INCENTIVES IN EXPERIMENTS: A THEORETICAL ANALYSIS[†]

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ABSTRACT. The purpose of an experiment is to observe choice in a controlled setting. When subjects are given multiple decisions, however, subjects' choices in one decision may be distorted by the choices made in others. Assuming only eventwise monotonicity (dominated gambles are never chosen), we prove that paying for one random problem—the *Random Problem Selection* (RPS) mechanism—is essentially the only incentive compatible payment mechanism. We also discuss situations where monotonicity may fail, and show that paying for every decision is incentive compatible if and only if a 'no complementarities at the top' (NCaT) condition is assumed.

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I Introduction

The value of laboratory experimentation lies in the researcher's ability to make observations in controlled environments. Chemists study pure samples at controlled temperatures. Physicists fire lasers through vacuums. In economics, experimenters observe choices in isolation, abstracting away the complexities of the field. When subjects make multiple decisions in a single experiment, however, this control may be lost because the payment from one decision may impact subjects' choices in another. Even randomized control trials in the field could be compromised by having multiple incentivized decisions. In this paper, we say that an experiment is incentive compatible if choices are not distorted by such complementarities. Whether this is satisfied depends both on subjects' preferences, and on the payment mechanism in use (for example, which problems are chosen for payment). Our goal is to provide a framework for analyzing experimental incentives, and a characterization of incentive compatible payment mechanisms using minimal assumptions about subjects' preferences.

The most common practice in modern experimental economics is to pay subjects for every decision (see Table I below). This mechanism is particularly vulnerable to complementarities such as wealth effects, portfolio effects, and hedging. A proposed alternative is to pay for one randomly-selected decision—what we call the Random Problem Selection (RPS) mechanism.[1] Although this apparently solves the complementarities problem, there are examples of preferences for which the RPS mechanism is not incentive compatible (Holt, 1986, e.g.). Thus, exact conditions under which this mechanism is incentive compatible are not well understood. Neither is it known whether other mechanisms can be used to guarantee truthful revelation of choices in experiments with multiple decisions.

In this paper, we develop a very general, choice-based theoretical framework for studying incentives in experiments. We do not require preferences to satisfy the independence axiom, or even that gambles have known probabilities. No structure is assumed on the set of choice objects: It could include consumption goods, objective lotteries, ambiguous acts, announcements of preferences, or strategies in a game. We assume subjects have underlying preferences over these choice objects. In an auction experiment, for example, choice objects are bidding strategies, and the experimenter wishes

[1]This is most often called the Random Lottery Incentive Mechanism (RLIM). Like Beattie and Loomes (1997), we avoid that term only because our subjective framework is more general than one that restricts randomness to be represented by objective lotteries.

to infer subjects' preferences (choices) over those strategies. We also assume that preferences extend naturally to random gambles over those choices. We prove that the RPS mechanism is incentive compatible if these extended preferences satisfy Savage's eventwise monotonicity axiom, which simply says that dominated gambles are never chosen. When nothing more than monotonicity is assumed, we prove that the RPS mechanism (or, a slightly generalized version of it) is the *only* incentive compatible mechanism. Thus, we provide a full characterization of incentive compatible payment mechanisms under the monotonicity assumption.

Our characterization identifies a small class of mechanisms other than the RPS mechanism that may also be incentive compatible in certain experiments. Essentially, the experimenter must either pay for one randomly-selected decision problem that the subject actually faces, or else a hypothetical decision problem for which the subject's optimal choice can always be inferred from their actual choices. Such hypothetical decision problems are called surely identified sets, and we refer to this broader family of mechanisms as Random Set Selection (RSS) mechanisms.[2] In practice, non-trivial surely identified sets rarely exist, in which case these mechanisms cannot be used. Therefore, the RPS mechanism is the unique incentive compatible mechanism in most experimental designs.[3]

Our framework follows Savage (1954), modeling random gambles as *acts* that map states of the world into outcomes. This allows for unobservable, subjective (and even non-probabilistic) beliefs by subjects, including ambiguity. Even when an experimenter draws balls from an urn or rolls a die, a subject who misunderstands the device or perceives a malevolent nature (Ozdenoren and Peck, 2008, e.g.) may have beliefs that differ substantially from the long-run frequency of the randomizing device's outcomes. The Savage framework naturally encompasses such preferences, but also includes the standard expected utility model as a special case.

In a companion paper (Azrieli et al., 2012), we explore incentives in experiments assuming subjects view gambles as lotteries with objective probabilities. Since experimenters frequently use objective randomizing devices such as die rolls, it may be appropriate to model gambles as having known probabilities. Monotonicity is stronger in that

[2]Incentive compatibility also requires that each decision problem's optimal choice be inferrable from the paid hypothetical problems, and that messages inconsistent with any preference never be optimal (see Theorem 1).

[3]To our knowledge, the only example of an RSS mechanism being used that pays based on a hypothetical decision is in Krajbich et al. (2010).

Payment Mechanism:	Only 1 Task	None Paid	One Random	Some Random	All Paid	Rank-Based	Total
	Individual Choice Experiments						
'Top Five' Journals	7	0	4	0	4	0	15
Experimental Economics	2	0	1	0	3	0	6
	Muti-Person (Game) Experiments						
'Top Five' Journals	6	0	0	0	9	0	15
Experimental Economics	8	1	3	3	5	1	21

TABLE I. Payment mechanisms used among (lab and field) experimental papers published in 2011. The 'top five' journals are *Econometrica*, *American Economic Review*, *Journal of Political Economy*, *Quarterly Journal of Economics*, and *The Review of Economic Studies*.

environment, requiring that subjects' preferences over gambles respect first-order stochastic dominance. Studying objective lotteries adds substantial structure to the problem, but—somewhat surprisingly—the results are qualitatively similar to those generated in the Savage framework. The set of incentive compatible mechanisms is slightly larger with objective lotteries, but the non-RPS mechanisms still require that choices from hypothetical decision problems be inferrable from actual decision problems. Again, this rarely occurs in practice, so the RPS mechanism is essentially the only one that is incentive compatible in either setting.

The incentive compatibility problem arising from complementarities across decisions has been known since at least Wold (1952) and Allais (1953). Yaari (1965, p.285) is one of the first examples of the RPS mechanism being used in practice with the intent of eliminating complementarities. Grether and Plott (1979) and Cox and Epstein (1989) both point to income effects (a type of complementarity) as a possible explanation for observed preference reversals in past work. Many experiments since have provided evidence of various types of complementarities, including portfolio effects (Laury, 2005), income effects (Lind and Plott, 1991; Kagel and Levin, 1991; Ham et al., 2005), and hedging (Blanco et al., 2010; Armantier and Treich, 2013).

Despite these concerns, paying for every decision is still the most frequently-used payment mechanism in economics experiments. A survey of experimental papers published in 2011 in top journals and in the field journal *Experimental Economics* (Table I) reveals that most experimentalists pay for every decision, with the RPS mechanism being used less than half as frequently—particularly in tests of multi-player games.[4]

[4]This includes both lab and field experiments. Most field studies only have subjects engage in one task, in which case subjects are either paid for the one task or not paid at all. 'Some Random' refers to experiments

One commonly-stated reason to pay for every decision is that complementarities are not expected to distort incentives in most environments. Thus, there is no need to use the RPS mechanism. This is a plausible argument, so we formalize exactly the condition on preferences necessary to make the pay-for-all mechanism incentive compatible. This condition—called No Complementarities at the Top (NCaT)—says that if any subset of decision problems is chosen, and the subject is paid their most-preferred item from each chosen problem, then that bundle is preferred to any other bundle they could receive from the same subset of problems. In other words, the bundle of favorites must also be the favorite bundle. Whether or not such an assumption is appropriate clearly depends on the nature of the decision problems under investigation.[5]

We stress that our paper informs but does not resolve the debate about which payment mechanism to use in practice. As emphasized by Proposition 1, any theoretical exploration of incentive compatibility must start with assumptions on subjects' preferences. Here we assume only monotonicity. This axiom dates back to Wald's (1939) notion of admissibility in statistical decision theory, and has been central to almost all models of choice under uncertainty, including Savage (1954); Luce and Raiffa (1957); Anscombe and Aumann (1963); Gilboa and Schmeidler (1989); Schmeidler (1989); Machina and Schmeidler (1992); and Maccheroni et al. (2006).[6]

Whether or not monotonicity holds in practice, however, is an empirical question, and the answer is likely to vary depending on the context. Although it appears innocuous in abstract settings, monotonicity often becomes quite restrictive in more structured environments when coupled with other commonly-assumed axioms. For example, with objective lotteries, monotonicity and the reduction of compound lotteries jointly imply the independence axiom. Various critiques of this axiom appear in the theory literature, and violations of monotonicity have been documented in certain laboratory experiments.[7] Thus, we cannot recommend the use of the RPS mechanism universally.

that pay 2, 3, 4, or 5 decisions, with some paid decisions possibly being fixed. Among 'Some Random' and 'All Paid' experiments, 18 showed subjects outcomes every period, while the other six only showed outcomes at the end; all nine of the 'Top 5' experiments showed outcomes every period. The rank-based payment mechanism gives payments to players based on their relative hypothetical earnings summed across all decisions. The one unpaid experiment used children as subjects. Several authors failed to describe the payment mechanism in the paper; in those cases it was either found in the instructions or inferred from conversion rates and average earnings.

[5]Paying for all decisions may still be acceptable in practice if violations of NCaT are 'small'. Our current framework does not include a metric for analyzing robustness to such small deviations, so we focus only on exact incentive compatibility. Robustness analysis is an interesting direction for future work.

[6]Monotonicity appears in Savage (1954) as Theorem 3, which is equivalent to his P3 under the sure-thing principle (P2).

[7]We review these critiques and documented violations in Section V.

Rather, the onus is on the experimenter to argue (or test) whether monotonicity is an appropriate assumption in the context of their experimental design. If so, then the RPS mechanism is clearly recommended. If NCaT is assumed, then paying for all decisions is also recommended. The 'right' payment mechanism to use depends crucially on the assumptions the experimenter is willing to make.

In our view, the most significant contribution of this work is in defining appropriate notions of payment mechanisms and incentive compatibility in a general framework that is tailored to experimental situations. This enables a more transparent dialogue on incentives in experiments. For example, Holt's (1986) criticism of the RPS mechanism can now be understood as a simple monotonicity violation, caused by the joint imposition of non-expected utility and the reduction of compound lotteries. The seemingly disparate concerns about subject wealth effects, portfolio effects, and hedging can now be viewed together as examples of complementarities. The one assumption used to rule out complementarities in the general framework can then be re-interpreted in the specific settings where each of these types of complementarities occur. One could explore incentive compatibility under more specific preference models such as cumulative prospect theory, regret aversion, or case-based decision theory. Alternatively, for any given mechanism, one could derive necessary and/or sufficient conditions on preferences for that mechanism to be incentive compatible.

We employ standard mechanism design techniques, drawing on the seminal works of Vickrey (1961), Hurwicz (1972), Green and Laffont (1977), Myerson (1981), and others. Our work shares much in spirit with these classical notions, but there are differences. First, we are not interested in eliciting an entire preference relation or implementing a social choice correspondence. Instead, we wish only to observe a collection of choices. This collection is chosen by the experimenter to address her particular research question, and is therefore treated as an exogenous constraint in our study. Furthermore, we desire a *strict* notion of incentive compatibility: It is not enough that a decision maker weakly prefers to announce her 'true' choices, but also that she would never be willing to announce false choices. Weak incentive compatibility may lead to incorrect inference if subjects deviate to non-truthful choices that make them no worse off.

The strict notion of incentive compatibility makes the incentive compatibility problem extremely difficult. As we establish, it is rare that a deterministic mechanism will allow for strict incentive compatibility. This is why we must work with random mechanisms. The papers closest to ours in this respect are Gibbard (1977), Barbera (1977), and Barbera et al. (1998). In fact, the idea of extending a given preference over outcomes to an extension over lotteries that respects monotonicity first appears in these works. Our main characterization result—placed in the domain of objective lotteries—is related to the main result of Gibbard (1977). Gibbard takes interest in a multiple-agent environment where payoffs are probabilistic. He seeks to elicit the entire preference relation, and only requires weak incentive compatibility. Gibbard's characterization is geometric, and he never explores the 'random selection' aspect of the rules. Our goal is to understand incentive compatible rules from an experimental design standpoint, so the practical and intuitive structure of the incentive compatible rules as random selection mechanisms is a crucial point.

