meaningful limit of a large finite economy. If one can not show the connection. then the results may just be artifacts of the mathematics, and give very misleading or incorrect insights about large finite economies. Unfortunately, the connection seems to be quite weak in the case of pure public goods economies as they are usually written. See Muench (1972), for the conical treatment and Berliant and Rothstein (2000) for a more recent treatment that discusses some of the issues outlined below.

The problem with the continuum approach to public goods economies is that private goods and public goods are measured in different ways that make them fundamentally incomparable. Since the level of a public good is determined by the integral over agents' contributions, one of two things must be true. If average contribution is strictly positive, then the ratio of the public to private goods is infinite for all consumers.[1] How would one compare two such allocations? Is it meaningful to say that one infinity of public goods is preferred to another? Would we be worse off if we halved our contributions given that we would still be funding

[1] If we each donate a dollar to public radio, and there are infinitely many of us, public radio has an infinite number of dollars to spend.

all public goods at infinite levels? On the other hand, the average contribution might be zero. Several things could be true in this case, but suppose that this was because agents became satiated in public goods an some finite level. What would their Lindahl taxes be in this case? Now we are trying to divide a finite public good cost over an infinite number of agents. Thus, average the contribution would have to be infinitesimal. How would agents express demand for public goods when faced with infinitesimal Lindahl prices? In any event, how would they distinguish between different allocations in which the public goods level is unmeasurable (almost zero)? For such allocations, the ratio of public to private goods consumption for each consumer could be bounded or undefined. Differentiating these two cases is important, but is not possible in a standard continuum economy. The upshot is that the direct extension of the Aumann approach to public goods creates significant mathematical and economic difficulties that make it hard to understand it as a limit of a large finite economy.

From a technical standpoint, these problems could be avoided using non-standard analysis (NSA), where infinitesimals and infinities are added to the set of real numbers. See Brown and Robinson (1972) and Khan (1974) for early examples of applications to economics and Rashid (1987) for a nice comprehensive reference. This approach allows for well-defined notions of the marginal willingness to pay and the Samuelson (1954) conditions for Pareto optimality, and makes it possible to compare different infinite levels of public goods. Most importantly, it becomes possible to see Lindahl equilibria in which the ratio of public goods to private goods is large, but finite, despite there being an infinity of agents, reflecting the limit of a large economy suggested in Conley (1994).

Although NSA provides a mathematical solution to these problems, it does not avoid the deeper economic issues with such models. Can agents really contemplate an infinity of any good much less think of one infinity as being twice as large as another? Can they really express a demand for a good with an infinitesimal price?

Perhaps most importantly, is it reasonable to suppose that the level of public goods for an economy truly diverges to infinity as the economy gets large? Would agents really demand an infinity of defense or knowledge? If not, does this necessarily imply the similarly implausible conclusion that the average contribution to public goods production should always go to zero in large economies?

In this paper we propose an entirely new approach to a continuum public goods economy which, we argue, provides a reasonable limiting model while matching many features of the large economy in which we actually live.

First, we restrict attention to pure public goods. If there is crowding or diminished service quality levels with distance, then we are in a local public goods economy, and the existence and core equivalence properties are well understood. (See Allouch, Conley and Wooders 2007 for a recent treatment and extensive discussion of the literature.) Even national defense falls into this category. The larger the country, the more there is to defend, so in a real sense, there is crowding for defense services.

Truly pure public goods are mostly in the category of knowledge and intellectual content. These are goods which are now most frequently delivered over the internet, radio, television, and, to a lesser extent, libraries and social networks. These types of goods share several features which we incorporate in our model:

1. They are differentiated. Each Jessica Simpson song is different (however subtly). We do not see economists or other scholars writing the same paper over and over again (Really! There are differences if you look hard enough). Each blog entry, poem, patent, movie, and Web page is a unique product
2. Each of the differentiated products is provided in clear, finite amounts. Most pop songs last about three and a half minutes. Even Wagner operas come to an end eventually. Authors of popular books or important articles do not respond to the high demand by adding pages to any great extent. Of course, they may write new contributions that are slightly differentiated and in the

same spirit, but these are newly created goods rather than additional quantity of the original good.

3. Agents have different tastes and like different items of content differently. If the market for content is thick, however, most agents can find items they like almost as much, and will share this taste with other agents, at least locally. That is, if two agents like one Jessica Simpson song, they are both likely to also enjoy another one almost as much. This implies nothing, however, about how much either agent feels about 50 Cent.
4. As the relevant market grows, the number of intellectual products is likely to increase. In the limit there may be an infinity of possible intellectual goods on offer. Despite this, it need not be the case that the number of agents who choose to consume any given piece of content becomes large. If too many agents consume one public good, it creates an market opportunity to create a slightly differentiated product that peels off some of the userbase and still make a profit. (Jessica Simpson in Spanish, Jessica Simpson as an animated mouse?)
5. Most critically, not only is it the case that agents do not consume an infinite amount of any given type of content but *they also do not consume an infinite number of types of content.* Agents have finite attention spans (or endowments of time). A person might listen to an hour of radio, study two articles, and read a chapter of a book, but he would not listen to one second of 3600 songs or read the first sentence of 1,000 books. Each time he consumes a public good, he incurs a cost of consumption. This may be the effort involved in clicking a mouse, retrieving a book from a shelf or just focusing his attention. Eventually this cost becomes prohibitive, and this implies that it can only be optimal to consume at most a finite set of goods each day.

Putting this together, we see the limit of a large pure public goods economy as having an infinity of slightly differentiated but discrete pieces of intellectual content all directed to relatively small, differentiated audiences. Thus, we propose a model in which average contributions to pure public goods production can be strictly positive and yet no public good is provided or consumed at infinite levels. Instead the contributions are absorbed by producing finite levels of an infinite number of pure public goods, each consumed by a finite number of agents. We argue that this closely reflects what we see in today's internet economy.

