Ec 181 — KC Border
Convex Analysis and Economic Theory — AY 2019–2020

Topic 11: Models of economies and equilibria

11.1 Models of economies

In this section, I shall describe a few simple models of the tastes, technology, and resources of an economy. For generalizations and more details on the interpretation Debreu [4] and Koopmans [6] are still among the best expositions. There are many aspects of economies that are not covered in these models, and these models are not suitable for analyzing every aspect of an economy. The models neglect the nature of contracts between economic agents, the network of who trades with whom, the internal structure of economic organizations, the power relationships between economic actors, etc. Nevertheless, they do provide insight into the rôle of prices in allocating resources, and have proven to fundamental to our understanding of economies.

11.1.1 Commodities

The first primitive concept is that of a **commodity**. A commodity is any good or service that may be produced, consumed, or traded. Commodities may distinguished not only by their physical characteristics, but also by their date, location, or state of the world. For mathematical simplicity, in this course we assume there is a finite number of commodities.

11.1.2 Consumption goods and factors of production

In this section we shall classify commodities into two categories, **consumption goods** or **final outputs** in one category, and **factors of production** or **inputs** in the other. This will make certain results easier to state and understand, but the distinction is seldom crucial. I shall try to remember to use y_j to denote a quantity of the j^{th} final output, and v_k to denote a quantity of the k^{th} factor. I shall also typically denote the number of final outputs by n, and the number of factors by ℓ.

In reality, many goods are **intermediate goods**, that is they are the final output of one production process, but an input into another process. For example, sheets of plywood are outputs of lumber mills, but inputs into the construction of houses, furniture, and even automobiles (Morgan roadsters) or airplanes. In a model of **household production**, much of what is bought by final consumers is used to produce meals or entertainment in the home.

11.1.3 Production functions

One approach to modeling production is to assume that each final output is produced separately according to an industry production function that relates quantities of inputs to quantities of outputs. So if we write $y_j = f_j(v) = f_j(v_1, \dots, v_\ell)$, it means that y_j units of the j^{th} final output can be produced using a combination of v_k units of the k^{th} factor, $k = 1, \dots, \ell$. Implicit in this approach is that there is no joint production of final outputs. One way to incorporate joint production is to use an **implicit transformation function** $T\colon \boldsymbol{R}^{\mathrm{n}}_+ \times \boldsymbol{R}^\ell_+ \to \boldsymbol{R}$, where $T(y, v) \geqslant 0$ means that is possible to produce the vector $y = (y_1, \dots, y_n)$ of output quantities from the vector $v = (v_1, \dots, v_\ell)$ of input quantities.

The virtue of the production function model is that if the production function is differentiable, we get to use the tools of the differential calculus, such as the Lagrange Multiplier Theorem, the Implicit Function Theorem, and the Envelope Theorem. If the production functions are assumed to be concave, then we can make use of the Saddlepoint Theorem.

11.1.4 Technology sets

Another approach treats commodities more symmetrically. There are ℓ commodities, and n **production units** or **enterprises**, indexed by j, each of which may convert inputs into outputs in accordance with its **technology set** $Y_j \subset \boldsymbol{R}^\ell$. A vector $y \in Y_j$ represents a feasible input/output plan, where $y_k < 0$ indicates that commodity k is used as an input and $y_k > 0$ indicates that it is an output.

The **aggregate production set** Y is defined to be $\sum_{j=1}^n Y_j$.[1]

A **pure trade economy** is an economy for which the aggregate production set Y is the singleton $\{0\}$.

11.1.5 Tastes

There are m idealized **consumers** or **households**. Each consumer i is partially described by a **consumption set** $X_i \subset \boldsymbol{R}^\ell$ and a **utility function** $u_i\colon X_i \to \boldsymbol{R}$. Elements x of X_i are ordered lists of quantities of commodities that are regarded as feasible "consumption bundles." For many cases, we assume $X_i = \boldsymbol{R}^\ell_+$.

Other times we may wish to allow x to have negative components. If $x_k > 0$, then commodity k is regarded as being "consumed." If $x_k < 0$, then this represents a supply of $|x_k|$ units of labor service k. There are other ways to model the supply of labor by the household. One is to treat leisure as part of the resource endowment and to treat unconsumed leisure as labor supplied. This latter treatment is less satisfactory when there are different kinds of labor services.