Finally, our paper resurrects a foundational issue in revealed preference theory, dating back at least to Wold (1952): Inferring an entire preference requires observing true choices from many different menus, but complementarities across these choices may render such observation impossible. We formalize this argument in our Proposition 1: Without additional assumptions, only one choice can be observed truthfully. Counterfactual choices (hence, preferences) cannot be inferred, and so novel predictions cannot be made. But if one is willing to make an assumption such as monotonicity, then incentive compatible mechanisms may exist. These mechanisms can then be used to infer preferences and make predictions about behavior. Thus, revealed preference theory has non-trivial testable implications only when incentive compatible methods for observing multiple choices are employed.

II The General Framework

The set of possible *choice objects* is given by the set X. The decision maker (also called the *subject*) has a complete, transitive preference relation $\succeq$ over X. For any $x \in X$, let $L(x,\succeq) = \{y \in X \ : \ x \succeq y\}$ and $U(x,\succeq) = \{y \in X \ : \ y \succeq x\}$ be the lower- and upper-contour sets of x according to $\succeq$, respectively. The $\succeq$-*dominant* elements of any set $E \subseteq X$ are denoted by

$$\mathrm{dom}_{\succeq}(E) = \{x \in E : (\forall y \in E) \ \ x \succeq y\}.$$

Where applicable, strict preference (the asymmetric part of $\succeq$) is denoted by $\succ$.

The researcher has an exogenously-given list of k decision problems, denoted $D = (D_1,\ldots,D_k)$, where each decision problem D_i is a finite set (or, menu) of choice objects from which the subject is asked to choose. Thus, $D_i \subseteq X$ for each $i \in \{1,\ldots,k\}$. Let $\mathscr{D} = \{D_1,\ldots,D_k\}$ represent the set of decision problems. Empty decision problems and decision problems with only one element have no impact on incentives, so, without loss, we assume $|D_i| > 1$ for all i.[8]

The researcher does not know the subject's preference relation $\succeq$, but wants to use the decision problems to observe some properties of $\succeq$. For example, a researcher interested in correlating risk preferences with time discounting may have D_1 be a set of lotteries used to assess risk aversion, and D_2 be a set of intertemporal choices.

Since choices are restricted to D, the choice data from the experiment can be thought of as an announced choice vector $m = (m_1, \ldots, m_k)$, with $m_i \in D_i$ for each i. We sometimes call this the subject's *message*. The space of all possible messages is $M = \times_i D_i$. For each $i \in \{1, \ldots, k\}$, let

$$\mu_i(\succeq) = \text{dom}_{\succeq}(D_i)$$

be the set of $\succeq$-dominant elements of D_i, and define $\mu(\succeq) = \times_i \mu_i(\succeq)$. We refer to $m \in \mu(\succeq)$ as a *truthful* message for $\succeq$.

The researcher pays the subject—based on their message—using objects from X. For example, if $D_1 = \{\text{cake,pie}\}$ and the subject announces $m_1 = \{\text{cake}\}$, he may be given a piece of cake as payment for that decision. If the subject is paid something outside of $\bigcup_i D_i$, then X can be expanded to include the payment objects as well as the choice objects. In particular, this framework allows for the payment to be a 'bundle' of choice objects, since such bundles may be incorporated directly into X. We discuss in detail the case of bundle payments in Section IV.

A randomizing device may be used to select payment objects from X. Let Ω be the state space containing all possible realizations ω of the randomization device. We adopt the subjective uncertainty framework of Savage (1954), modeling a random payment as an *act*—a mapping from Ω into X. The space of all acts is X^Ω. We assume throughout that Ω is finite.[9] A constant act is one that pays the same object in X regardless of the state. For example, if $f(\omega) = \{\text{cake}\}$ for every $\omega \in \Omega$, then f is a constant act.

In an experiment, payments depend on the subject's announced choices. A *(payment) mechanism* ϕ therefore takes the announced choice m and awards the subject with an act in X^Ω. Thus, $\phi : M \to X^\Omega$. If ϕ is a payment mechanism, then $\phi(m)(\omega)$ identifies the choice object in X that is paid if the subject announces choice vector m and state ω obtains.

[8]It is possible that such decision problems do affect preferences through a *framing* effect. Framing effects, however, do not affect incentives *within* a given experiment. We discuss framing in the sequel.

[9]The results here continue to hold as stated for infinite state spaces. However, our notion of monotonicity does not allow for "null" states and so may be less compelling in such environments.

We refer to the pair (D,ϕ) as an *experiment*; D completely specifies the choices the subject must face, and ϕ describes how they are paid for those choices. Since ϕ includes a description of its domain D, there is little distinguishing an experiment (D,ϕ) from its associated mechanism ϕ; when it causes no confusion, we refer to experiments and mechanisms interchangeably.[10]

The original preference $\succeq$ is defined over elements of X, not X^Ω. But payments are objects in X^Ω, and these payments must be evaluated by the subject when making their choices. To this end, we assume that the subject has a complete and transitive preference $\succeq^*$ on the space X^Ω which "extends" $\succeq$ in a sense to be made precise. To avoid confusion, we henceforth refrain from calling $\succeq^*$ a preference relation; instead, we refer to it as an extension of $\succeq$. The asymmetric relation $\succ^*$ denotes the asymmetric part of $\succeq^*$.

A given experimenter may view some extensions as plausible ('admissible'), and others not. In general, let $\mathscr{E}(\succeq)$ be the experimenter's set of admissible extensions for any given preference relation $\succeq$. Thus, $\mathscr{E}$ is a correspondence mapping preferences into admissible extensions. At this point, we only assume that all admissible extensions $\succeq^* \in \mathscr{E}(\succeq)$ agree with $\succeq$ on the space of constant acts. Formally, if $x \succeq y$, and if f and g are constant acts giving x and y, respectively ($f(\omega) = x$ and $g(\omega) = y$ for every ω), then $f \succeq^* g$ for all $\succeq^* \in \mathscr{E}(\succeq)$.[11] Additional restrictions on $\mathscr{E}$ will be discussed throughout. The incentive properties of a given mechanism depend critically on what extensions the experimenter considers admissible, and so all of our key concepts and results directly depend on $\mathscr{E}$.

A successful experiment is one in which the payment mechanism always induces the subject to announce their choices truthfully, regardless of $\succeq$ and $\succeq^* \in \mathscr{E}$. We refer to this as *incentive compatibility* (with respect to $\mathscr{E}$) of the experiment.[12]

Definition 1 (Incentive Compatibility). A mechanism ϕ is incentive compatible with respect to $\mathscr{E}$ if, for every preference $\succeq$, every extension $\succeq^* \in \mathscr{E}(\succeq)$, every truthful message $m^* \in \mu(\succeq)$, and every message $m \in M$, we have that $\phi(m^*) \succeq^* \phi(m)$, with $\phi(m^*) \succ^* \phi(m)$ whenever $m \notin \mu(\succeq)$.

[10]We do not consider the mechanism to be fixed or exogenous from the viewpoint of the experimenter. Rather, the mechanism is a variable chosen by the experimenter, and we study which mechanisms have desirable incentive properties. This follows the standard mechanism design approach.

[11]We could equivalently view $\succeq^*$ as primitive, and $\succeq$ as the restriction of $\succeq^*$ to constant acts.

[12]Starmer and Sugden (1991) and Bardsley et al. (2010) refer to incentive compatible experiments as *unbiased*. Cox et al. (2012a) say they satisfy the *isolation hypothesis*. Starmer and Sugden (1991) and Cubitt et al. (1998) refer to complementarities as *contamination*.

In other words, incentive compatible experiments induce the subject to announce truthfully, treating each decision problem as though it were in isolation. When there is no confusion, we drop the reference to $\mathscr{E}$ and simply refer to ϕ as incentive compatible.

Model Interpretation

Our framework is quite general, covering nearly any choice-based, incentivized experiment.[13] It covers laboratory experiments, field experiments, consumer focus groups, and essentially any other incentivized choice elicitation procedure. This breadth of applicability arises because no structure is assumed on the set of choice objects X. Elements of X may be consumption goods. They may represent monetary payments to the subject, or vectors of payments to all subjects. If D_i is a game, then each $x \in D_i$ would represent a strategy choice, possibly modeled as an act that maps others' strategies into payoffs.[14] If the game is extensive-form, then x represents a behavioral strategy. In preference elicitation tasks (Becker et al., 1964, e.g.), each x would represent a possible announcement of the subject's value, which leads to a particular lottery payment. In the field, x may represent a labor decision, a portfolio allocation, a decision to exercise, the online purchase of a good, or any other choice that can be made.

Because X lacks structure, preferences over X are completely general. Following the revealed preference tradition, $\succeq$ simply represents choices from X. This can capture, for example, the idea that people care about others' outcomes. And that they care about the actions taken that lead to those outcomes (Falk et al., 2008, e.g). In games, we make no assumption of earnings maximization or equilibrium play. Preferences are unrestricted, and the researcher seeks to uncover interesting regularities in those preferences.

Preferences (and their extensions) may depend on the experiment. Incentive compatibility only requires that preferences in the current experiment be truthfully elicited. It does not require that preferences be stable across experiments. For example, suppose $D_1 = \{\text{soda,milk}\}$, $D_2 = \{\text{salad, fish}\}$, and $D'_2 = \{\text{cheeseburger, pizza}\}$. Even in an incentive compatible mechanism, we might find different announced choices in D_1 in experiment $D = (D_1, D_2)$, compared to experiment $D' = (D_1, D'_2)$. This is because the underlying preferences are actually changed by the content of the other decision problem; the mere

[13]Experiments that only gather non-choice data such as neural activations, eye movements, pupil dilation, or galvanic skin response are excluded.

[14]This does require that we model strategy choice as a decision problem (as in Aumann, 1987), thus ignoring any 'interactive epistemology'.

discussion of healthy food alternatives may prompt a healthy drink choice. The presence of such *framing effects* is not an indictment of the experiment's incentives, but rather a suggestion that preferences over D_1 are highly sensitive to the context of the decision environment and, therefore, that laboratory results for this decision may be difficult to generalize.

Decision problems may be given simultaneously or sequentially. As long as the composition of decision problems cannot be affected by subjects' choices, the incentive compatibility of the RPS mechanism (Corollary 1) is robust to timing.[15] This is true even if preferences change over time, perhaps due to framing effects or learning. If the choice in one decision problem affects the alternatives offered in later problems, however, then we must view them together as one large, dynamic decision problem. Our results would not hold otherwise.

Recall that decision problems are units of observation meant to be viewed as independent. Thus, the content of the decision problems can vary depending on the research question. For example, suppose a subject is asked to play a sequence of 20 one-shot prisoners' dilemma games with randomly-drawn opponents and feedback after each period. A researcher interested in one-shot prisoners' dilemma games—and how strategy choices (preferences) change over time—would view these as 20 separate decision problems. A researcher interested in the 'contagion' folk theorem of Kandori (1992), however, would view this as one large decision problem in which every period is part of a larger supergame. In this case, payments for the 20-period supergame should mimic the accrual of payments in the repeated game model being tested (see Chandrasekhar and Xandri, 2011, e.g.).

Restrictions on Admissible Extensions: Monotonicity and Richness

We first consider the case where the researcher is not willing to make *any* assumptions about the set of admissible extensions.

Proposition 1. If every extension is admissible, then there exists an incentive compatible payment mechanism if and only if $k = 1$ (the experiment has only one decision problem).

Proofs not provided in the text appear in appendix.

[15] We would extend the state space to capture uncertainty about future decision problems, and choice at each time t would represent an 'uncertain payment act' in which future choices are unknown. Monotonicity (defined below) would extend naturally to this domain.

Most experiments collect data from multiple decision problems. Proposition 1 verifies that, in these experiments, incentive compatibility is never free: The experimenter must make *some* assumptions about subjects' possible extensions. This implies that every multiple-decision experiment necessarily represents a joint test of both the main research question and the assumptions necessary to guarantee incentive compatibility of the payment mechanism.

One natural restriction on extensions is that they respect dominance. Given the underlying preference $\succeq$ on X, act f is said to *dominate* act g (written $f \sqsupseteq g$) if, for every $\omega \in \Omega$, $f(\omega) \succeq g(\omega)$. If $f \sqsupseteq g$ and $f(\omega) \succ g(\omega)$ for some ω then f strictly dominates g ($f \sqsupset g$).[16] An extension that respects the dominance relation is said to be *(eventwise) monotonic*.

