# Assignment 0: Coin flipping

**Please read the directions, including the experimental protocol!**
**Due: Thursday, January 9, 8:00 p.m., on line.**

During the first lecture I will distribute dollar coins, obtained from the Caltech Employees Federal Credit Union. **I ask that you use one of these coins, to make the coins as uniform as possible.** Your first assignment is to flip the coin 128 $(= 2^7)$ times and record the sequence of results (Heads or Tails), using the **protocol described below**.

If you miss the first lecture, you may obtain an official coin from Barbara Estrada in 111 Baxter during normal business hours, starting after 1:00 p.m. on January 7. You may keep your coin as partial compensation for your help in compiling a database of over 100,000 coin flips to date. Or I'll always take them back. (The coins are provided by me personally, not the Math Department.)

In order to save time and minimize transcription errors, I have created a form to submit your results at

[http://www.coyote.caltech.edu/Ma3/CoinTossSubmissionForm.html](http://www.coyote.caltech.edu/Ma3/CoinTossSubmissionForm.html)          .

# Experimental Protocol

1. **It is faster and more fun to work in pairs or larger groups.** Assign about half of the group a starting position of Heads and the others Tails. Half the group can record while the other half flips. Then switch. Even if you work in a group, **you should flip your own coin and report your 128 tosses in the sequence they occur.**

   You might find that imbibing a beverage such as espresso will make the task more enjoyable and elevate the conversation. If you record the results on a computer, you can just cut and paste the results into the web form.

2. Part of this experiment is to test the claim by Diaconis–Holmes–Montgomery [1] that a coin is more likely to come up the way it begins. In order to do this, **I ask that you choose either Heads or Tails and start all you tosses with this side facing up.** See the tossing protocol below.

3. Diaconis–Holmes–Montgomery are not explicit about the exact protocol for flipping a coin, but based on [1, § 5.1 and § 6.5] here is my version:

   - Make a fist with your thumb tucked slightly inside. Position the coin on top of your thumb-fist with Heads or Tails facing up, **depending on your chosen starting position**.

   - Give the coin a "normal flip" with your right hand if you are right-handed (left hand if you are left-handed), catching it in the same hand (without bouncing) at approximately the same height. Try to flip it high enough and/or with enough spin so it flips several times.

   - Open you hand palm up, and look at which side of the coin is facing up and record it. Do *not* flip the coin over after catching it to put it on you arm, as many people do. (Or if you do flip it over, report the reverse side.)

   - Don't worry too much about your flipping and catching technique. This is not a test of your coordination or dexterity. If you miss catching the coin, pretend it never happened.

4. Record the results of 128 flips **in the order that they occur**. Do not sort them! **Please encode the results as sequence of 0s and 1s, with 0 = Heads, and 1 = Tails.** ("Heads" is the side of the coin that shows someone's head, often a president, or Susan B Anthony or Sacajawea, but not the Statue of Liberty. "Tails" is something else, often the Statue of Liberty. If in doubt, ask around for the consensus opinion.)

   Do not include any white space. (Actually the web site will drop spaces and translate H's into 0's and T's into 1's.)

5. Submit the results via the form at
       http://www.coyote.caltech.edu/Ma3/CoinTossSubmissionForm.html.
   Note that this is not on the course web site itself.

   This year the web site will inform you immediately if submit the wrong number of flips, or if any information is missing. It will also let you know if your submission

has been accepted. There may be a short lag in getting a response. (I don't know what will happen if everyone tries to submit at the same instant.)

If there is anything you do not understand, ask me or your TA for clarification.

# 1   Why are we doing this?

Coin flipping is the canonical example of a random experiment. A number of authors have written on why this completely deterministic physical process can appear to be random. An understandable example is a paper by Karl Menger [2] who argues that in the space of initial conditions (height and rotational velocity) approximately equal areas give rise to Heads and to Tails. More recently, Persi Diaconis, Susan Holmes, and Richard Montgomery [1], using a more elaborate physical model that allows for wobbling, and high-speed photography, argue that a coin that is flipped and caught should turn up the same way as it started about 51% of the time.

This is not an idle exercise. The data from these sequences will be used for a number of purposes. First, I hope to illustrate that the Laws of Chance enable us to predict with reasonable accuracy such things as the number of Heads and the number of runs of length 1, 2, or 3. We will also use the results to generate approximately uniform random variables that can then be used to generate any other kind of random variable. When we start on statistics, we will use these data to test the Diaconis–Holmes–Montgomery hypothesis.

# References

[1] P. Diaconis, S. Holmes, and R. Montgomery. 2007. Dynamical bias in the coin toss. *SIAM Review* 49(2):211–235.                               DOI: 10.1137/S0036144504446436

[2] K. Menger. 1954. Tossing a coin. *American Mathematical Monthly* 61(9):634–636.
                                                    http://www.jstor.org/stable/2307682