[1] This definition of the aggregate production set is not without restriction. It assumes that the production plans of one producer do not restrict the feasibility of any other's. That is, it presupposes that there are no **externalities** in production.

The utility of a consumption bundle represents its comparative desirability to the consumer. The set $\{y \in X : u(y) = u(x)\}$ is the **indifference class** of x or the **indifference curve** through x. The set $U(x) = \{y \in X : u(y) \geqslant u(x)\}$ is the **upper contour set**, or weak superlevel set, at x, and $P(x) = \{y \in X : u(y) > u(x)\}$ is the **strict upper contour set**, or strict superlevel set, at x.

11.1.1 Definition *A utility $u \colon X \to \boldsymbol{R}$ is*

- ***monotonic*** *if*
$$x \gg y \implies u(x) > u(y).$$
- ***strictly monotonic*** *if*
$$x > y \implies u(x) > u(y).$$
- ***nonsatiated*** *is u has no maximum on X.*
- ***locally nonsatiated*** *if*
$$(\forall x \in X)\ (\forall \varepsilon > 0)\ (\exists y \in X)\ [\,\|x - y\| < \varepsilon \ \&\ u(y) > u(x)\,].$$

The term local nonsatiation, which is the same as local nonmaximization introduced in Topic 7, is traditional in economics, so I'll use it here. Note that this is a joint condition on X and u. In particular, if X is nonempty, it must be that for each point $x \in X$ and every $\varepsilon > 0$ there is a point $y \neq x$ belonging to X with $\|x - y\| < \varepsilon$. That is, X may have no isolated points.

Strict monotonicity implies monotonicity, which implies local nonsatiation. Also, local nonsatiation does not imply monotonicity, but from the point of view of our theory the added generality is mostly irrelevant. See Exercise 12.9.1.

11.1.6 Resources

The final element in the description of an economy is the **aggregate resource endowment** ω, a vector in $\boldsymbol{R}^\ell_+$.

11.1.7 Allocations

An **economy** is thus summarized by a list What about the production function case?

$$E = \Big(X_i, u_i)_{i=1}^m, (Y_j)_{j=1}^n, \omega \Big).$$

A **pure trade economy** E may be described by the abbreviated list

$$E = \Big((X_i, u_i)_{i=1}^m, \omega \Big).$$

An **allocation** for the economy E is a list

$$(x^1, \dots, x^m, y^1, \dots, y^n)$$

satisfying

$$\begin{aligned} x^i &\in X_i \quad i = 1, \dots, m \\ y^j &\in Y_j \quad j = 1, \dots, n \\ \sum_{i=1}^m x^i &= \omega + \sum_{j=1}^n y^j. \end{aligned}$$

An allocation for a pure trade economy may omit the y^j's since they all are zero.

A natural question is whether allocations exist at all. That is, is $\sum_{i=1}^m X_i \cap (Y + \omega) \neq \varnothing$? One way to guarantee this is to assume $0 \in Y$ and $\omega \in \sum_{i=1}^m X_i$.

11.2 The production possibility set: I

If the technology is described with production functions and separate factor and final goods, then we make the following definition.

11.2.1 Definition *The **production possibility set** (PPS) is the set of feasible final outputs,*

$$PPS = \left\{ y \in \boldsymbol{R}^{\mathrm{n}} : 0 \leqslant y^j \leqslant f^j(v^j),\ v^j \geqq 0,\ j = 1, \dots, n, \text{ and } \sum_{j=1}^n v^j \leqq \omega \right\}.$$

11.2.2 Proposition *The PPS is compact.*

Proof: Since the f^j's are assumed continuous and monotonic, the PPS is the continuous image of the compact set

$$\left\{ (v^1, \dots, v^n) \in \boldsymbol{R}^{\ell \mathrm{n}} : v^j \geqq 0,\ j = 1, \dots, n, \text{ and } \sum_{j=1}^n v^j \leqq \omega \right\}.$$

Recall (Lemma A.7.15) that continuous images of compact sets are compact. ∎

11.2.3 Proposition *If each f^j is concave, then the PPS is convex.*

Proof: Exercise. Hint: Consider the set $\hat{H}$ in the proof of Theorem 10.1.1. ∎

11.2.4 Definition *More precisely, we say that output vector y in the PPS is **technically inefficient** if there is some y' in the PPS such that $y' > y$. A point is **technically efficient** if it is not technically inefficient. The **production possibility frontier** (PPF) set of efficient points in the PPS.*

Every point on the PPF is a support point. That is, if y belongs to the PPF, then there is a vector p of nonnegative prices such that y maximizes p over the PPS. This follows from the separating hyperplane theorem applied to the PPS and $\{z : z \gg y\}$. In this case the PPF can be parametrized by p.