Definition 2 (Monotonicity). The extension $\succeq^*$ is a monotonic extension of $\succeq$ if $f \sqsupseteq g$ implies $f \succeq^* g$, and $f \sqsupset g$ implies $f \succ^* g$.

For each $\succeq$, let $\mathscr{E}^{\text{mon}}(\succeq)$ be the set of all monotonic extensions of $\succeq$.[17] Our main results concern the case where all admissible extensions are monotonic ($\mathscr{E}(\succeq) \subseteq \mathscr{E}^{\text{mon}}(\succeq)$). Here, a sufficient condition for incentive compatibility is that acts resulting from truth-telling messages dominate all acts resulting from any other message, with strict dominance whenever the other message is not truthful. This fact is used throughout our paper, so we prove it formally in the following lemma.

Definition 3 (Truth Dominates Lies). A mechanism ϕ has the *truth-dominates-lies (TDL) property* if, for every $\succeq$, every $m^* \in \mu(\succeq)$, and every $m \in M$, we have that $\phi(m^*) \sqsupseteq \phi(m)$, with $\phi(m^*) \sqsupset \phi(m)$ whenever $m \notin \mu(\succeq)$.

Lemma 1. If $\mathscr{E}(\succeq) \subseteq \mathscr{E}^{\text{mon}}(\succeq)$ for every $\succeq$ and ϕ has the TDL property then ϕ is incentive compatible with respect to $\mathscr{E}$.

We will also study cases where *every* monotonic extension is admissible ($\mathscr{E}(\succeq) = \mathscr{E}^{\text{mon}}(\succeq)$), or at least that the class of admissible extensions is "sufficiently rich".

Definition 4 (Richness). The correspondence $\mathscr{E}$ is *rich* (or, satisfies *richness*) if, for every $\succeq$,

(1) $\mathscr{E}(\succeq) \subseteq \mathscr{E}^{\text{mon}}(\succeq)$, and

[16] Technically, $\sqsupseteq$ and $\sqsupset$ depend on $\succeq$. Because it will always be obvious, we use a notation which suppresses this dependence.

[17] This definition of monotonicity implicitly assumes that all states $\omega \in \Omega$ are non-null under each $\succeq^* \in \mathscr{E}^{\text{mon}}(\succeq)$, which essentially means that the subject views every state as having some (possibly tiny) chance of occurring. See Savage (1954, p.24) for a formal definition.

(2) if $f \succeq^* g$ for every $\succeq^* \in \mathscr{E}(\succeq)$, then $f \sqsupseteq g$.

In addition to monotonicity, our definition of richness requires that each $\mathscr{E}(\succeq)$ be sufficiently large so that if f is preferred to g for every admissible extension, then we can conclude that f dominates g. This is satisfied in many standard domains of extensions, including the following cases:

- Every monotonic extension is admissible ($\mathscr{E} = \mathscr{E}^{\text{mon}}$).
- Every (subjective) expected utility extension is admissible.
- For every state ω, there is an admissible expected utility extension that puts subjective probability on ω arbitrarily close to one.
- Every probabilistically sophisticated extension is admissible.
- Every multiple priors extension is admissible.

Lemma 2. Suppose $\mathscr{E}$ is rich. A mechanism ϕ is incentive compatible with respect to $\mathscr{E}$ if and only if it has the TDL property.

Thus, our characterization theorem—which assumes richness—is simply a characterization of mechanisms satisfying the TDL property.

Incentive Compatibility of the RPS Mechanism

The following definition formalizes the RPS mechanism in our framework.

Definition 5 (Random Problem Selection Mechanisms). A payment mechanism ϕ is a *random problem selection* (RPS) mechanism if there is a fixed partition $\{\Omega_1, \dots, \Omega_k\}$ of Ω with each Ω_i non-empty such that, for each $r \in \{1, \dots, k\}$ and $m \in M$, $\omega \in \Omega_r$ implies that

$$\phi(m)(\omega) = m_r.$$

It is well known that the RPS mechanism is incentive compatible when all admissible extensions satisfy the expected utility axioms. What is not well understood in the existing literature is whether the RPS mechanism is incentive compatible under more general preferences. With our framework, it is easy to see that the RPS mechanism satisfies the TDL property: deviating from truth-telling to a less-preferred option in some D_i results in an act that gives a worse outcome in the state where D_i is chosen for payment. If extensions are monotonic, this property is sufficient for incentive compatibility. This simple argument proves the following result.

Corollary 1. If $\mathscr{E} \subseteq \mathscr{E}^{\text{mon}}$ then every RPS payment mechanism is incentive compatible with respect to $\mathscr{E}$.

In the appendix, we provide a (mechanism-specific) weakening of monotonicity—which we call ϕ-monotonicity—and show that an RPS mechanism ϕ is incentive compatible with respect to $\mathscr{E}$ if and only if every extension in $\mathscr{E}$ satisfies ϕ-monotonicity. In this sense, ϕ-monotonicity characterizes incentive compatibility of the RPS mechanisms.[18] Since monotonicity implies ϕ-monotonicity, Corollary 1 follows immediately.

III A Characterization of Incentive Compatible Mechanisms

To obtain a full characterization of incentive compatible mechanisms, we assume in this section that only strict preferences on X are admissible. We discuss the case of weak preferences after the main theorem.

With strict preferences, the most-preferred element in each decision problem is unique ($|\mu_i(\succeq)| = 1$). In that case, Lemmas 1 and 2 would be modified slightly to say that (D,ϕ) is incentive compatible with respect to a rich $\mathscr{E}$ if and only if, for every preference $\succeq$ and every message $m \neq \mu(\succeq)$,

$$\phi(\mu(\succeq)) \sqsupset \phi(m).$$

Another consequence of assuming strict preferences is that there may be messages that cannot be truthful for any preference relation. For example, if $D_1 = \{x, y\}$, $D_2 = \{y, z\}$, and $D_3 = \{z, x\}$, then $m = (x, y, z)$ cannot be *rationalized* by any $\succeq$, since it would imply $x \succ y \succ z \succ x$. We therefore need to distinguish messages that can be rationalized from those that cannot. Let

$$M_R = \left\{m \in M : (\exists \succeq)\ m = \mu(\succeq)\right\}$$

be the set of *rationalizable* messages. $M_{NR} = M \setminus M_R$ is then defined as the set of non-rationalizable messages. One immediate necessary condition for incentive compatibility with respect to a rich $\mathscr{E}$ is that, for any $\succeq$, all non-rationalizable messages be dominated by the truthful message.

To understand how incentive compatibility can extend beyond the RPS mechanism, consider a mechanism ϕ and suppose for the moment that the subject's submitted announcement vector m^* is truthful, meaning $m^*_i = \mu_i(\succeq)$ for each i. Given any other set $E \subseteq X$, we can also ask whether m^* reveals the subject's true favorite element in E. If ϕ pays this 'inferred favorite' element from E in some state of the world, incentive compatibility may still be maintained.

[18]The ϕ-monotonicity assumption is a form of local monotonicity that applies only for specific acts which depend on the particular experiment under consideration. We also show in Proposition 3 that if one requires ϕ-monotonicity for all possible experiments that use the RPS mechanism then the (global) monotonicity axiom is implied.

To construct such mechanisms, we must first characterize the sets E whose favorite element can always be inferred from the subject's choices in $D = (D_1, \dots, D_k)$. To do this, we must identify the preferences that are revealed by any truthful announcement $m \in M_R$. For two distinct choice objects $x, y \in X$, say that x is *directly revealed preferred* to y under announced choices $m = (m_1, \dots, m_k)$ if there is $1 \le i \le k$ such that $m_i = x$ and $y \in D_i$. Denote the transitive closure of this relation by $R(m)$, and say that x is *revealed preferred* to y under choices m if $x R(m) y$.[19]

The relation $R(m)$ is transitive and asymmetric since $m \in M_R$, but it need not be complete. Denote by $L(x,m) = \{y \in X : x R(m) y\}$ and $U(x,m) = \{y \in X : y R(m) x\}$ the sets of elements that are revealed to be worse than x and better than x under choices m, respectively. Clearly, $L(x,m) \subseteq L(x, \succeq)$ and $U(x,m) \subseteq U(x, \succeq)$ when $m = \mu(\succeq)$, with strict inclusions for some x when $R(m)$ is not a complete relation. It is well known (Richter, 1966, e.g.) that m is rationalizable ($m \in M_R$) if and only if $R(m)$ is acyclic.

Let

$$\mathrm{dom}_m(E) = \{x \in E : (\forall y \in E) \;\; x R(m) y\}$$

be the set of $R(m)$-*dominant* elements of E. If E has no element that is $R(m)$-dominant (meaning m does not reveal the most-preferred element of E), then $\mathrm{dom}_m(E) = \varnothing$. Otherwise, $\mathrm{dom}_m(E)$ contains a unique element since preferences are strict. If $m = \mu(\succeq)$, then either $\mathrm{dom}_m(E) = \varnothing$ or else $\mathrm{dom}_m(E) = \mathrm{dom}_{\succeq}(E)$.

We can now describe the sets $E \subseteq X$ whose most-preferred elements are always revealed when the subject submits a truthful message.

Definition 6 (Surely Identified Sets). A non-empty set $E \subseteq X$ is *surely identified (SI)* if, for every $m \in M_R$,

$$\mathrm{dom}_m(E) \neq \varnothing.$$

In other words, E is SI if, for any order $\succeq$, the message $m = \mu(\succeq)$ identifies the most-preferred element of E, so that $\mathrm{dom}_m(E) = \mathrm{dom}_{\succeq}(E)$.

Let $SI(\mathscr{D})$ be the collection of sets surely identified from the given set of decision problems $\mathscr{D}$.[20] Obviously, any D_i is in $SI(\mathscr{D})$, but there may be other sets in $SI(\mathscr{D})$. For instance, if $D_1 = \{x,y\}$, $D_2 = \{y,z\}$, and $D_3 = \{z,x\}$, then $\{x,y,z\} \in SI(\mathscr{D})$. Also, any singleton set $\{x\}$ is trivially SI. A full characterization of SI sets is given by Proposition 4 in the appendix.

[19] Formally, $x R(m) y$ if there is a chain $x = z_1, \dots, z_l = y$ such that z_i is directly revealed preferred to z_{i+1} for every $i = 1, \dots, l_1$.

[20] Recall that $\mathscr{D} = \{D_1, \dots, D_k\}$ is the collection of decision problems, while $D = (D_1, \dots, D_k)$ is the ordered list of decision problems.

We wish to discuss mechanisms that choose sets in $SI(\mathscr{D})$ for payment; to this end, define the *payment set* of a mechanism ϕ at each state ω by

$$P^{\phi}(\omega) = \{\phi(m)(\omega)\}_{m\in M},$$

and the collection of all payment sets by

$$\mathscr{P}^{\phi} = \{P^{\phi}(\omega)\}_{\omega\in\Omega}.$$

In an RPS mechanism, $\mathscr{P}^{\phi} = \mathscr{D}$. The following definition generalizes RPS mechanisms to allow other surely identified sets to be used as payment sets.

Definition 7 (Random Set Selection Mechanisms). A mechanism ϕ is a *random set selection* (RSS) mechanism if

(1) $\mathscr{P}^{\phi} \subseteq SI(\mathscr{D})$, and
(2) if $m \in M_R$ then for each $\omega \in \Omega$, $\phi(m)(\omega) = \mathrm{dom}_m(P^{\phi}(\omega))$

The first condition requires that every payment set be surely identified, and the second requires that most-preferred elements are chosen from each payment set whenever messages are rationalizable. No restrictions are placed on the acts chosen at non-rationalizable messages.

In an RPS mechanism, each payment set is a decision problem, and each decision problem corresponds to at least one payment set ($\mathscr{D} = \mathscr{P}^{\phi}$). Since decision problems are surely identified, and since each m_i is the revealed-most-preferred outcome in D_i, RPS mechanisms are special cases of RSS mechanisms.[21] Our main theorem shows that a particular subclass of RSS mechanisms (which includes the RPS mechanisms) fully characterizes the set of incentive compatible mechanisms when the set of admissible extensions is rich.

Theorem 1. Let the admissible extensions $\mathscr{E}$ satisfy richness. A mechanism ϕ is incentive compatible with respect to $\mathscr{E}$ if and only if it is a random set selection (RSS) mechanism in which

(1) $D_i \in SI(\mathscr{P}^{\phi})$ for each $D_i \in \mathscr{D}$, and
(2) $\phi(M_R) \cap \phi(M_{NR}) = \varnothing$.