Using this approach to public goods, we show that the set of Pareto optimal allocations is not empty. More importantly, we show that at any Pareto optimal (or approximately Pareto optimal) allocation, agents consume a finite number of public goods and that a finite number of agents consume each good. This is consistent with the five desiderata outlined above. We then explore two possible price systems in an attempt to decentralize these Pareto optimal allocations, in much the same way that Lindahl equilibrium decentralizes optimal allocations in economies with a finite number of public goods. The first price system assigns one (anonymous) price to each public good. These prices necessarily do not reflect the differing marginal willingness to pay for public goods across agents, and thus fail to generate the standard welfare theorems for the same reasons they do in standard public goods economies. We then proceed along the lines of Lindahl (1919) by allowing personalized prices. Despite the additional informational requirements of this price system, the first welfare theorem continues to fail. A second welfare theorem, however, holds in this case.

The inability to identify a price system with reasonable informational requirements that delivers the first welfare theorem suggests that this environment has inescapable complexities that prevent market forces from removing arbitrage opportunities. For example, equilibrium prices may not support the production of a particular public good, but introducing that good into production and switching some agents to consuming the good might improve overall welfare. Unlike standard models, this economy provides opportunities for agents to reorganize and realize mutual gains. What this suggests to us is that the internet economy is fundamentally entrepreneurial and creative in nature. Production, in particular, cannot be decentralized by any reasonable price system. Thus, economic profits are to be expected. There will be five dollar bills on the ground in the information age and very little content production will take place under the more familiar industrial age system of competitive markets.

T'he paper proceeds as follows: We give a brief review of some of the related literature in the section 2. We describe our model in detail in Section 3. We show our results in Section 4, and Section 5 concludes.

2. A brief literature review

A key point that quickly emerges when trying to model the internet economy is that any reasonable treatment must take the commodity space as endogenous. Thousands of web sites, songs, movies, books, and so on appear every day, and an Arrow-Debreu-MacKenzie type fixed commodity space in no way approximates this reality. Exactly what kind of market signals content creators are responding to is not clear. Certainly, it is not a set of Walrasian prices. More likely it relates to some estimate on the part of entrepreneurs of the demand for given types of content based on current and projected consumer behavior (dare we say, on marketing studies).

Of course the process of new product development that we implicitly assume has been considered by other contributers to the literature. Although the Schumpeterian view emphasizes the importance of entrepreneurial activity (Schumpeter 1947) and empirical work demonstrates the importance of entrepreneurship in growing economies (McMillan and Woodruff 2002), Baumol (1968) argues that such models are unlikely to develop in the modern maximization paradigm where managers are cold calculating machines with no capacity for the inspired creation of new ideas or products. In short, this is a very difficult problem to model in a general way that gives clear results. Some theoretical work in this direction has been attempted (see Nelson and Winter 1974, 1977, or 1978, for example) but it centers around off-equilibrium dynamic selection rather than neoclassical equilibrium analysis and is therefore more difficult to analyze with standard economic tools. Other models explore the self-selection of entrepreneurs based on skill (Lucas 1978 or Calvo and Wellisz 1980) or risk aversion (Kihlstrom and Laffont 1979), or analyze the decision to enter into the production of a new good (as in Aron and Lazear 1990), but these models do not predict what types of new goods one might expect to develop. Similarly, Grossman and Helpman (1991) look at the innovation of existing products, but not the development of completely new goods. In this paper, we treat this problem very abstractly and focus only on the limit of this process. Not surprisingly, it is difficult to get as many results as we might like under these conditions, but we do make progress which we will argue gives real insights into the information economy.

Although the product development process is not well understood, ours is certainly not the first general equilibrium model with an infinite number of commodities. Classic models of savings and consumption allow agents to choose infinite streams of future consumption levels. For example Mas-Colell (1975) and Jones (1983) model private goods as a vector of characteristics, so that consumers' consumption bundles are represented by measures over the set of characteristics. These models assume an infinite number of agents and no production. We will not attempt to survey this very large literature here, but see Podczecka and Yannelis (2007) for a recent treatment with references and Aliprantis and Brown and Burkinshaw (1990) for an older, but more comprehensive treatment of the literature. As far as we are aware, none of the above models include public goods.

Expanding on the Lindahl approach for pricing public goods, Mas-Colell (1980) defines a valuation equilibrium and a cost-share equilibrium for partial equilibrium settings with a single private good and a finite set of public projects with no mathematical structure; this was generalized by Anna De Simone and Maria Gabriella Graziano (2004) who include production and Riesz space of private goods. The standard welfare theorems are proved in these settings (using appropriate assumptions), showing, for example, that the valuation equilibrium concept decentralizes the Pareto efficient allocations. These models differ substantially from the current paper in that they assume a finite number of agents and allow for only one public project to be chosen from the set of available projects. In this way they avoid the difficulties of large economies with public goods.

Despite the proliferation of the Internet and related research in fields such as computer science, surprisingly few theoretical papers in economics have examined the knowledge goods created by the Internet and on World Wide Web. Cremer, Rey and Tirole (2000), Besen *et al.* (2000), and Laffont *et al.* (2001) study issues of pricing and bargaining between Internet service providers and Internet backbone providers, and Jackson and Rogers (2007) explore a simple model of network formation that captures observed regularities in the network of links between Web sites. However, none of these studies focus on the actual provision of content by Web site operators. Perhaps the closest work to ours in this respect is by Harper *et al.* (2005), who model the incentives for users to contribute to an Online movie review Web site, though their model is purely decision-theoretic and does not consider market or strategic interactions.

3. The Model

Consider an economy with a continuum of agents:

$$i \in [0,1] \equiv \mathcal{I}$$

one private good

$$x \in \Re_+$$

and a continuum of potential pure public goods:

$$w \in [0, W] \equiv \mathcal{W}$$

where W is finite and is meant to remind us that these are “Web Projects”. Thus, the public goods here are pure public projects without Euclidian structure. Of course, one might prefer instead to make the level of public goods provision part of the producer's decision (as is traditional). Although our approach is more general, it forecloses studying the implications of such quantity decisions.[2]

The cost of producing a public project in terms of private good is given by

$$c : \mathcal{W} \rightarrow [0, \bar{C}].$$

The public goods are consumed (or *Subscribed* to) by agents. Let this *subscription map* which is denoted

$$S : \mathcal{W} \times \mathcal{I} \rightarrow \{0, 1\}$$

be an indicator function such that $S(w, i) = 1$ if agent i subscribes to good w and $S(w, i) = 0$ if he does not.