11.3 The production possibility set: II

If we describe the technology in terms of production sets Y_j, the aggregate production set is $Y = Y_1 + \cdots + Y_n$.

11.3.1 Definition *The* ***production possibility set*** *(PPS) is the set of feasible final outputs,*

$$PPS = \omega + Y.$$

There is a potential difficulty in this version of the model. We saw (cf. Topic 20) that even if each Y_j is a closed set, their sum Y need not be closed. We shall deal with this complication later.

11.4 Efficiency

An allocation $(\bar{x}^1, \ldots, \bar{x}^m, \bar{y}^1, \ldots, \bar{y}^n)$ is **inefficient**, or **Pareto dominated**, if there is some other allocation $(x^1, \ldots, x^m, y^1, \ldots, y^n)$ such that

$$u_i(x^i) \geqslant u_i(\bar{x}^i) \quad \text{for all} i,$$

and

$$u_(x^i) > u_i(\bar{x}^i) \quad \text{for at least one} i.$$

An allocation is **efficient** (or **Pareto efficient** or **Pareto optimal**) if it is not inefficient.

11.5 Models of equilibrium

11.5.1 Private property

In an economy with the social convention of **private property**, the aggregate endowment and all the enterprises are wholly owned by the consumers. To completely describe such an economy and its property system we need to specify who owns what.

A **private ownership economy** is described by a list

$$\Big((X_i, u_i, \omega^i)_{i=1}^m, (Y_j)_{j=1}^n, (\theta_j^i)_{j=1,\ldots,n}^{i=1,\ldots,m} \Big),$$

where ω^i is a list of consumer i's **private resource endowment** of each commodity and labor service, so

$$\omega = \sum_{i=1}^m \omega^i,$$

and θ_j^i is the share of firm j owned by consumer i. These shares are nonnegative and sum to unity:

$$\theta_j^i \geqslant 0, \text{ all } i, j \qquad \text{and} \qquad \sum_{i=1}^m \theta_j^i = 1 \text{ all } j.$$

11.5.2 Walrasian equilibrium

Léon Walras in his *Elèments D'Èconomie Politique Pure* [9] initiated the mathematical study of equilibrium in multiple markets. In his honor, the outcome of competitive markets in a private ownership economy is usually modeled as a **Walrasian equilibrium**, which is an allocation together with a **price system** that is characterized by three properties.

1. Each enterprise maximizes profits, taking prices as given.
2. Each consumer maximizes their utility subject to their budget constraint.[2]
3. All markets clear.

Due to our sign conventions on inputs and outputs, the profit generated by the input-output plan y at price vector p is $p \cdot y$. So formally:

11.5.1 Definition *A* ***Walrasian equilibrium*** *of a private ownership economy is a list*

$$(\bar{x}^1, \ldots, \bar{x}^m, \bar{y}^1, \ldots, \bar{y}^n, \bar{p}),$$

where

1. *(Profit Maximization) For every enterprise j,*

$$\bar{y}^j \in Y_j \quad \text{and } \bar{p} \cdot \bar{y}^j \geqslant \bar{p} \cdot y^j \text{ for all } y^j \in Y^j.$$

2. *(Utility Maximization) For every consumer i,*

$$\bar{x}^i \in \beta_i = \left\{ x^i \in X_i : \bar{p} \cdot x^i \leqslant \bar{p} \cdot \omega^i + \sum_{j=1}^n \theta^i_j \bar{p} \cdot \bar{y}^j \right\} \quad \text{and}$$
$$u_i(\bar{x}^i) \geqslant u_i(x^i) \text{ for all } x^i \in \beta_i.$$