In other words, a mechanism is incentive compatible if and only if it is an RSS mechanism (rationalizable messages pay the most-preferred element from some randomly-selected, surely-identified set); each decision problem can be surely identified from the

[21] RPS mechanisms completely specify the payments from non-rationalizable messages, while RSS mechanisms do not. Thus, the family of RSS mechanisms for which $\mathscr{P}^{\phi} = \mathscr{D}$ can be strictly larger (in the sense of inclusion) than the family of RPS mechanisms.

payment sets; and non-rationalizable messages map to different acts than rationalizable messages.

The intuition for the above result is quite simple. First, to show that any such mechanism is incentive compatible, recall that incentive compatibility is satisfied when an agent with a preference $\succeq$ receives the best option in each state of the world according to $\succeq$. But this is precisely how our mechanisms are defined: ϕ chooses, for each m, an optimal element for any $\succeq$ which rationalizes m. Furthermore, condition (1) of the theorem guarantees that, if the subject misrepresents his preferences in any one of the decision problems, then this will affect the resulting output of the mechanism. Since truth-telling gives the best outcome in every state, this change will result in a dominated act. Condition (2) guarantees that reporting non-rationalizable messages is never optimal. The proof of the other direction is again quite simple, since the TDL property almost directly implies that the criteria for an RSS mechanism as well as conditions (1) and (2) of the theorem are satisfied.

Conditions for Uniqueness of the RPS Mechanism

Theorem 1 shows that non-RPS mechanisms (in which $\mathscr{P}^{\phi} \neq \mathscr{D}$) can be incentive compatible. Whether such mechanisms exist depends on $\mathscr{D}$. In practice, they are almost never observed, suggesting that most experiments naturally rule out these mechanisms. We now identify conditions on $\mathscr{D}$ under which these other incentive compatible mechanisms do not exist. For simplicity, we assume here that $D_i \neq D_j$ for all i, j.

First, if the decision problems are sufficiently distinct and small in number, then announced choices will not reveal the most-preferred element of any other set $E \notin \mathscr{D}$. In other words, every surely identified set will either be a decision problem or a singleton. Since every incentive compatible mechanism is an RSS mechanism (Theorem 1), and the payment sets of an RSS mechanism are all surely identified, it follows that $\mathscr{P}^{\phi} \subseteq \mathscr{D}$.[22]

Furthermore, it seems unlikely that an experimenter would want to give subjects a *redundant* decision problem—one that is surely identified by the remaining decision problems—, since there is no new information that can be obtained from that choice. Using condition (1) of Theorem 1, this implies that every decision problem must be a payment set at some state ($\mathscr{P}^{\phi} \supseteq \mathscr{D}$). Combining these two observations, we get that $\mathscr{P}^{\phi} = \mathscr{D}$, which essentially means that any incentive compatible mechanism is an RPS mechanism.

[22] There may be singleton payment sets which are not included in $\mathscr{D}$. We ignore this insignificant complication in this informal discussion.

To formalize this intuition we need the following definition.

Definition 8 (Redundant Decision Problems). Fix $D = (D_1, \ldots, D_k)$. The decision problem D_i is redundant in D if $D_i \in SI(\mathscr{D} \setminus \{D_i\})$. Say that D_i is non-redundant otherwise. The collection of all non-redundant decision problems is denoted $\mathrm{NRed}(\mathscr{D})$

We say that a payment mechanism ϕ is *essentially identical to an RPS mechanism* if there is an RPS mechanism ϕ^* such that $\phi(m)(\omega) = \phi^*(m)(\omega)$ for every $m \in M_R$ and ω at which $\mathscr{P}^{\phi}(\omega)$ is non-singleton. Such a mechanism can differ from an RPS mechanism in at most two ways: (1) ϕ may pay differently for non-rationalizable messages, and (2) there may be some states ω where ϕ pays a constant payment regardless of m.

Corollary 2. Let $\mathscr{E}$ satisfy richness. Every incentive compatible mechanism with respect to $\mathscr{E}$ is essentially identical to an RPS mechanism if and only if

$$SI(\mathscr{D}) \setminus \{\{x\}_{x \in X}\} = \mathrm{NRed}(\mathscr{D}).$$

In words, if the decision problems are non-redundant and there are no other non-singleton surely-identified sets, then the only incentive compatible mechanisms are essentially identical to RPS mechanisms. The following corollary gives a more easily-tested sufficient condition for RPS mechanisms to be the only incentive compatible mechanisms.

Corollary 3. Suppose $\mathscr{E}$ is rich and D is such that $M_{NR} = \varnothing$. If ϕ is incentive compatible with respect to $\mathscr{E}$ then it is essentially identical to an RPS mechanism.

Environments in which Corollary 3 are satisfied abound in experiments. For example, any experiment in which the D_i are pairwise disjoint satisfies the hypothesis. Since most experimental designs satisfy the conditions of this corollary, RPS mechanisms are, in practice, the only incentive compatible mechanisms.

Weak Preferences

We do not have a full characterization of the set of incentive compatible mechanisms when weak preferences are allowed, but we do know that it will be strictly between the set of RPS mechanisms and the mechanisms characterized by Theorem 1.

To see that non-RPS mechanisms can be incentive compatible with weak preferences, suppose $D_1 = \{x, y\}$, $D_2 = \{y, z\}$, and $D_3 = \{x, z\}$. Let $\Omega = \{\omega_1, \ldots, \omega_4\}$. For each $i \in \{1, 2, 3\}$, let $\phi(m)(\omega_i) = m_i$. Set $\phi(m)(\omega_4) = \mathrm{dom}_{\succ}(\{x, y, z\})$ if m can be rationalized by a strict preference relation $\succ$, and $\phi(m)(\omega_4) = z$ otherwise. Verifying incentive compatibility is easy, noting that the 'non-rationalizable' messages are truthful only for subjects with complete indifference.

But not all mechanisms identified by Theorem 1 are incentive compatible with weak preferences. Suppose D_1, D_2, and D_3 are as before, and we add $D_4 = \{x, y, z\}$. Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$. If m can be rationalized by a strict preference, set $\phi(m)(\omega_i) = m_i$ for each $i \in \{1,2,3\}$. Otherwise, let $\phi(m)$ pick x in ω_1, y in ω_2, and z in ω_3. The conditions of Theorem 1 are satisfied, so this is incentive compatible with strict preferences. But consider a subject with preferences $x \sim y \succ z$ for whom both $m = (x, y, x, x)$ and $m' = (x, y, x, y)$ are truthful. Because m can be rationalized and m' cannot, $\phi(m')$ is strictly dominated by $\phi(m)$. Thus, the subject has a truthful message that is strictly dominated, violating incentive compatibility. The mechanism does satisfy a weaker notion of incentive compatibility, however, in which at least one truthful message dominates all others.

IV Paying for Multiple Decisions

Theorem 1 has an important corollary: Paying for multiple bundles is generally not incentive compatible for a rich $\mathscr{E}$. If multiple decisions are paid, the resulting bundle payments are typically not elements of any one decision problem. The payment sets live outside of $\bigcup_i D_i$, and therefore cannot be surely identified by $\mathscr{D}$ unless they are singleton (*i.e.*, don't depend on the subject's message). But incentive compatibility on a rich $\mathscr{E}$ requires an RSS mechanism, and RSS mechanisms can only use payment sets that are surely identified by $\mathscr{D}$. Thus, paying in bundles can only be incentive compatible if bundle payments don't depend on the subject's message.

To formalize this argument, we first identify the set of outcomes in X that only belong to singleton payment sets, and thus are unaffected by the subject's announcement.

Definition 9 (Invariance). A mechanism ϕ is *invariant* on a set $E \subseteq X$ if, for every $\omega \in \Omega$ and $m, m' \in M$, $\phi(m)(\omega) \in E$ implies that $\phi(m')(\omega) = \phi(m)(\omega)$.

We conclude that incentive compatible mechanisms must be invariant on bundle payments (or, anything outside of $\bigcup_i D_i$).

Corollary 4. If $\mathscr{E}$ is rich and a mechanism ϕ is incentive compatible with respect to $\mathscr{E}$, then ϕ is invariant on $X \setminus \bigcup_{i=1}^k D_i$. In particular, it must be invariant on bundle payments that are not part of any one decision problem.

Corollary 4 shows that additional assumptions on preferences are necessary to justify bundle payments as incentive compatible. We now explore the strength of such assumptions.[23]

To formalize mechanisms that pay in bundles, let Y be a set of 'singleton items' so that each D_i is contained in Y (for instance, we may take Y to be equal to the union $\bigcup_{i=1}^{k} D_i$). Let $|Y|$ be the number of elements in Y. A bundle can now be described by a vector $x \in \mathbb{R}^{|Y|}$. Each $y \in Y$ is identified with a particular coordinate. A bundle $x \in \mathbb{R}^{|Y|}$ means that the agent consumes x_y units of alternative y. Admitting bundle payments means setting $X = \mathbb{R}^{|Y|}$. This represents a special case of our general framework, so all of the above results continue to hold.

This construction mimics the representation of commodity bundles in general equilibrium models, for example. If $Y = \{\text{pen}, \text{pencil}, \text{sheet of paper}\}$, then the vector $(1,2,3)$ represents consumption of one pen, two pencils, and three sheets of paper. In this context, the alternative y can be identified with 1_y, which is the vector that puts one on the yth coordinate and zeroes elsewhere.

Almost all experiments that pay in bundles can be characterized as randomly choosing a subset of the decision problems (or, all decision problems) and paying the bundle of messages announced in those chosen problems. We refer to these as Random *Multiple* Problem Selection (RMPS) mechanisms. To formalize these mechanisms, let $\mathscr{I}$ be the set of all non-empty subsets of $\{1,\dots,k\}$. An RMPS mechanism randomly selects one set of decision problem indices from $\mathscr{I}$ for payment.

Definition 10 (Random Multiple Problem Selection Mechanisms). For any experiment (D,ϕ), the payment mechanism ϕ is a *Random Multiple Problem Selection* (RMPS) mechanism if there is a function $I : \Omega \to \mathscr{I}$ mapping each state $\omega \in \Omega$ into a subset $I(\omega) \in \mathscr{I}$ of decision problems such that, for each $m \in M$,

$$\phi(m)(\omega) = \sum_{i \in I(\omega)} 1_{m_i}.$$

Definition 11 (No Complementarities at the Top). Fix the list of decision problems $D = (D_1,\dots,D_k)$ and let $\mathscr{F} \subseteq \mathscr{I}$ be a collection of non-empty subsets of $\{1,\dots,k\}$. The preference $\succeq$ over $X = \mathbb{R}^{|Y|}$ satisfies no complementarities at the top (NCaT) relative to $\mathscr{F}$ if, for every $F \in \mathscr{F}$ and every $m \in M$,

$$\sum_{i \in F} 1_{\mu_i(\succeq)} \succeq \sum_{i \in F} 1_{m_i},$$

[23]We emphasize that the assumptions we study in this section are restrictions on preferences $\succeq$ and not on extensions $\succeq^*$. Extensions are still assumed to be rich, as in the rest of the paper.

with a strict preference if there exists $i \in F$ for which $1_{\mu_i(\succeq)} \succ 1_{m_i}$.

To interpret the NCaT condition, think of each set $F \in \mathscr{F}$ as a collection of decision problems that could be paid together as a bundle. For example, an RMPS mechanism randomly picks the collection $F = I(\omega)$ at state ω. NCaT requires that, for any feasible collection F, the bundle of most-preferred items ($\sum_{i \in F} 1_{\mu_i(\succeq)}$) is also the most-preferred bundle. There is no way to bundle together items that would be less-preferred individually, and end up with a more-preferred bundle. In other words, there are no complementarities strong enough to overwhelm the subject's top-ranked alternatives.

Proposition 2. Given is an experiment (D, ϕ), where ϕ is an RMPS mechanism with associated function $I : \Omega \to \mathscr{I}$. Mechanism ϕ is incentive compatible with respect to a rich $\mathscr{E}$ if and only if any admissible preference $\succeq$ on X satisfies NCaT with respect to $I(\Omega)$.

The proof is immediate from the definition of incentive compatibility.

There are many reasons to suspect that NCaT will not be satisfied in certain settings, due to the known and observed presence of complementarities such as wealth effects, portfolio effects, and hedging opportunities. In developing prospect theory, however, Kahneman and Tversky (1979) argue that subjects isolate each decision, even when all are paid (Tversky and Kahneman, 1981). Thus, the isolation effect often used to justify use of the RPS mechanism (Cubitt et al., 1998; Wakker et al., 1994) can equally be applied to justify paying for multiple decisions. Several other models of risky choice implicitly satisfy NCaT. For example, if subjects learn about payments after every period, then NCaT is satisfied if they have reference-dependent preferences with rapidly-updating reference points (as discussed by Cox et al., 2012b), or separable expected utility over earned income rather than terminal wealth (see Cox and Sadiraj, 2006).