[2] Note also, one might pursue an hedonic approach along the lines of Lancaster (1966) and Rosen (1974). Here the new commodities would be composed of different proportions of an underlying set of characteristics that are valued by consumers. Leaving aside the real world difficulties of understanding what these hedonic characteristics might be (more cowbell?) or if they would even be finite in dimension, there is a deeper problem. It is very unlikely that preferences would be convex or even monotonic in the underlying hedonic space (is the average of Jessica Simpson and Prince better than either artist taken separately?). Thus, optimal commodity creation could not be decentralized by linear prices, and thus, this structure adds complexity without yielding new results. We therefore opt for the more abstract approach.

Given a subscription map S, let S_i denote the list of public goods subscribed to agent i:

$$S_i \equiv \{w \in \mathcal{W} \mid S(w,i) = 1\}.$$

In a similar way, denote the set of projects that have at least one subscriber (and thus are constructed) under S by:

$$\mathcal{W}^S \equiv \{w \in \mathcal{W} \mid \exists\ i \in \mathcal{I} \text{ s.t. } S(w,i) = 1\}$$

Agents have a utility function of the form:[3]

$$U_i(x, S_i) = x + V_i(S_i) - a_i(\mid S_i \mid).$$

Thus, the utility function is quasilinear. We interpret $a_i(n)$ as the attention cost of subscribing to n different public projects. We are therefore embedding two important assumptions: (a) it is equally costly in terms of attention to consume any given public project, and (b) there is no "intensity of consumption" decision. The first can be motivated as thinking of a as a search or transaction cost of calling up any given set of web pages. The fact that it takes more attention to consume some web pages, songs broadcast on the radio, television programs or books than others can be rolled in the net utility that agents get from consumption. The second implicit assumption, however, does have bite. It would be interesting to generalize this by allowing all agents to receive both an increased flow of utility and an increased attention cost as they put more time or effort into consuming a given public project. One could also model this as an explicit time cost of consumption.

[3] Since the norm $\mid \bullet \mid$ is only well defined for finite sets, the utility functions also is only well defined for this case. Thus, we are building in an inability for agents to contemplate consuming an infinite number of public goods. While it would be hard to imagine that this is possible in any event, we will see below that we make assumption (2, and 3) which render such consumption levels strongly suboptimal. Thus, we do not see this as restrictive, and do not choose to further burden the notation by allowing for this possibility.

This would allow us to consider the often neglected but important fact that one has only 24 hours a day to allocate not only to work and leisure, but also to consumption activities of all kinds. Consuming a leisurely dinner is different from wolfing one down. It may be more pleasurable to do the former, even with the same food in front of you, but it means one must curtail how long one plays Halo 3 later and may prevent one from editing one's MySpace page as much as one might wish. We will leave these considerations for future research.

Assume the following:

Assumption 1: For all $i \in \mathcal{I}$, $V_i(\emptyset) = 0$ and $a_i(0) = 0$.

Assumption 2: There exists a bound $B > 0$ such that for all $i \in \mathcal{I}$ and all possible subscription lists (finite or infinite), $V_i(S_i) \leq B$

Assumption 3: There exists $n \in \mathbf{N}$ (the natural numbers) such that for all $\bar{n} > n$ and all $i \in \mathcal{I}$, $a_i(\bar{n}) > B$

Assumption 4:[4] There exist $\epsilon > 0$ and $\delta \in (0, 1]$ such that for all S and all $w \in \mathcal{W}^S$ there exists a group of agents $g \subseteq \mathcal{I}$ satisfying

1. $g \subseteq \{i \in \mathcal{I} \mid w \in S_i\}$,

2a. if $\{i \in \mathcal{I} \mid w \in S_i\}$ is finite then $\mid g \mid \geq \text{Int}(\delta \mid \{i \in \mathcal{I} \mid w \in S_i\} \mid)$ (where $\text{Int}(z)$ is the largest integer weakly less than z),

2b. if $\{i \in \mathcal{I} \mid w \in S_i\}$ is countably infinite then $\mid g \mid \geq \text{Int}(\bar{C}/\epsilon + 1)$,

2c. if $\{i \in \mathcal{I} \mid w \in S_i\}$ is uncountable then g has positive measure, and

[4] There is at least an informal relationship between this assumption and *small group effectiveness* originally defined for local public goods economies in Wooders (1978) and used extensively in various forms in the large subsequent literature which we will not survey here. Assumption 4 implies (see the results below) that no pure public project will be subscribed to by a large number of agents in any Pareto optimal allocation. In this sense, only a small (finite) group is needed to fund any given at an optimal allocation. However, large (positive measure) groups may still be able to improve on the welfare obtainable by any measure zero set of agents due to overlapping multiple subscriptions that might be required to find enough subscribers to support an equilibrium subscription structure. In this sense, small groups are not effective in our context.

3. there exists some $\bar{w} \in \mathcal{W}$ such that for each $i \in g$, $U_i(x, S_i) = x_i + V_i(S_i) - a_i(\mid S_i \mid) < x_i + V_i(S_i \cup \bar{w} \setminus w) - a_i(\mid S_i \cup \bar{w} \setminus w \mid) - \epsilon$.

Assumption 1 is just a normalization that says if agents do not subscribe to any public goods, they receive no consumption benefits and pay no attention costs.

Assumption 2 says that there is an upper limit on the utility that agents can get from any set of subscriptions. To allow otherwise would be to imagine that one either achieves Nirvana while consuming a finite list of goods, or can approach it as one consumes public goods without bound. While it may be that agents might attain enlightenment by reading a book that contains the universal truth, or that listening to the *Dark Side of the Moon* enough times brings one close to a bliss point, the economic problem disappears in these cases.

Assumption 3 says that at some point, the attention cost of consuming one more webpage exceeds any possible gain.