Note that if $\bar{x}^i$ exhausts the budget, that is, if $\bar{p} \cdot \bar{x}^i = \bar{p} \cdot \omega^i + \sum_{j=1}^n \theta^i_j \bar{p} \cdot \bar{y}^j$, then utility maximization is equivalent to

$$u(x^i) > u(\bar{x}^i) \quad \implies \quad p \cdot x^i > p \cdot \bar{x}^i.$$

3. *(Market clearing) $(\bar{x}^1, \ldots, \bar{x}^m, \bar{y}^1, \ldots, \bar{y}^n)$ is an allocation, that is,*

$$\sum_{i=1}^m \bar{x}^i = \sum_{i=1}^m \omega^i + \sum_{j=1}^n \bar{y}^j.$$

(In other words, in an allocation, total consumption equals total resources plus total production.)

[2] Some pedants will claim that the use of *they* or *their* as an ungendered singular pronoun is a grammatical error. There is a convincing argument that they are wrong. See, for instance, Huddleston and Pullum [5, pp. 103–105].

11.5.3 Walrasian quasi-equilibrium

A closely related concept is that of a **Walrasian quasi-equilibrium**, in which the utility maximization property is replaced by an expenditure minimization property.

2′. (Expenditure minimization) For every consumer i,

$$u(x^i) \geqslant u(\bar{x}^i) \quad \implies \quad p \cdot x^i \geqslant p \cdot \bar{x}^i.$$

11.5.4 Valuation equilibrium and quasi-equilibrium

A valuation equilibrium, introduced by Debreu [2] captures most of the properties of a Walrasian equilibrium, but does not require a specification of private ownership property rights.

11.5.2 Definition *A **valuation equilibrium** of an economy is a list*

$$(\bar{x}^1, \dots, \bar{x}^m, \bar{y}^1, \dots, \bar{y}^n, \bar{p}),$$

where

1. *(Profit Maximization) For every enterprise j,*

$$\bar{y}^j \in Y_j \quad \text{and} \quad \bar{p} \cdot \bar{y}^j \geqslant \bar{p} \cdot y^j \text{ for all } y^j \in Y^j.$$

2. *(Utility Maximization)*

$$u(x^i) > u(\bar{x}^i) \quad \implies \quad p \cdot x^i > p \cdot \bar{x}^i.$$

3. *(Market clearing) $(\bar{x}^1, \dots, \bar{x}^m, \bar{y}^1, \dots, \bar{y}^n)$ is an allocation, that is,*

$$\sum_{i=1}^m \bar{x}^i = \sum_{i=1}^m \omega^i + \sum_{j=1}^n \bar{y}^j.$$

*A **valuation quasi-equilibrium** replaces the maximization property with an expenditure minimization property.*

2′. *(Expenditure minimization) For every consumer i,*

$$u(x^i) \geqslant u(\bar{x}^i) \quad \implies \quad p \cdot x^i \geqslant p \cdot \bar{x}^i.$$

To convert a valuation equilibrium to a Walrasian equilibrium, let

$$\alpha_i = \frac{p \cdot \bar{x}^i}{\sum_{k=1}^m p \cdot \bar{x}^k},$$

the fraction of the total value of consumption accruing to consumer i. Set $\bar{\omega}^i = \alpha_i \omega$, and $\bar{\theta}^i_j = \alpha_i$ for all $i = 1, \dots, m$ and $j = 1, \dots, n$. Then a valuation equilibrium is a Walrasian equilibrium for the private ownership economy with ownership shares $(\bar{\omega}^i, \bar{\theta}^i)$.

11.6 Properties for utilities

Given a utility function u on a set X, define the **strict** and **weak superlevel sets**

$$P(x) = \{y \in X : u(y) > u(x)\} \qquad \text{and} \qquad U(x) = \{y \in X : u(y) \geqslant u(x)\}.$$

We also define the **strict** and **weak sublevel sets**

$$P^{-1}(x) = \{y \in X : u(y) < u(x)\} \qquad \text{and} \qquad U^{-1}(x) = \{y \in X : u(y) \leqslant u(x)\}.$$

Note that u is **upper semicontinuous** if for each x, the weak superlevel set $U(x)$ is closed, or equivalently, the strict sublevel set $P^{-1}(x)$ is open in X. Similarly, u is **lower semicontinuous** if for each x, the weak sublevel set $U^{-1}(x)$ is closed, or equivalently, the strict superlevel set $P(x)$ is open in X. Recall that u is continuous if and only if it is both upper and lower semicontinuous.