Show-Up Fees

Experimenters often pay subjects a (state-independent) show-up fee for their participation, in addition to their earnings from their choices. Technically, paying a show-up fee turns each payment into a bundle. For example, if x_0 is the show-up fee and an RPS mechanism is used, then the realized payment is the bundle $(1_{x_0} + 1_{m_i})$ when decision problem D_i is randomly chosen. Without ruling out any form of complementarities, it is possible that the show-up fee may distort incentives. As an extreme example, paying a $1,000 show-up fee is likely to generate wealth effects and reduce risk aversion.

In practice, however, we agree that relatively small show-up fees are unlikely to distort incentives. Formally, this is equivalent to assuming that $x \succeq y$ implies $1_{x_0} + 1_x \succeq 1_{x_0} + 1_y$ for every $x, y \in X$. This assumption represents a very weak form of translation invariance—restricted only to additions by x_0—and could be termed *show-up fee invariance*.

V MONOTONICITY VIOLATIONS & FAILURE OF THE RPS MECHANISM

Monotonicity is tightly linked with the incentive compatibility of the RPS mechanism. Whether one should use an RPS mechanism in practice depends on the empirical validity of the monotonicity assumption. In the abstract, monotonicity has strong appeal and is often viewed as a normative criterion (Wald, 1939). In many specific settings, however, monotonicity becomes much less compelling. In this section, we review plausible violations of monotonicity, as well as experiments that test directly the incentive compatibility of the RPS mechanism. These represent settings where one should be wary of the RPS mechanism, and perhaps search for other incentive compatible mechanisms under a different set of assumptions.

Order Reversal. When the choice objects in X are acts that map some state space S into outcomes in Z, random payment mechanisms such as the RPS result in gambles over these acts. We typically think of the resolution of the payment mechanism uncertainty (*e.g.*, the draw of which decision problem to pay) as the 'first stage', and the resolution of the chosen act as the 'second stage'. Since our definition of monotonicity operates on choice objects before their uncertainty is resolved, we refer to it here as *first-stage monotonicity*. With this ordering, a subject would evaluate the act $f \in D_i$ conditional on D_i already being chosen for payment. But it is plausible that a subject may reverse the order of conditioning, fixing some state $s \in S$ and then considering the gamble over chosen acts' payments at s induced by the payment mechanism. This can drastically alter the resulting incentives of the mechanism.

To illustrate, consider the following example, adapted roughly from Seo (2009): A subject is asked to bet on whether her opponent will play U or D in a two-by-two game. Let f be the bet on U (specifically, $f(U) = \$1$ and $f(D) = \$0$) and g be the bet on D ($g(U) = \$0$ and $g(D) = \$1$). Suppose $f \succ g$ and she is asked to announce her preferred bet *twice*, so that $D_1 = D_2 = \{f, g\}$.[24] A fair coin is flipped to determine which is paid, and we assume here the subject views the coin as an objective lottery with probabilities equal

[24] More realistically, the subject might be asked to bet on the strategy and to play the game. The game play simply represents another bet on the opponent's strategy.

to 1/2. If the subject announces $m = (f, g)$, then $\phi(f, g) = (1/2, f; 1/2, g)$, which is a lottery paying acts f and g with equal probability. Similarly, $\phi(f, f) = (1/2, f; 1/2, f) = f$. Under (first-stage) monotonicity, we must have $\phi(f, f) \succ^* \phi(f, g)$ because $f \succ g$.

Now suppose the subject reverses the order of conditioning, fixing first the opponent's strategy and then evaluating the lottery given by the RPS mechanism. In state U, the payment $\phi(f, g)$ pays $(1/2, f(U); 1/2, g(U) = (1/2, \$1; 1/2, \$0)$. In state D it pays $(1/2, f(D); 1/2, g(D)) = (1/2, \$0; 1/2, \$1)$. Thus, regardless of the state, the subject receives the same objective lottery. The coin flip provide a hedge against ambiguity. The payment $\phi(f, f)$, on the other hand, results in the ambiguous act f regardless of the order of conditioning. If the subject is ambiguity averse, she may strictly prefer $\phi(f, g)$ over $\phi(f, f)$. But $f \succ g$, so this would be a violation of first-stage monotonicity.

In many models of multi-stage gambles it is common to assume an *order reversal* axiom that subjects are indifferent over the order of randomization (e.g., Anscombe and Aumann, 1963). Formally, order reversal requires that the subject be indifferent between the lottery $(\alpha, f; 1-\alpha, g)$ and the act $\alpha f + (1-\alpha)g$ that pays the lottery $(\alpha, f(s); 1-\alpha, g(s))$ in each state $s \in S$.

Unfortunately, the joint imposition of order reversal and monotonicity leads to a surprisingly strong restriction that underlying preferences must satisfy the independence axiom ($f \succeq g$ implies $\alpha f + (1-\alpha)h \succeq \alpha g + (1-\alpha)h$ for any act h and any $\alpha \in [0, 1]$). Indeed, if $f \succeq g$ then monotonicity implies that $(\alpha, f; 1-\alpha, h) \succ^* (\alpha, g; 1-\alpha, h)$ and by order reversal this implies that $\alpha f + (1-\alpha)h \succeq \alpha g + (1-\alpha)h$. Since the literature on ambiguity aversion is based on weakening the independence axiom, we get the following observation:

Observation 1. If a subject is ambiguity averse and satisfies order reversal, then they must violate (first-stage) monotonicity. Thus, there exist experiments for which the RPS mechanisms will not be incentive compatible.

We must emphasize, however, that ambiguity aversion *by itself* does not contradict first-stage monotonicity. Most ambiguity models of the last twenty years do not focus on two-stage gambles, and are therefore silent on how individuals should evaluate first-stage mixing. For example, it is perfectly consistent to assume that subjects have maxmin preferences of Gilboa and Schmeidler (1989) in the second stage (when choosing from X), and that the extensions $\succeq^*$ on lotteries over acts satisfy first-stage monotonicity. In that situation, the RPS mechanism would be incentive compatible even though subjects are ambiguity averse on X. In fact, the prominent theoretical model that formally studies lotteries over acts in the context of ambiguity (Seo, 2009) allows for ambiguity-averse preferences on X while explicitly assuming the independence axiom on lotteries over acts—a much stronger assumption than first-stage monotonicity.[25] In contrast, Kuzmics (2013) assumes order reversal but not first-stage monotonicity, and shows that choices of an ambiguity averse decision maker can always be rationalized as those of an expected utility maximizer.

Dominiak and Schnedler (2011) find, however, that subjects who exhibit ambiguity aversion in a two-color Ellsberg-urn experiment do not generally prefer a coin flip between the two possible bets over each one separately. A plausible explanation for this result is that these subjects do not view the coin as providing a hedge against ambiguity, because they only view the coin's randomization as being performed 'before' the realization of the urn draw.

Reduction of Compound Lotteries. When second-stage choice objects are viewed as objective lotteries, the order reversal axiom is identical to the familiar *reduction of compound lotteries* (ROCL) axiom. Again, the joint imposition of ROCL and first-stage monotonicity leads to the requirement that preferences over lotteries be linear. In other words, ROCL and monotonicity jointly imply the independence axiom.

Observation 2. If a subject reduces compound lotteries but does not satisfy the independence axiom, then they must violate (first-stage) monotonicity. Thus, there exist experiments for which RPS mechanisms will not be incentive compatible.

Holt (1986)—commenting on the preference reversal experiments of Grether and Plott (1979) (among others)—shows by example how the RPS mechanism may not be incentive compatible with non-expected utility preferences (using the models of Machina, 1982, Quiggin, 1982, or Yaari, 1987) when ROCL is assumed.[26] Similar arguments are found in Cox et al. (2012b) and Harrison and Swarthout (2011). Cubitt et al. (1998) prove that, under ROCL, the RPS is incentive compatible if and only if the independence axiom is satisfied. Our results show that it is the resulting incompatibility with monotonicity that gives this result.

[25]Saito (2012) also discusses a preference for randomization after (versus before) the state is realized. His axiomatic characterization allows for different degrees to which subjects prefer *ex-post* randomization.

[26]Several similar rationalizations of the Grether-Plott results have been provided. Karni and Safra (1987) shows that the Becker et al. (1964) (BDM) elicitation mechanism used by Grether and Plott (1979) is not incentive compatible if subjects satisfy ROCL but not independence. Segal (1988) shows that the BDM mechanism is not incentive compatible if subjects satisfy independence, but not reduction. (These conclusions also follow from our results because the BDM mechanism is a type of RPS mechanism.) Loomes and Sugden (1983) alternatively rationalize preference reversal using regret aversion (see also Loomes and Sugden, 1982).

Although violations of the independence axiom are well-documented, ROCL has more limited support. Loomes et al. (1991); Starmer and Sugden (1991); Cubitt et al. (1998, Experiment 1); and Beattie and Loomes (1997) all run experiments using the RPS mechanism in which two different messages m and m' lead to the same simple lottery if reduction is assumed. In their data, subjects choose one message significantly more often than the other, clearly indicating that m and m' are evaluated differently in many subjects' preferences. Furthermore, Halevy (2007) shows that those who perform reduction tend to satisfy independence. Thus, subjects who respect reduction seem to be rare, and seem to be exactly those for whom independence (and, therefore, monotonicity) is a reasonable assumption.

In a companion paper (Azrieli et al., 2012), we show that if all rank-dependent preferences are admissible and subjects reduce compound lotteries, then there are experiments for which no incentive compatible mechanism exists. In other words, if one assumes reduction but not expected utility, then incentive compatibility may be impossible to achieve. On a positive note, Cox et al. (2012b) find a mechanism that is incentive compatible under linear cumulative prospect theory (Schmidt and Zank, 2009) when reduction is assumed and lotteries are cosigned.[27] Thus, incentive compatible mechanisms can exist in some settings where first-stage monotonicity fails. Given Proposition 1 above, however, different mechanisms will have to be found for different domains of non-monotonic extensions.

Correlated Randomizing Devices. If choice objects in X are uncertain prospects with state space S, a subject may believe that there is 'correlation' between the processes generating S and Ω, whether or not this correlation is justified empirically. For example, suppose D_i is chosen for payment if a coin flip lands heads, and $x, y \in D_i$ are acts whose payments depend on the same coin flip. Clearly, subjects picking between x and y should condition on the fact that they will only be paid for this choice in the event that the coin comes up heads. Preferences over x and y may be altered as a result. This would represent a violation of monotonicity in our general framework, because monotonicity requires that preferences over X be state-independent. In general, if preferences over X can depend on the realization of $\omega \in \Omega$, then incentive compatibility can fail.

Since the experimenter can choose the randomizing device used to choose D_i, care should be taken to ensure that the subject will behave as if S and Ω are independent.

[27] A set of lotteries is cosigned if, in each state, either all lotteries give positive payments, or all lotteries give negative payments.

Even computer-generated randomization may allow room for distrusting subjects to perceive correlation. Using a physical, objective randomizing device such as a coin flip or die roll should mitigate problems with perceived correlation.[28]

Decision Overload. If subjects are asked to make many decisions, they may not expend much cognitive effort on each choice. The result may be dominated choices in some decision problems. This is particularly plausible when there are effort-minimizing choices m^0 to which subjects can default when cognitively overloaded. In an RPS mechanism, suboptimal choices of m_i^0 over the truthful favorite m_i^* would represent a violation of monotonicity. Thus, the (positive) appeal of the monotonicity axiom reduces as the number of decision grows.This concern has been expressed by Harrison (1994), Beattie and Loomes (1997), and others. Experimental results by Wilcox (1993), Moffatt (2005), and Baltussen et al. (2010) suggest that incentive compatibility of the RPS mechanism may indeed fail if decisions are chosen with small probabilities or if the decision problems are sufficiently complex. It is plausible, however, that cognitive overload would have a detrimental impact on *any* payment mechanism. Simply put, paying subjects \$20 for 100 decisions may lead to noisy or biased results, regardless of the specific payment rule.