Assumption 4 is a weak way of capturing the idea of the existence of close substitutes for any public project. Specifically, the assumption says the following: consider any subscription system S and set of agents consuming any given public project w. Then there will exist a close substitute $\bar{w}$ in the following sense: We can always select a group g, from the set of agents consuming the good w, which is at least a fraction δ as big as the original group such that these agents prefer by at least ϵ private good, an allocation in which another public project $\bar{w}$ has been exchanged for w. To be more precise, the coalition has to contain at least *the integer part of* δ times the number of agents who were originally consuming w. Thus, if the number of such agents is small, or δ is small, this rounds off to zero and g may be empty. Assumption 4 only need have real bite if a really large number of agents are consuming a given project.

It will turn out that assumption 4 is the key property that generates the equilibrium structure we describe in the introduction. To be a bit less formal, this says, no good is a first choice of a large number of agents. If a lot of agents are consuming a single good, there is a similar good that at least a fraction of them prefer by ϵ. Thus, assumption 4 ensures both a diversity of tastes and the existence of commonly agreed upon substitutes. Note this is is done without forcing preferences to lie in a metric space (see Allouch and Wooders 2007), assuming any kind of convexity or monotonicity, or imposing a norm on the goods space. Agents who agree on the value of one good, may totally disagree on the value of others, and there is no sense in which goods are similar just because they are close together in the project space $[0, W]$.

Agents are initially endowed with an amount of private good, ω_i which we assume is bounded. The endowment for the economy is given by

$$\Omega : \mathcal{I} \to \Re_+$$

A *feasible allocation* is a private good allocation $X : \mathcal{I} \to \Re_+$ and a subscription function S such that

$$\int_{i \in \mathcal{I}} \Omega(i) - \int_{i \in \mathcal{I}} X(i) - \int_{w \in W^S} c(w) \geq 0$$

A feasible allocation X, S is *Pareto Optimal* if there does not exist another feasible allocation $\bar{X}, \bar{S}$ such that for a group $g \subseteq \mathcal{I}$, of full measure and for all $i \in g$:

$$U_i(\bar{x}_i, \bar{S}_i) > U_i(x_i, S_i).$$

Note that the assumption of quasilinearity allows us to state the definition this way. If it is possible to make some agents better off while leaving the rest just as well off, we can transfer some of the surplus to make all agents strictly better off. The definition of feasibility for a group $g \subset \mathcal{I}$ is immediate: the integration above is simply taken over g instead of $\mathcal{I}$. In the case that g is countable or finite, the sum is taken instead.

A feasible allocation X, S is ϵ-*Pareto Optimal* if there does not exist another feasible allocation $\bar{X}, \bar{S}$ such that for a group $g \subseteq \mathcal{I}$ of full measure and for all $i \in g$:

$$U_i(\bar{x}_i, \bar{S}_i) > U_i(x_i, S_i) + \epsilon.$$

Now we define two notions of equilibrium, one anonymous and one nonanonymous.

Anonymous Equilibrium has a great deal of real world appeal in this context. It is quite intuitive that content providers are not be able to identify agents by name or preferences and charge discriminatory prices as a result. This is especially true given that there is a continuum of agents, so even knowing what price to charge everyone is an informationally daunting task. Thus, for agents, a price system consists of a simple mapping from public goods produced in an allocation to the reals.

$$P : \mathcal{W}^S \rightarrow \Re_+,$$

where $P(w)$ is the subscription price to good w. We will assume that equilibrium prices are only well defined for $w \in \mathcal{W}^S$. Alternatively, one may choose to let $P(w) = B$ for all $w \notin \mathcal{W}^S$ since given assumption 1, no agent would choose to consume such a public project. This is more correct from a formal standpoint, but is needlessly pedantic.

Now consider this from the standpoint of firms. In the standard public goods economy, all agents consume the total amount produced of each public good. In turn, they pay their Lindahl price times the quantity produced. Thus, all the firm needs to know is the sum of the Lindahl prices and the cost of production to make a profit maximizing decision. Things are quite different here. First, prices must be anonymous as we mention above. Second, *only a fraction of the agents in the economy will purchase any given public good,* and the number that do depends on the price that is charged. Thus, an entrepreneur considering whether he should produce a public good must know the number of agents who will choose to subscribe to this service at any given price he might charge. The price alone is not enough, he must also know the number of times he will sell the good.

In the interests of simplicity, we will assume that each public good can be produced by one specific firm. Each firm also needs to have a conjecture of the number of subscriptions he will get at any price p. This is given by:

$$N : \mathcal{W} \times \Re_{+} \rightarrow \mathbf{N}.$$

where $N(w, p)$ is the number of subscriptions the firm producing good w expects to see at anonymous price p. In equilibrium, these projections must agree with the set of subscriptions chosen by optimizing agents at market prices. Thus, we will need to know the *equilibrium demand* which we define as follows:

$$D(w, S) \equiv \mid \{ i \in \mathcal{I} \mid w \in S_i \} \mid$$

Since firms may make profits, these must be returned to agents in equilibrium. It would be messy and pointless to define ownership shares for each firm for each agent since there is a continuum of each. Thus, we will simply assume each agent owns a share of the economy denoted θ_i. The ownership function for the economy is given by

$$\Theta : \mathcal{I} \rightarrow \Re_{+},$$

where $\int_{i \in \mathcal{I}} \Theta(i) = 1$. Note that this may take values above one for any given agent (which would mean that this agent is getting a higher than average share of profits), but we assume that there is strict upper bound on the "share" of the economy any agent can own. This is necessary to make sure that all agents have negligible market power.

Given this, a feasible allocation X, S and a price and demand system P, N for an endowment and ownership system Ω, Θ is an *Anonymous Equilibrium* if

1. There does not exist a set of agents $g \subseteq \mathcal{I}$ of positive measure such that for all agents $i \in g$ there exists $\bar{S}_i$ such that [5]

$$\omega_i + V_i(\bar{S}_i) - a_i(\mid \bar{S}_i \mid) - \sum_{w \in \bar{S}_i} P(w) + \theta_i \int_{w \in W^S} (P(w)N(w, P(w)) - c(w)) >$$

$$\omega_i + V_i(S_i) - a_i(\mid S_i \mid) - \sum_{w \in S_i} P(w) + \theta_i \int_{w \in W^S} (P(w)N(w, P(w)) - c(w)) =$$

$$X_i + V_i(S_i) - a_i(\mid S_i \mid).$$

2. For almost all $w \notin W^S$ and all $p \in \Re_+$

$$pN(w, p) - c(w) \leq 0$$

3. For almost all $w \in W^S$ it holds that[6]

$$P(w)N(w, P(w)) - c(w) \geq 0$$

[5] Note that agents assume that the "profit" income they have to spend in equilibrium does not depend on their own consumption choices. This is because each agent is infinitesimal and does not affect the economy-wide profits.