Ex-Ante Fairness. Diamond (1967) demonstrates that certain fairness concerns can lead to monotonicity violations using an example similar to the following:[29] Suppose a subject is presented with two decision problems: (1) give \$10 to person A or B, and (2) give \$10 to person A or C. A coin flip will determine which problem is chosen. It may be that giving \$10 to A is strictly preferred to giving it to either B or C but, because of ex-ante fairness, the subject would rather choose a fair coin flip between A and C.[30] Such a choice would violate monotonicity and cause the RPS mechanism not to be incentive compatible. The NCaT assumption is also not particularly appealing in this case (\$10 each for A and C may be preferred to giving all \$20 to A), so paying for both decisions may also not be incentive compatible. In general, experiments with

[28]Bade (2012) models the ambient probability space and the space corresponding to the randomization device as one and the same. For an RPS mechanism, Ω is partitioned into $\{\Omega_1,\ldots,\Omega_k\}$, decision problem D_i is chosen if $\omega \in \Omega_i$, and the choices in D_i are all acts with Ω_i as their domain. Bade fixes a preference relation and characterizes when the σ-algebra generated by the decision problems and the σ-algebra generated by the mechanism are independent (for the given preference relation). This independence turns out to be equivalent to a weak form of incentive compatibility. Here, instead of fixing preferences and varying σ-algebras, we fix the σ-algebras and study restrictions on preferences.

[29]Diamond actually argues against the independence axiom; Machina (1989) explains that monotonicity itself is the source of difficulty. This example is now known as “Machina’s mom”.

[30]Evidence of this kind of preference has been documented by Bolton and Ockenfels (2010) and Cappelen et al. (2013), for example.

difficult moral or ethical choices may be plagued by this persistent incentive problem when multiple decisions are given.

Irrational Diversification. Loomes (1998) and Rubinstein (2002) document a violation of monotonicity caused by 'irrational diversification'. Rubinstein's experiment can be summarized by the following stylized example: Subjects are asked to place five bets on the color of five independent draws (with replacement) from an urn containing six red and four blue balls. They are paid for all five bets. Assuming subjects employ probabilities and believe that draws are independent and identically distributed, the stochastically dominant choice is to bet all five times on their more-likely outcome. In fact, more than half of the subjects diversify their choices by sometimes betting on the other outcome. Thus, paying for every bet is not incentive compatible because NCaT is violated: the 'bundling' of identical bets leads to diversification. Although this is not a violation of monotonicity in the subjective world, it is in the world of objective lotteries (Azrieli et al., 2012), and so the RPS mechanism may not be incentive compatible when objective lotteries are assumed and subjects face the same decision problem multiple times. It is unknown whether payment mechanisms exist that can avoid this problem.

Direct Experimental Evidence. There exist several direct tests of the incentive compatibility of the RPS mechanism. Results are generally mixed, though some convincing failures of incentive compatibility have been observed.

Camerer (1989) allows subjects in an RPS mechanism to change their choice after learning which decision problem would be paid. Less than three percent of subjects opt to change, suggesting that the RPS mechanism is incentive compatible.[31]

Studying lottery choices and assuming ROCL, Hey and Lee (2005b) statistically test the extreme theories that subjects either treat each choice in isolation or combine them to form one large lottery. Using multiple functional forms for preferences over lotteries and two different criteria, they find the data fit better the hypothesis that each decision is treated in isolation.[32]

Starmer and Sugden (1991) find that choice frequencies differ in a decision problem if it is given as part of an RPS mechanism or in isolation, though statistical significance is marginal due to small sample sizes. In two out of four problems, Beattie and Loomes (1997) find noticeable but insignificant differences between the RPS mechanism and

[31] This mechanism is not strictly incentive compatible because initial choices (thus, all unpaid choices) are not consequential. But Camerer's results suggest subjects did not deviate from truthful announcements.

[32] Hey and Lee (2005a) find a similar conclusion when subjects are given problems sequentially and future problems are not known.

giving only the one problem. Cubitt et al. (1998, Experiment 2) also find virtually no difference between a single paid problem and that problem as part of an RPS mechanism.

Cox et al. (2012b) compare choices in five lottery decision problems across seven different payment mechanisms. Significant differences in behavior are observed. Subjects behave similarly whether facing only one decision problem or being paid for each problem sequentially, but are significantly more risk averse (in some problems) if the RPS mechanism is used or they are paid at the end for all decisions.[33]

Similarly, Harrison and Swarthout (2011) estimate various utility functionals using both the RPS mechanism and data from subjects who make only one choice. Non-expected utility functional estimates differ across payment mechanisms (mostly in their estimated probability weights), but the expected utility functional estimates do not.

In all of the these direct-comparison studies, subjects who are given a single choice do not see the other decision problems. Thus, behavior differences may be attributed to framing effects (causing a change in underlying preferences) rather than monotonicity violations. For example, Cox et al. (2012a) compare a lottery-choice experiment with only $D_1 = \{\overline{x}, \underline{y}\}$, to an RPS experiment with $D_1 = \{\overline{x}, \underline{y}\}$ and $D_2 = \{\underline{x}, \overline{y}\}$, where $\overline{x}$ dominates $\underline{x}$ and $\overline{y}$ dominates $\underline{y}$. More subjects announce $m_1 = \overline{x}$ in the RPS experiment. Though this may be a monotonicity violation, it seems more likely that it is an application of the well-known asymmetric dominance (or, decoy) effect, in which choices change when dominating alternatives are present (see Huber et al., 1982 or Herne, 1999). Adding D_2 simply makes $\overline{x}$ look more appealing to subjects. If such framing effects are present, lab results may not generalize to field settings. If monotonicity is violated, then the RPS is not correctly eliciting preferences. Disentangling these confounded explanations is clearly important.

In our view, Cubitt et al. (1998, Experiment 3) provides the cleanest test of incentive compatibility of the RPS mechanism because the confound with framing is eliminated. Subjects are randomly assigned to one of three groups. All groups are given the same 20 decision problems. The first group is paid only for D_1; the second group is paid only for D_2; and the third group is paid for one randomly selected problem out of the twenty, each selected with equal probability. The order of the problems is randomized for each subject, but subjects can backtrack and change any decision at any time before

[33]A pilot experiment on lists of problems by Brown and Healy (2013) corroborates the claim that RPS subjects are more risk averse, though a new study by Freeman et al. (2012) shows *less* risk aversion under RPS using Mechanical Turk (online) subjects. These lists hold constant one choice while the other becomes progressively more (or less) attractive, and the differing results can be explained by subjects simply 'switching too late' when working down the list. Similar biases in list elicitation tasks are explored by Andersen et al. (2006); Beauchamp et al. (2012); Sprenger (2011); Castillo and Eil (2013); and others.

a problem is chosen for payment. Choice frequencies in D_1 do not significantly differ between groups one and three (χ^2 p-value 0.355) and choice frequencies in D_2 do not differ between groups two and three (χ^2 p-value 0.285). Thus, incentive compatibility of the RPS mechanism holds for that experiment.[34]

VI Discussion

Incentives Within Decisions. Our paper focuses on how to pay for multiple decision problems, assuming simply that subjects have preferences over the exogenously-given choice objects. But in many cases the choice objects themselves are designed by the experimenter for a specific purpose, and the proper structuring of the choice objects for the question at hand can be a non-trivial problem.

For example, suppose an experimenter wants to study play of an infinitely repeated game. The standard methodology—established by Roth and Murninghan (1978)—is to pay for each period, terminating the repeated game with a fixed probability after each period. Such games are often called *indefinitely repeated*. Each $x \in D_i$ represents a possible strategy in this randomly-terminating game, and each strategy choice x is an act that maps the play of others into a payment equal to the sum of all periods' earnings. Charness and Genicot (2009), Fischer (2011), Chandrasekhar and Xandri (2011), and Sherstyuk et al. (2011) all recognize that this payment induces the same incentives as the infinitely-repeated game model only if subjects have linear utility. With concavity, accumulated (unconsumed) earnings through the game can alter preferences going forward. Paying for one randomly-selected period clearly induces a myopic bias. Paying only for the final period induces the correct incentives, regardless of the subjects' risk preferences. Sherstyuk et al. (2011) and Frechette et al. (2011) test these payment schemes and find support for these predictions.[35]

Many researchers elicit preferences directly as part of a larger experiment. For example, the well-known Becker et al. (1964) (BDM) mechanism is often used to elicit the subjective value of an object. Grether (1981) and Karni (2009) describe a similar mechanism for eliciting belief probabilities. Although we think of these as individual decision

[34]In a similarly-structured pilot study of list elicitation tasks (Sprenger, 2011 and Freeman et al., 2012), Brown and Healy (2013) do find significant differences in choice frequencies, even when those paid for one fixed decision are shown all decision problems.

[35]Frechette et al. (2011) also test payment mechanisms of Cabral et al. (2011) and Cooper and Kuhn (2010).

problems, their incentive properties can be analyzed easily through our framework because they are random payment mechanisms in which subjects announce information about their preferences.

In the BDM mechanism, for example, a subject is usually endowed with k dollars and asked how many dollars she would give up to obtain the object. Assuming only integer-valued responses, we could think of there being k decision problems, each of the form $D_i = \{(0,k),(1,k-i)\}$, where $(0,k)$ means not getting the object and keeping k dollars, and $(1,k-i)$ means getting the object and keeping $k-i$ dollars. When the subject announces a value of v, this is treated as identical to the (rationalizable) message m in which $m_i = (1,k-i)$ if $i \leq v$ and $m_i = (0,k)$ if $i > v$.[36] The BDM mechanism is then an RPS mechanism in which one D_i is randomly chosen and the inferred message m_i is paid. It is incentive compatible assuming monotonicity.[37] Karni (2009) proves this same result for the probability-elicitation version of the BDM mechanism. The second-price auction of Vickrey (1961) can also be thought of as an RPS mechanism using exactly the same framework, except that the random choice of the decision problem is determined by the second-highest bidder's strategy, rather than a computer. In the context of games, the assumption of monotonicity is identical to the assumption that subjects never play dominated strategies.

Payment Inequality. Suppose an experiment has subjects play 10 matching pennies games against varying opponents. If only one game is drawn for payment, then half the subjects will receive no earnings beyond their show-up fee. Subjects dislike such payment inequality, and future participation may be affected. Paying for every game would mitigate this problem. But the matching pennies game is *supposed to be* a game of unequal payments; if subjects leave the lab with roughly equal earnings, then they have not played a true, one-shot matching pennies game, and complementarities are likely to have distorted choices. One possibility is to use the RPS mechanism with lower payments for the game, and a higher show-up fee. Obviously, the experimenter must trade off greater payment equality against 'watered down' incentives, and also should worry about wealth effects that may arise from the larger show-up fee.

Framing Effects & Learning. As discussed above, framing effects occur when the mere presence of decision problems alters underlying preferences. For example, the inclusion of dominated lotteries in one problem may impact choices in another. Machina

[36]The mechanism therefore assumes the *single-crossing* property that $(1,k-i) \succ (0,k)$ implies $(1,k-j) \succ (0,k)$ for all $j < i$, and $(0,k) \succ (1,k-i)$ implies $(0,k) \succ (1,k-j)$ for all $j > i$.

[37]Bohm et al. (1997) find that elicited values in the BDM correlate with the upper bound of the range of prices, suggesting that a framing effect may alter preferences with these mechanisms.

(1987) reviews ways in which the presentation of a gamble can affect subjects' choices. If preferences are changed by the presence of another decision problem or by the presentation of choices, then an incentive compatible mechanism will still correctly elicit those changed preferences.[38] The generalizability of observed choices, however, can be called into question, because they are apparently sensitive to the surrounding context. This sensitivity is to the inclusion of other decision problems, but not to the way in which they are incentivized. Thus, the payment mechanism is not responsible for (and cannot mitigate) framing effects.

It is well known that subjects often 'learn' over time in an experiment (Camerer, 2003, Ch. 6). For example, Garvin and Kagel (1994) show that subjects learn to avoid the winner's curse mistake in experimental auctions simply by observing others make the mistake. Weber (2003) even shows evidence of 'non-observational' learning across decision problems when no feedback is given. Learning represents another example of a framing effect. The choice behavior in the kth decision problem can be affected by the presence of prior problems, regardless of the way in which decisions are paid. Thus, learning represents another way in which laboratory results can be difficult to generalize.

Confusion. A subject may misunderstand the structure of the choice objects, perhaps due to poorly-written experimental instructions. This may lead them to choose items from X differently than if they were properly informed. Formally, one could define an 'informed' preference relation $\succeq_1$ and a 'confused' preference relation $\succeq_2$. An incentive compatible payment mechanism will elicit whichever preference relation currently describes the subject's choices. An experimenter who observes 'confused' choices (according to $\succeq_2$) may be misled when attempting to rationalize those choices assuming no confusion. As with framing, this is not rectified through the appropriate choice of payment mechanisms.