[6] One might also add "for all $p \in \Re_+$" to the condition of this statement. We don't do this for two reasons. First, we are looking for a price taking equilibrium, and this would put producers in the position of being price makers. Second, and even more troublesome, is that it is not clear what reasonable equilibrium demand speculations would be for projects that are produced. Holding the demand for other goods constant, one could require that $N(w, p)$ is consistent for with actual demand for w at every p. But even in this case, who claim consumers who are indifferent between two projects at a given price, and how does one maintain the consistency of demand conjectures when other prices change at the same time? To summarize, there did not appear to be a way to impose much market discipline on these equilibrium demand conjectures without introducing other inconsistencies. Thus, we have chosen to maintain a more traditional price taking approach here. We thank Marco Castanada for pointing this out to us.

4. For almost all $w \in W^S$

$$N(w, P(w)) = D(w, S)$$

Condition 1 says that taking the subscription prices as given, almost all agents choose an affordable, utility maximizing set of projects.

Condition 2 says that taking demand as given, almost no public project which is not produced could generate positive profits at any possible price.

Condition 3 says that for almost all projects produced in the equilibrium, taking price and demand as given, costs are at least covered.

Condition 4 says that for almost all projects produced in equilibrium, the equilibrium demand system N posited by producers agrees with the actual subscriptions demanded at the equilibrium price.

Remark 1. This is a very appealing equilibrium, however, it is easy to see that it will not be able to decentralize the Pareto optimal allocations in general. Imagine the following somewhat informal counterexample. Suppose there is a pair of agents interested in a given public good w which costs c to produce. The first agent values it at $.4c$ and the second at $.8c$. Neither agent gets utility from any other public good. Now suppose all the agents in the interval $[0, 1]$ form such pairs of other public goods. Then it is clear that if attention costs are low enough, the only PO allocation would have the agents form similar pairs and subscribe to the relevant goods. However, just as clearly, it is not possible to decentralize this with anonymous prices. If the price is $.4c$ or below, both agents may subscribe (depending on the attention costs), but then prices do not cover the costs of the public goods. If they are $.5c$ or above, one agent will certainly not subscribe, and again, costs would not be covered.

Given this more or less abject failure, we are forced to consider nonanonymous prices:

$$P_i : \mathcal{W}^S \to \Re_+,$$

where $P_i(w)$ is the subscription price to good w paid by agent i.

Again, consider this from the standpoint of firms. The conjectures of the firms are much more complicated here. Firms now take the whole infinite dimensional, personalized price vector as given, and simply make a revenue estimate based on this. Formally $R : (w, P_i) \to r \in \Re_+$ is the net revenue expected which is equal to the sum of the personalized subscription prices for all agents who subscribe to a given public good in equilibrium.

Given this, a feasible allocation X, S and a price and demand system P, R for an endowment and ownership system Ω, Θ is an *Nonanonymous Equilibrium* if:

1. There does not exist a set of agents $g \subseteq \mathcal{I}$ of positive measure such that for all agents $i \in g$ there exists $\bar{S}_i$ such that

$$\omega_i + V_i(\bar{S}_i) - a_i(\mid \bar{S}_i \mid) - \sum_{w \in \bar{S}_i} P_i(w) + \theta_i \int_{w \in W^S} (R(w, P) - c(w)) >$$

$$\omega_i + V_i(S_i) - a_i(\mid S_i \mid) - \sum_{w \in S_i} P_i(w) + \theta_i \int_{w \in W^S} (R(w, P) - c(w)) =$$

$$X_i + V_i(S_i) - a_i(\mid S_i \mid).$$

2. For almost all $w \notin W^S$ and all $p \in \Re_+$

$$R(w, P) - c(w) \leq 0$$

3. For almost all $w \in W^S$

$$R(w, P) - c(w) \geq 0$$

4. For almost all $w \in W^S$

$$\sum_{i \in \mathcal{I} \text{ s.t. } w \in S_i} P_i(w) = R(w, P)$$

These conditions say essentially the same thing as they do for anonymous equilibrium but for non-anonymous prices.

Remark 2. This equilibrium solves the problem pointed out in Remark 1. However, it is now the case that many non-Pareto optimal allocations can be decentralized. This means that the First Welfare Theorem is not true and the Second Welfare Theorem is much less meaningful. To see this, imagine the following somewhat informal counterexample. Suppose there is a pair of agents interested in a given public good w which costs c to produce. The first agent values it at $.6c$ and the second at $.7c$. They are also interested in a second public good $\bar{w}$ which costs c to produce. The first agent values it at $.8c$ and the second at $.9c$. They get no utility from any other good, and the utility for w is zero when $\bar{w}$ is consumed. Finally suppose all the agents in the interval form similar pairs for other sets of public goods. Then it is clear that if attention costs are low enough, the only PO allocation would have the agents form these pairs and consume the second public good. However, suppose we decided to price the second good at B. Then no agent would find it optimal to consume the second good in equilibrium and the revenue would be projected at zero under these prices. On the other hand, suppose we priced the first (less desirable good) at $.5c$. All pairs of agents will then choose to subscribe to this good and revenue will be correctly projected at c. Since utility is transferable, one can then adjust the endowments to achieve the private goods consumption specified in the allocation. Thus, we see that we can decentralize non-Pareto optimal allocations. Notice also that we did this with anonymous prices, so the counterexample also applies to Anonymous Equilibrium.

We see that our notions of equilibrium, while having a very strong information requirement, still do give us the kinds of welfare theorems we might have hoped for. Of course there are many other potential ways of defining equilibrium and our less than fully satisfying conclusions do not by any means logically exclude the possibility that an alternative formulation might do better. We also tried many alternatives but attempted to stick to price systems that reflected either traditional equilibrium notions or the information/decision precesses we see available to internet users and entrepreneurs. We strongly encourage other authors to work on developing new and better equilibrium notions for this important economic environment.

4. Results

In this section we give our results. We are able to show that all Pareto optimal allocations have certain properties, that Pareto optimal allocations exist, in a sense, and that they can be decentralized, in a sense.

The first lemma shows that in essence, in any PO allocation, agents will choose to consume at most a finite number of public goods.

Lemma 1. *There is an upper bound on how many subscriptions almost every agent will have in any PO allocation X, S. In addition, for any PO allocation there exists a Pareto indifferent allocation in which no agent has more than a strictly bounded number of subscriptions.*

Proof/

By assumption 3, there exists $n \in \mathbf{N}$ such that for all $\bar{n} > n$ and all $i \in \mathcal{I}$, $a_i(\bar{n}) > B$. Suppose that a group $g \subset \mathcal{I}$ of positive measure were to subscribe to more than n goods. Recall that by assumption 2, there exists a bound $B > 0$ such that for all $i \in \mathcal{I}$ and all possible subscription lists (finite or infinite), $V_i(S_i) \leq B$. Thus, consider an alternative allocation $\bar{X}, \bar{S}$ such that $\bar{X} = X$ and $\bar{S}$ is constructed so that all agents in g subscribe to no public goods while agents not in g continue to subscribe to the projects specified for them in S.

For agents not in g, the two allocations are equally good. For agents in g, the utility benefits of consuming S_i are bounded above by B, while the attention cost is bounded below by B. Thus, consuming S_i is worse than doing nothing (even while continuing to pay for the subscriptions given in S), and so agents in g are better off consuming the (null) subscriptions given in $\bar{S}$.

Note that we assume that all the same public goods are produced in both cases. This is feasible since the same net amount of private good is distributed to agents in both cases. Of course, some public goods may not be needed at all under $\bar{S}$, in which case it is possible to define a feasible $\bar{X}$ that further improves the welfare of agents. It follows that $\bar{X}, \bar{S}$ Pareto dominates X, S.

Finally, while allowing a group of zero measure to inefficiently consume more than n public goods might be Pareto optimal, forcing them to drop all their subscriptions is also feasible and Pareto indifferent (since they are zero measure).

■

The next Lemma shows that, in essence, in any PO allocation, all public projects will have only a finite number of subscribers.

Lemma 2. *There is an upper bound on how many agents will subscribe to almost every public good in any PO allocation X, S. In addition, no public good will be subscribed to by a set of agents of positive measure at any PO allocation. Finally, for any PO allocation X, S, there exists a Pareto indifferent allocation in which all* *public goods that are produced are consumed by a finite set of agents.*

Proof/

We claim that

$$n = \frac{\bar{C} + \epsilon}{\epsilon\delta}$$

is the upper bound of PO subscription levels mentioned in the hypothesis. Suppose instead there existed a set of public projects $\bar{\mathcal{W}} \subseteq \mathcal{W}$ of positive measure, each of which is consumed by a set of agents larger than this bound.

Recall by assumption 4, there exist $\epsilon > 0$ and $\delta \in (0, 1]$ such that for all S and all $w \in \mathcal{W}^S$, there exists a group $g \subset \mathcal{I}$ where $w \in S_i$ for all $i \in g$, the size of g is at least $\text{Int}(\delta \mid \{i \in \mathcal{I} \mid w \in S_i\} \mid)$ if the number of subscribers of w is finite or has at least $\text{Int}(\bar{C}/\epsilon + 1)$ members if the number of subscribers of w is infinite, and there exists some $\bar{w} \in \mathcal{W}$ such that for all $i \in g$,

$$U_i(x, S_i) = x_i + V_i(S_i) - a_i(\mid S_i \mid) < x_i + V_i(S_i \cup \bar{w} \setminus w) - a_i(\mid S_i \cup \bar{w} \setminus w \mid) - \epsilon$$

Now consider one $w \in \bar{\mathcal{W}}$, and the implied group g above. For this group there exists some $\bar{w} \in \mathcal{W}$ such that for all $i \in g$,

$$U_i(x, S_i) = x_i + V_i(S_i) - a_i(\mid S_i \mid) < x_i + V_i(S_i \cup \bar{w} \setminus w) - a_i(\mid S_i \cup \bar{w} \setminus w \mid) - \epsilon$$

Suppose now that all agents in g dropped their subscriptions to w, picked up a subscription to $\bar{w}$ and reduced their consumption of private good by ϵ. Then from the equation above, all agents in g would be strictly better off. The number of members of g is at least

$$\text{Int}\left(\delta \frac{\bar{C} + \epsilon}{\epsilon\delta}\right) = \text{Int}(\bar{C}/\epsilon + 1),$$

which is at least as large as $\bar{C}/\epsilon$. Thus, the total additional private good collected is at least $\bar{C}$, more than enough to fund the production of $\bar{w}$. Since agents in g are still making implicit contributions to the production of the remaining public goods (including w) there is no difficulty in funding the existing set of public goods. We conclude that at least a finite set of agents g are strictly better off as a result while all other agents are at least as well off. Since this argument can be repeated for all $w \in \bar{\mathcal{W}}$ we find that a set of agents of positive measure can be made strictly better off, while the remaining agents are just as well off at the new feasible allocation. This contradicts the hypothesis that X, S is a PO allocation.

Next suppose that there was at least one public good w that had a set of agents g subscribing where g had positive measure. By the argument above, as long as this is the case, it is possible find a coalition of agents who prefer and can fund alternative public goods. Since this set of agents who are better off will also have positive measure by assumption 4, this is a Pareto improvement. This contradicts the hypothesis that X, S is a PO allocation.

Finally, given the two facts just proved, it still might be the case that a set of public goods with measure zero has an unbounded, but measure zero set subscriptions. If one simply desubscribes all these agents to their current public goods, one affects only a measure zero of agents. Thus, there exists an allocation that is Pareto indifferent to X, S in which all public goods are consumed by a finite set of agents.

■

Put together, Lemmas 1 and 2 say that the equilibria of an economy that satisfies assumptions 1 through 4 will have the properties outlined in the introduction. In particular, the equilibria will involve an infinite number of public goods, each of which is consumed by a finite number of agents, each of whom in turn consumes at most a finite number of public goods.