Subjects may also be confused about the payment mechanism. The mechanism ϕ may be viewed incorrectly by the subject as ϕ', resulting in a loss of control by the experimenter. The only practical solution is to ensure that the payment mechanism is explained well and the randomizing devices are simple, physical devices that are observable by subjects. It may also be worthwhile to test subjects' understanding of the experiment before proceeding. If the RPS mechanisms if found to be too complex, for example, then paying for every decision may be justified.

[38]Different presentations of the same gamble might more properly be modeled as different choice objects in X.

Irrational Choice. We assume choice can always be rationalized by a preference relation. Modeling incentives in experiments under irrational choice (as in the model by Masatlioglu et al., 2012, or others) is well beyond the scope of our current paper, but would be an interesting extension for future research.

APPENDIX A: ϕ-MONOTONICITY

Here we show that incentive compatibility of the RPS mechanisms obtains if and only if a weakened version of monotonicity—called ϕ-monotonicity—is assumed. ϕ-monotonicity requires that $\succeq^*$ respect dominance only when comparing a payment that results from a truthful report to the payments that could result from other reports.

Axiom 1 (**ϕ-Monotonicity**). Fix an experiment (D,ϕ). For any preference $\succeq$, the extension $\succeq^*$ is ϕ-monotonic if, for every $f \in \phi(\mu(\succeq))$ and $g \in \phi(M)$, $f \sqsupseteq g$ implies $f \succeq^* g$, and $f \sqsupset g$ implies $f \succ^* g$.[39]

Let $\mathscr{E}^{\phi\text{mon}}$ identify the ϕ-monotonic extensions for each $\succeq$. The following shows that ϕ-monotonicity applied to all experiments using an RPS mechanism is equivalent to the global monotonicity requirement.

Proposition 3. Fix X, $\succeq$, and extension $\succeq^*$. If $\succeq^*$ satisfies ϕ-monotonicity for every experiment (D,ϕ) in which ϕ is an RPS mechanism, then $\succeq^*$ satisfies monotonicity.

Proof. Consider any two acts f and g such that $f \sqsupseteq g$. Let $\{\Omega_1,\ldots,\Omega_k\}$ be the coarsest refinement of the pre-images of f and g. For each Ω_i, pick any $\omega_i \in \Omega_i$ and let $D_i = \{f(\omega_i), g(\omega_i)\}$. Consider an RPS mechanism ϕ for this $D = (D_1,\ldots,D_k)$. Let $m^* = (f(\omega_1),\ldots,f(\omega_k))$ and $m = (g(\omega_1),\ldots,g(\omega_k))$. Since $f \sqsupseteq g$, we have that $m^* \in \mu(\succeq)$. Thus, $f = \phi(m^*) \in \phi(\mu(\succeq))$ and $g \in \phi(M)$, and so, by ϕ-monotonicity, $f \succeq^* g$. If $f \sqsupset g$ then $f(\omega_i) \succ g(\omega_i)$ for at least one i, so $m \notin \mu(\succeq)$. By ϕ-monotonicity, $f \succ^* g$. Thus, monotonicity holds. □

We now demonstrate that ϕ-monotonicity is equivalent to incentive compatibility of the RPS mechanisms.

Theorem 2. An RPS mechanism ϕ is incentive compatible with respect to $\mathscr{E}$ if and only if $\mathscr{E} \subseteq \mathscr{E}^{\phi\text{mon}}$.

Proof. The proof of the theorem follows from the following lemma:

[39]For any set $S \subseteq M$, $\phi(S) = \{\phi(m)\}_{m \in S}$.

Lemma 3. Let ϕ be an RPS mechanism, $f \in \phi(\mu(\succeq))$ and $g \in \phi(M)$.

(1) $f \sqsupseteq g$.

(2) $g \notin \phi(\mu(\succeq))$ if and only if $f \sqsupset g$.

Proof. Let $f = \phi(m^*)$ for some $m^* \in \mu(\succeq)$ and $g = \phi(m)$ for some $m \in M$. For each $i \in \{1,\ldots,k\}$, m_i^* and m_i are both elements of D_i and $m_i^* \succeq m_i$. If $g \notin \phi(\mu(\succeq))$ then $m \notin \mu(\succeq)$, and so there is some i for which $m_i^* \succ m_i$.

Since ϕ is an RPS mechanism and $f = \phi(m^*)$ for some $m^* \in \mu(\succeq)$, it must be that for each $i \in \{1,\ldots,k\}$ and $\omega \in \Omega_i$, $f(\omega) = m_i^* \succeq m_i = g(\omega)$. Thus, $f \sqsupseteq g$, proving part (1) of the lemma. If $g \notin \phi(\mu(\succeq))$ then $m \notin \mu(\succeq)$, and so there is some j where, for each $\omega \in \Omega_j$, $f(\omega) = m_j^* \succ m_j = g(\omega)$. In that case, $f \sqsupset g$. Conversely, if $f \sqsupset g$ then there is at least one state ω where $f(\omega) \succ g(\omega)$. Let i be such that $\omega \in \Omega_i$. Thus, $f(\omega) = m_i^* \succ m_i = g(\omega)$, proving that $m \notin \mu(\succeq)$ and, therefore, $g \notin \phi(\mu(\succeq))$. □

Returning to the theorem, assume incentive compatibility with respect to some $\mathscr{E}$, and pick an arbitrary $\succeq$. If $f \in \phi(\mu(\succeq))$, $g \in \phi(M)$, and $f \sqsupseteq g$, then incentive compatibility guarantees that $f \succeq^* g$ for every $\succeq^* \in \mathscr{E}(\succeq)$. (This is true regardless of ϕ, and the fact that $f \sqsupseteq g$.) Now assume $f \in \phi(\mu(\succeq))$, $g \in \phi(M)$, and $f \sqsupset g$. By part (2) of Lemma 3, $g \notin \phi(\mu(\succeq))$, so incentive compatibility guarantees that $f \succ^* g$ for all $\succeq^* \in \mathscr{E}(\succeq)$. Thus, $\mathscr{E} \subseteq \mathscr{E}^{\phi\text{mon}}$.[40]

Conversely, assume $\mathscr{E} \subseteq \mathscr{E}^{\phi\text{mon}}$, and pick an arbitrary $\succeq$ and extension $\succeq^* \in \mathscr{E}(\succeq)$. Pick any $f \in \phi(\mu(\succeq))$ and $g \in \phi(M)$. By part (1) of Lemma 3, we have that $f \sqsupseteq g$. Since $\succeq^* \in \mathscr{E}^{\phi\text{mon}} \subseteq \mathscr{E}^{\text{mon}}$, we have $f \succeq^* g$, as needed. If, in addition, $g \notin \phi(\mu(\succeq))$, then, by part (2) of Lemma 3, $f \sqsupset g$. Here, $\succeq^* \in \mathscr{E}^{\phi\text{mon}} \subseteq \mathscr{E}^{\text{mon}}$ guarantees that $f \succ^* g$, proving that incentive compatibility holds.

□

Appendix B: Proofs of the Main Results

B.1: Proof of Proposition 1

Let $x \in X$ denote both the choice object x, and the constant act that pays x at every state $\omega \in \Omega$. For sufficiency, if $k = 1$ then the mechanism in which $\phi(m) = m$ for each $m \in M$ is clearly incentive compatible. The proof of necessity proceeds in several steps. In each, assume the hypothesis that ϕ is incentive compatible and $\mathscr{E}$ has no restrictions.

[40] Finding an example of a non-monotonic extension where incentive compatibility fails (as in Holt, 1986) is not sufficient to prove this direction of the theorem, as it must be shown for *all* possible counter-examples.

Step 1: $|\text{Range}(\phi)| > 1$.

If $x, y \in D_i$ (with $x \neq y$) then consider a preference $\succeq$ where $x \succ z$ for all $z \neq x$ and a preference $\succeq'$ where $y \succ' z$ for all $z \neq x$. Let $m = \mu(\succeq)$ and $m' = \mu(\succeq')$, and note that $m \neq m'$ since $m_i = x$ and $m'_i = y$. Incentive compatibility therefore requires $\phi(m) \succ^* \phi(m')$, which implies $\phi(m) \neq \phi(m')$. Thus, $|\text{Range}(\phi)| > 1$.

Step 2: $\text{Range}(\phi) \subseteq X$ (that is, $\phi(m)$ is a constant act for every $m \in M$).

Suppose not. Then there is some $m' \in M$ such that $\phi(m')$ is not a constant act. Using step 1, let $m \neq m'$ be such that $\phi(m) \neq \phi(m')$, and then pick any $\succeq$ such that $m \in \mu(\succeq)$. Since there are no restrictions on $\mathscr{E}$, pick an extension $\succeq^* \in \mathscr{E}(\succeq)$ such that $\phi(m') \succ^* f$ for every act $f \neq \phi(m')$. But then $\phi(m') \succ^* \phi(m) \in \mu(\succeq)$, contradicting incentive compatibility.

Step 3: $\text{Range}(\phi) \subseteq D_i$ for every $i \in \{1, \ldots, k\}$.

Suppose not. Then there is some $a \in \text{Range}(\phi)$ (by step 2) and some D_j with $x, y \in D_j$ ($x \neq y$) where $a \notin D_j$. Now pick a preference $\succeq$ where $a \succ x \succ z$ for every $z \in X \setminus \{a, x\}$, and a strict preference $\succeq'$ where $a \succ' y \succ' z$ for every $z \in X \setminus \{a, y\}$. Let $m = \mu(\succeq)$ and $m' = \mu(\succeq')$, and note that $m_j = x$ and $m'_j = y$, so $m \neq m'$. Incentive compatibility requires that $\phi(m) = a$ and $\phi(m') = a$. But incentive compatibility also requires that $\phi(m) \succ^* \phi(m')$, which is a contradiction.

Step 4: $\text{Range}(\phi) = D_i$ for every $i \in \{1, \ldots, k\}$.

Suppose not. Then there is some D_i and some $a \in D_i$ such that $a \notin \text{Range}(\phi)$. Let $\succeq$ be a preference where $a \succ z$ for every $z \neq a$, and let $m = \mu(\succeq)$. Let $b = \phi(m)$, and note that $b \in D_i$ by step 3. Now let $\succeq'$ be a strict preference where $b \succ' z$ for every $z \neq b$, and let $m' = \mu(\succeq')$. Since $m'_i = b$ and $m_i = a$, we have $m' \neq m = \mu(\succeq)$. Therefore, incentive compatibility requires that $\phi(m) \succ \phi(m')$. But incentive compatibility also requires that $\phi(m') = b$, so that $\phi(m') = \phi(m)$, a contradiction.

Step 5: $k = 1$.

(If we assume no two decision problems are identical, then this step is unnecessary.) Suppose not. By step 4, we have $D_1 = D_2 = \text{Range}(\phi)$. Pick any m' such that $m'_1 \neq m'_2$, and let $x = \phi(m')$. Now consider the preference $\succeq$ where $x \succ z$ for every $z \neq x$, and let $m = \mu(\succeq)$. Since $m_1 = m_2 = x$, we have that $m \neq m'$. Incentive compatibility requires that $\phi(m) \succ \phi(m')$, but also that $\phi(m) = x = \phi(m')$, a contradiction.

B.2: Proof of Lemma 1

Fix a preference $\succeq$, a truthful message $m^* \in \mu(\succeq)$, and an arbitrary message m. If $\phi(m^*)$ dominates $\phi(m)$ under $\succeq$ then, under monotonicity, $\phi(m^*) \succeq^* \phi(m)$ for any extension

$\succeq^* \in \mathscr{E}(\succeq) \subseteq \mathscr{E}^{\text{mon}}(\succeq)$ (with strict orderings when $m \notin \mu(\succeq)$). Since this holds for all $\succeq$, the experiment is incentive compatible.