Our next theorem shows that there will always exist a Pareto Optimal allocation, at least in an ϵ sense.

Lemma 3. *There exists feasible $\epsilon-$Pareto optimal allocation.*

Proof/

let X^0, S^0 be any feasible allocation. If X^0, S^0 is not PO, then there must a Pareto dominant allocation X^1, S^1. Continue to form this sequence of Pareto dominating feasible allocations and denote it by $\{X^k, S^k\}$. If this converges in a finite number of steps, then the last element of the sequence is Pareto optimal.

Suppose instead that $\{X^k, S^k\}$ does not converge in a finite number of steps. Consider the aggregate utility of the sequence:

$$\int_{i \in \mathcal{I}} \left(x_i^k + V_i(S_i^k) - a_i(\mid S_i^k \mid)\right) \in \Re^1_+$$

Note that by assumptions 2 and 3, we can conclude that this is bounded below by $\int_{i \in \mathcal{I}} \Omega(i)$ and bounded above by $\int_{i \in \mathcal{I}} \Omega(i) + B$. Thus, the sequence

$$\left\{ \int_{i \in \mathcal{I}} (x_i^k + V_i(S_i^k) - a_i(\mid S_i^k \mid)) \right\}$$

is bounded and so must have a supremum.

Since the sequence is also monotonic by construction, for any $\epsilon > 0$ there is a k such that for all subsequent elements of the sequence, the value is within ϵ of the supremum. Thus, for any ϵ there exists an $\epsilon-$Pareto optimal allocation.

■

Note that if there are a finite number of agents and public goods, existence of exact Pareto optimal allocations can be obtained by the Bolzano-Weierstrass Theorem. In our case, the sequence of allocations is an infinite dimensional vector (alternatively, they are functions of two variables that take values zero or one). Unfortunately, the theorem only applies to finite dimensional spaces.

Our next theorem is a version of the Second Welfare Theorem. As we mention in Remark 2, above, a SWT is not as impressive in this context as many things can be decentralized that are not PO. We will add two additional assumptions to show this result. We will require that the incremental utility to consuming more projects diminishes in a certain weak sense, and that the attention cost is increasing in the number of projects consumed. These are sufficient to show the Second Welfare Theorem, but may not be necessary.

Assumption 5: For all $i \in \mathcal{I}$, all $w \in \mathcal{W}$ and any two subscription lists $S_i, \bar{S}_i$ such that $S_i \subset \bar{S}_i$,

$$V_i(S_i \cup w) - V_i(S_i) > V_i(\bar{S}_i \cup w) - V_i(\bar{S}_i)$$

Assumption 6: For all $i \in \mathcal{I}$ and all $n \in \mathbf{N}$

$$a_i(n+2) - a_i(n+1) > a_i(n+1) - a_i(n)$$

We will also need to define a refinement of Pareto optimality which we call Strict Pareto Optimality (SPO) as follows: A feasible allocation X, S is *Strictly Pareto Optimal* if there does not exist another feasible allocation $\bar{X}, \bar{S}$ such that for a group $g \subseteq \mathcal{I}$, of full measure and for all $i \in g$:

$$U_i(\bar{x}_i, \bar{S}_i) > U_i(x_i, S_i).$$

and in addition: for all (not just almost all) $w \in \mathcal{W}^S$

$$\sum_{i \in \mathcal{I} | w \in S^i} [V_i(S_i) - a(\mid S_i \mid)] - [V_i(S_i \setminus w) - a(\mid S_i \mid -1)] \geq c(w).$$

This says that, in addition to X, S being PO in the ordinary way, *every* public project that is produced has the property that the net increment to utility summed over all the agents who consume it is at least as large as the cost of producing it. Otherwise, the consumption benefits of producing the project do not repay the expenditure needed to produce it. Under the usual definition of PO for continuum economies, this might not be satisfied for a measure zero set of projects. Of course, when a finite economy is considered, SPO and PO are identical.

Theorem 1. *Let X, S be a PO allocation. Then there exists P, R that decentralizes it as an Nonanonymous Equilibrium for some ownership and endowment system $\bar{\Theta}, \bar{\Omega}$.*

Proof/

For all $i \in \mathcal{I}$, all $w \in S_i$, let the personalized price the maximum that any given agent consuming the project would pay and still not want to not drop the subscription:

$$P_i(w) = [V_i(S_i) - a(\mid S_i \mid)] - [V_i(S_i \setminus w) - a(\mid S_i \mid - 1)]$$

Let the price of all other public projects be B. Let the ownership shares be identically 1, and adjust the endowments of each agent so that when he pays the prices specified for the subscriptions in S_i, he gets the same utility as at the PO allocation X, S assuming that agents continue to subscribe as specified by S. Note this is feasible since all agents consume the same subscriptions and the same public projects are produced. Thus, the same net amount of economy-wide transferable utility is available at both allocations.

Note that no agent would ever choose to buy anything under these prices that he was not consuming in the SPO allocation. This is because the price of these other goods is B, and so by assumptions 1 and 2, must lower his utility if purchased.

Thus, agents will not add subscriptions not in S. We must also show that they will not choose to drop any subscriptions. That is, we must show that under these prices, there does not exist a subset of the subscriptions given in S_i that increases the utility of agent i.

To see this, note that it would not be in any agent's interest to drop any single subscription $w \in S_i$. This is because the $P_i(w)$ is chosen to equal exactly the utility difference of the subscription bundle with and without w. Could an agent be better off if he dropped any two subscriptions $w, \bar{w} \in S_i$? Note that by assumptions 5 and 6, the net utility lost from dropping $\bar{w}$ *after* dropping w strictly increases and thus must exceed the personalized price $P_i(\bar{w})$. He is therefore indifferent about not purchasing w, but is made strictly worse off if he subsequently decides not to purchase $\bar{w}$. The utility loss just increases, by assumption, the more public project subscriptions he drops before he decides to drop $\bar{w}$, and this argument holds for any order of dropping subscriptions. We conclude that he cannot do better under these prices than by going ahead and purchasing all the public projects given in S_i. Thus, condition 1 of the definition of equilibrium is therefore satisfied.

To ensure that firms cannot make profits by producing an alternative bundle of goods in equilibrium, we construct the revenue projection functions as follows: For all $w \notin \mathcal{W}^S$, let $R(w,p) = 0$. Clearly, no positive profits can be made on such projects in this case. Thus, condition 2 of the definition of equilibrium is satisfied. For all $w \in \mathcal{W}^S$, let $R(w,P) = \sum_{i \in \mathcal{I} \text{ s.t. } w \in S_i} P_i(w)$. Thus, condition 4 of the definition of equilibrium is satisfied.

It only remains to show that condition 3 of the definition of equilibrium is satisfied. Recall that by assumption X, S is SPO and so for *every* $w \in \mathcal{W}^S$

$$\sum_{i \in \mathcal{I} | w \in S^i} [V_i(S_i) - a(\mid S_i \mid)] - [V_i(S_i \setminus w) - a(\mid S_i \mid -1)] \geq c(w).$$

Also recall that by construction,

$$P_i(w) = [V_i(S_i) - a(\mid S_i \mid)] - [V_i(S_i \setminus w) - a(\mid S_i \mid -1)].$$

It follows that for every (and so trivially, for almost every) $w \in W^S$

$$\sum_{i \in \mathcal{I} \text{ s.t. } w \in S_i} P_i(w) \geq c(w).$$

Therefore, any SPO allocation can be decentralized as an Nonanonymous Equilibrium for some ownership and endowments $\bar{\Theta}, \bar{\Omega}$.

∎

As we mention above, since SPO is the same as PO in the case of finite economies, a stronger Second Welfare Theorem holds for in this case. We also note that if assumptions 5 and 6 were strengthened so that "=" replaced ">" in the displayed equations, then the full set of PO allocations could be decentralized for the continuum economy. Economically, these strengthenings would require that V_i is additively separable and a_i is linear for all agents. These seem unduly restrictive, so we prefer the form of the SWT given here.

5. Conclusions

In this paper, we were motivated by two concerns. First, we wanted to provide a model of a continuum public goods economy that was an economically and mathematically meaningful limit of a large finite public goods economy. Second, we wanted to provide a positive analysis of the properties of such an economy based as much as possible on the institutional details and constraints we observed in the real world.

We argued that it was unlikely that the levels of public goods consumed by agents would grow without bound as an economy gets large. At some point, agents cease to be able to even contemplate the vastness of the consumption bundle. Certainly, agents would not be able to contemplate infinities of public goods. As long as the number of public goods is fixed, this necessarily implies that the private good contribution levels to public goods production (Lindahl taxes, for example) would have to converge to zero. This clearly is not happening in any real world economy we see, no matter how large.

We proposed an alternative: as the economy gets large, the number of public goods (which we think of as internet or information goods) also gets large. Thus, the commodity set is not fixed as it is in traditional Arrow-Debreu-MacKenzie/Samuelson economies. We showed that under fairly mild conditions, the Pareto optimal allocations of this economy will involve an infinite number of public projects being produced and that each of these projects being consumed by a finite number of agents. In addition, each agent would only consume a finite number of public projects.

What drives these results are two basic assumptions: (a) agents have limited attention spans and it costs an agent a certain amount of attention to consume public projects and (b) there are likely to be close substitutes for any project that are at least slightly preferred, all else equal, by a fraction of any set of consumers.

The first assumption could also be imposed on private goods economies, of course. However, the bite is not as strong because income runs out as well as time when consuming private goods. In the case of large numbers of public goods offered at low price, the attention/time constraint is binding. This is the reason that we do not see agents aggregating in their heads all the knowledge of universe.

The second assumption implies the existence of potential “forks”, if we may use a term from the open source software movement. A fork occurs when a group of software developers contributing to a project decide they would be happier contributing to a version of the project that goes in a slightly different direction. Thus, the development path takes a fork, with some developers going one direction and the rest in another. In our case, we mean that if a large number of agents are consuming a given public project, there will exist another, slightly different project that a significant fraction of these agents prefer by at least a small amount. You can’t please all the people all the time. Tastes are diverse enough that you will not get universal agreement on the very best form for a public project.

Although the optimal allocations are exactly what we believe we see in the real world, things get tricky when the question of how to support these allocations as equilibrium outcomes is considered. We defined two notions of equilibrium, one anonymous and one nonanonymous. Although both have extremely (and unrealistically) large information requirements, neither satisfied a First Welfare Theorem. Obviously, relaxing the information requirements would only make matters worse. While this might be viewed as a negative result, we think it is better viewed as a positive conclusion. Information goods are infinitely variable, and have the kind of discrete, non-metrizable structure we describe in the paper. There is no sense in which a Jessica Simpson song can be compared to a Bach cantata. At least we know of no hedonic or other quantitative metric that could do so. They are simply different. The market, therefore, has a hard time signaling that a certain new public project should be produced. Entrepreneurs take educated guesses about what will succeed but there are no arbitrage opportunities implied by disparities in the cost/revenue signals from equilibrium price system that are visible to all. Thus, we argue that the conclusion should be that we generally do not see first best outcomes in such an economy. It is possible to get rich (that is, make economic profits) if you happen to stumble on a public project that you can produce cheaply and is in high demand. It is not at all surprising that no one beat you to it. There may indeed be five dollar bills laying on the ground in the new information economy.

On the other hand, we also show that Pareto optimal allocations exist for the finite case, and exist in the ϵ sense for the continuum case. In addition, any strictly Pareto optimal allocation can be supported as an equilibrium for the continuum case, and any Pareto optimal allocation can be supported as an equilibrium for the finite case. Thus, the lemmas we show that characterize the Pareto optimal allocations are not vacuous. In addition, if do we happen to find ourselves at a Pareto optimal allocation, the price system can at least support it.

There is clearly room for much additional work in this area. To mention only two questions: it would be desirable to have an equilibrium notion for which a First Welfare Theorem does hold. This would have to have extremely high information requirements and would most likely not have much real world appeal, however. We have also not considered the core or its relationship equilibrium allocations. We do think that the basic model we propose correlates reasonably well to a significant class of public goods consumed today. We hope that exploring variations of this approach will yield some insight into how the internet economy should be managed and governed.

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