B.3: Proof of Lemma 2

Let $\succeq$ be a preference, and let $m^* \in \mu(\succeq)$. We claim that $\phi(m^*)$ dominates $\phi(m)$ under $\succeq$. To this end, we know that for all $\succeq^* \in \mathscr{E}(\succeq)$, we have $\phi(m^*) \succeq^* \phi(m)$. Because of our hypothesis that $\bigcap_{\succeq^* \in \mathscr{E}(\succeq)} \succeq^* = \sqsupseteq$, this implies that $\phi(m^*) \sqsupseteq \phi(m)$. Now, suppose that $m \notin \mu(\succeq)$. Then, for all i, it follows by definition that $m_i^* \succeq m_i$, and that there exists j for which $m_j^* \succ m_j$. Therefore, for all $\succeq^* \in \mathscr{E}(\succeq)$, we have $\phi(m^*) \succ^* \phi(m)$. Consequently, by our hypothesis that $\bigcap_{\succeq^* \in \mathscr{E}(\succeq)} \succeq^* = \sqsupseteq$, we know that $\phi(m) \sqsupseteq \phi(m^*)$ is false. Further, we know that $\phi(m^*) \sqsupseteq \phi(m)$ is true. Conclude that $\phi(m^*) \sqsupset \phi(m)$.

B.4: Proof of Theorem 1

We start by showing that any RSS mechanism that satisfies conditions (1) and (2) is incentive compatible (with respect to $\mathscr{E}^{\text{mon}}$). Let $\succeq$ be arbitrary, $m^* = \mu(\succeq)$ and let $\succeq^*$ be some (monotonic) extension of $\succeq$. We claim that $\phi(m^*) \succeq^* \phi(m')$ for any $m' \neq m^*$. This follows since, for each ω, $\phi(m^*)(\omega) = \text{dom}_{m^*}(P^\phi(\omega))$ and $\phi(m')(\omega) \in P^\phi(\omega)$, so $\phi(m^*)(\omega) \succeq \phi(m')(\omega)$. Since $m' \neq \mu(\succeq)$, we must also show that there exists $\omega \in \Omega$ for which $\phi(m^*)(\omega) \succ \phi(m')(\omega)$. Suppose not, so that $\phi(m^*)(\omega) \sim \phi(m')(\omega)$ at each ω. Because $\succeq$ is a linear order, this implies that $\phi(m^*) = \phi(m')$. Recalling condition (2) of the hypothesis, this implies that $m' \in M_R$, so there exists $\succeq'$ for which $m' = \mu(\succeq')$. Since $\phi(m^*) = \phi(m')$, both acts pick the same elements from every $P^\phi(\omega)$. Condition (1) requires $D_i \in SI(\mathscr{P}^\phi)$ for every i, so that $\mu_i(\succeq) = \mu_i(\succeq')$ for every i. But $\mu(\succeq) = \mu(\succeq')$ contradicts $m^* \neq m'$.

Conversely, let ϕ be an incentive compatible mechanism for $(D_1, \ldots, D_k)$. Recall that, for each $\omega \in \Omega$, $P^\phi(\omega) = \phi(M)(\omega)$. Let $m^* \in M_R$, and let $\succeq$ such that $m^* = \mu(\succeq)$. By incentive compatibility (recall Lemma 2), it follows that for all $m \in M$, we have $\phi(\mu(\succeq)) \sqsupseteq \phi(m)$. In particular, this implies that, for all $\omega \in \Omega$, $\phi(\mu(\succeq))(\omega) \succeq \phi(m)(\omega)$ (by definition of $\sqsupseteq$), or $\phi(m^*)(\omega) \succeq \phi(m)(\omega)$. That is, $\phi(m^*)(\omega) \succeq y$ for all $y \in P^\phi(\omega)$. Since $\succeq$ was arbitrary, this establishes both that $P^\phi(\omega) \in SI(\mathscr{D})$, and that $\phi(m)(\omega) = \text{dom}_m(P^\phi(\omega))$ whenever $m \in M_R$. Hence, ϕ is an RSS.

We claim now that for all i, $D_i \in SI(\mathscr{P}^\phi)$. If not, then by definition, there exists D_i, and preferences $\succeq, \succeq'$ for which for all $\omega \in \Omega$, $\text{dom}_{\succeq} P^\phi(\omega) = \text{dom}_{\succeq'} P^\phi(\omega)$, but for which $\text{dom}_{\succeq} D_i \neq \text{dom}_{\succeq'} D_i$. Hence, $\mu(\succeq) \neq \mu(\succeq')$. Since ϕ is an RSS mechanism, for all $\omega \in \Omega$, $\phi(\mu(\succeq))(\omega) = \text{dom}_{\succeq} P^{\phi}(\omega) = \text{dom}_{\succeq'} P^{\phi}(\omega) = \phi(\mu(\succeq'))(\omega)$. Consequently, $\phi(\mu(\succeq)) = \phi(\mu(\succeq'))$, but $\mu(\succeq) \neq \mu(\succeq')$. In particular, since $\mu(\succeq)$ and $\mu(\succeq')$ are each single-valued, incentive compatibility implies that there exists $\omega \in \Omega$ for which $\phi(\mu(\succeq))(\omega) \succ \phi(\mu(\succeq'))(\omega)$, a contradiction.

Finally, suppose that there are $m \in M_R$ and $m' \in M_{NR}$ such that $\phi(m) = \phi(m')$. Let $\succeq$ be such that $\mu(\succeq) = m$. Incentive compatibility requires that $\phi(m) \sqsupset \phi(m')$ with respect to $\succeq$, which contradicts $\phi(m) = \phi(m')$.

Appendix C: Uniqueness of the RPS Mechanism

We begin with a useful characterization of surely identified sets, which is used to prove the subsequent lemma.

Proposition 4. $E \in SI(\mathcal{D})$ if and only if for every pair $\{x, y\} \subseteq E$, there exists $D \in \mathcal{D}$ for which $\{x, y\} \subseteq D \subseteq E$.

Proof. Suppose that $E \in SI(\mathcal{D})$, and that E is not a singleton. Let $\{x, y\} \subseteq E$ be arbitrary. Consider two linear orders, $\succeq$ and $\succeq'$, which are identical except in their ranking of x and y (which are adjacent). They rank all elements of $X \setminus E$ above all elements of E, and they rank x and y above all elements of $E \setminus \{x, y\}$. However, $x \succ y$ and $y \succ' x$. It is clear that if there is no $D \in \mathcal{D}$ such that $\{x, y\} \subseteq D \subseteq E$, then for all $D \in \mathcal{D}$, we have $\text{dom}_{\succeq} D = \text{dom}_{\succeq'} D$, yet $\text{dom}_{\succeq} E = x \neq y = \text{dom}_{\succeq'} E$, contradicting sure identification.

Conversely, suppose that for every pair $\{x, y\} \subseteq E$, there exists $D \in \mathcal{D}$ for which $\{x, y\} \subseteq D \subseteq E$. Suppose by means of contradiction that there exist $\succeq$ and $\succeq'$ for which for all $D \in \mathcal{D}$, $\text{dom}_{\succeq} D = \text{dom}_{\succeq'} D$, but $\text{dom}_{\succeq} E \neq \text{dom}_{\succeq'} E$. Let $w = \text{dom}_{\succeq} E$ and $z = \text{dom}_{\succeq'} E$. There exists $D' \in \mathcal{D}$ for which $\{w, z\} \subseteq D' \subseteq E$. As a consequence, $w = \text{dom}_{\succeq} D'$ and $z = \text{dom}_{\succeq'} D'$, contradicting the fact that $\text{dom}_{\succeq} D = \text{dom}_{\succeq'} D$ for all $D \in \mathcal{D}$. $\square$

Lemma 4. Let ϕ be an RSS mechanism that satisfies condition (2) of Theorem 1 (that $\phi(M_R) \cap \phi(M_{NR}) = \varnothing$). Then ϕ is incentive compatible if and only if $\text{NRed}(\mathcal{D}) \subseteq \mathcal{P}^{\phi}$.

Proof. The proof follows immediately from Theorem 1; all we need to show is that condition (1) of that theorem ($\mathcal{D} \subseteq SI(\mathcal{P}^{\phi})$) is equivalent to $\text{NRed}(\mathcal{D}) \subseteq \mathcal{P}^{\phi}$.

Suppose first that any decision problem is surely identified from $\mathcal{P}^{\phi}$, and let D_i be non-redundant. Assume by contradiction that $D_i \notin \mathcal{P}^{\phi}$. By Proposition 4, for each pair $\{x, y\} \subseteq D_i$ there is some $\{x, y\} \subseteq P^{\phi}(\omega_{(x,y)}) \subsetneq D_i$. Now, as each $P^{\phi}(\omega_{(x,y)}) \in SI(\mathcal{D})$, and since each $P^{\phi}(\omega_{(x,y)}) \subsetneq D_i$, it follows by Proposition 4 that $P^{\phi}(\omega_{(x,y)}) \in SI(\mathcal{D} \setminus \{D_i\})$. This tells us that there exists $D_{(x,y)}$ for which $\{x, y\} \subseteq D_{(x,y)} \subseteq P^{\phi}(\omega_{(x,y)})$. Further, we know that $\{x,y\} \subseteq D_{(x,y)} \subsetneq D_i$. By Proposition 4, it follows that D_i is surely identified by the $D_{(x,y)}$, contradicting the fact that D_i is non-redundant.

Conversely, suppose that $\text{NRed}(\mathscr{D}) \subseteq \mathscr{P}^\phi$ and let D_i be some decision problem. If D_i is non-redundant then obviously it is surely identified from the payment sets, so assume D_i is redundant. Then by Proposition 4 for every $\{x,y\} \subseteq D_i$ there is $D_{(x,y)}$ such that $\{x,y\} \subseteq D_{(x,y)} \subsetneq D_i$. For each $\{x,y\}$ choose a minimal (with respect to inclusion) set $D_{(x,y)}$ with the above property. Then each $D_{(x,y)}$ is non-redundant and this collection of sets surely identifies D_i. It follows that D_i is surely identified from the payment sets, so ϕ is incentive compatible. □

C.1: Proof of Corollary 2

First, suppose that $SI(\mathscr{D})\setminus\{\{x\}_{x\in X}\} = \text{NRed}(\mathscr{D})$. We know from Theorem 1 that any incentive compatible mechanism ϕ is an RSS mechanism, and from Lemma 4 that any non-redundant set must be a payment set at some state. It follows from the assumption that $\mathscr{D} = \text{NRed}(\mathscr{D})$, so every decision problem is a payment set at some state. Furthermore, since any payment set is in $SI(\mathscr{D})$, it follows that whenever $\mathscr{P}^\phi(\omega)$ is not a singleton it is in $\mathscr{D}$. This proves that ϕ coincides with an RPS mechanism when restricted to M_R and to states where the payment sets of ϕ are not singletons.

Now, suppose that any incentive compatible mechanism ϕ coincides with an RPS mechanism restricted to M_R and to states where the payment sets of ϕ are not singletons. If $\text{NRed}(\mathscr{D}) \subsetneq \mathscr{D}$ then, by Lemma 4, there are incentive compatible mechanisms in which $\mathscr{P}^\phi \subsetneq \mathscr{D}$, contradicting the assumption. Thus, we must have $\text{NRed}(\mathscr{D}) = \mathscr{D}$. Similarly, we must also have $SI(\mathscr{D}) \setminus \{\{x\}_{x\in X}\} = \mathscr{D}$, and the result follows.

C.2: Proof of Corollary 3

We show that the condition from Corollary 2 is satisfied. To this end, we will show that all non-singleton elements of $SI(\mathscr{D})$ are elements of $\mathscr{D}$, and that all elements of $\mathscr{D}$ are non-redundant.

For the first claim, suppose by means of contradiction that $E \in SI(\mathscr{D})$, but that $E \notin \mathscr{D}$. Let $x,y \in E$. By Proposition 4, there is $D_{(x,y)} \in \mathscr{D}$ for which $\{x,y\} \subseteq D_{(x,y)} \subsetneq E$. There exists $z \in E\setminus D_{(x,y)}$. Let $D_{(x,z)} \in \mathscr{D}$ be such that $\{x,z\} \subseteq D_{(x,z)}$. If $y \in D_{(x,z)}$, then an irrational message exists (namely, announcing x from E and y from $D_{(x,z)}$. If $y \notin D_{(x,z)}$, then let $D_{(y,z)} \in \mathscr{D}$ for which $\{y,z\} \subseteq D_{(y,z)}$. Then, an irrational message exists, namely, announcing x from E, y from $D_{(y,z)}$ and z from $D_{(x,z)}$.

For the second claim, suppose that there is a redundant D_i. Then, similarly to the preceding paragraph, we can construct an irrational message from the elements of $\mathscr{D}\backslash D_i$.

Now, since $M_{NR} = \emptyset$, we do not need to add the qualification that there is an agreeing RPS mechanism on M_R (since $M = M_R$).

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