11.6.1 Lemma *If u is continuous and locally nonsatiated, then $U(x)$ is the closure of $P(x)$.*

Proof: $\overline{P(x)} \subset U(x)$: Let y belong to $\overline{P(x)}$. That is, there is a sequence y_n in $P(x)$, that is, $u(y_n) > u(x)$, with $y_n \to y$. So by continuity, $u(y) \geqslant u(x)$, that is, $y \in U(x)$.

$U(x) \subset \overline{P(x)}$: Let y belong to $U(x)$, that is $u(y) \geqslant u(x)$. By local nonsatiation, for each n there is a y_n satisfying $d(y_n, y) < \frac{1}{n}$ and $u(y_n) > u(y) \geqslant u(x)$, so $y_n \in P(x)$. But $y_n \to y$, so $y \in \overline{P(x)}$. ∎

11.6.2 Lemma *If X is convex, and u is quasiconcave, continuous, and locally nonsatiated, then $P(x)$ is the interior of $U(x)$.*

Proof: This is just Proposition 7.4.2. ∎

11.7 Utility maximization and expenditure minimization

Fix a price vector $p \in \boldsymbol{R}^\ell$ and income level $w \in \boldsymbol{R}$. Define the **budget set** by

$$\beta = \{x \in X : p \cdot x \leqslant w\}.$$

Assume that w is chosen large enough so that $\beta \neq \varnothing$.

In a Walrasian equilibrium we require that each consumer maximizes their utility over the budget set, so assume

$$x^* \text{ maximizes } u(x) \text{ over } \beta.$$

A sufficient condition for the existence of a minimizer is that β be compact, and u be upper semicontinuous.

In a Walrasian quasiequilibrium we require that each consumer minimizes their expenditure at price p subject to achieving a utility level $u(\hat{x})$. That is,

$$\hat{x} \text{ minimizes expenditure } p \cdot x \text{ subject to } u(x) \geqslant u(\hat{x}).$$

The next two lemmas give conditions under which $\hat{x} = x^*$.

11.7.1 Lemma (LNS and utility max imply expenditure min) *Assume u is locally nonsatiated, and x^* maximizes u over β, then*

1. *$p \cdot x^* = w$ (**budget exhaustion**),*
2. *and x^* minimizes expenditure $p \cdot x$ subject to $u(x) \geqslant u(x^*)$.*

Proof: We start by showing that if $y \in X$, then

$$u(y) \geqslant u(x^*) \implies p \cdot y \geqslant w. \tag{$\star$}$$

To see this, suppose by way of contradiction that there is some $\in X$ with $u(y) \geqslant u(x^*)$ and $p \cdot y < w$. By continuity of $p \cdot x$ with respect to x, there is some $\varepsilon > 0$ such that $\|z - y\| < \varepsilon$ implies $p \cdot z < w$ (so that $z \in \beta$). By local nonsatiation, one such z satisfies $u(z) > u(y) \geqslant u(x^*)$, which contradicts the maximality of x^* in β. This proves ($\star$).

Part 1 now follows by taking $y = x^*$, so ($\star$) implies $p \cdot x^* \geqslant w$, but $x^* \in \beta$ implies $p \cdot x^* \leqslant w$, so

$$w = p \cdot x^*,$$

which coupled with ($\star$) implies Part 2. ∎

11.7.2 Lemma (When expenditure min implies utility max) *Assume X is convex, and u is lower semicontinuous. Assume $\hat{x}$ minimizes $p \cdot x$ over $U(\hat{x})$, and the **cheaper-point assumption** holds, that is, there exists $\tilde{x} \in X$ satisfying $p \cdot \tilde{x} < p \cdot \hat{x}$.*

Then $\hat{x}$ maximizes u over $\beta = \beta(p, p \cdot \hat{x})$.

Proof: Suppose by way of contradiction that there is some $y \in \beta$ satisfying $u(y) > u(\hat{x})$, that is, $y \in P(\hat{x}) \subset U(\hat{x})$. Then $p \cdot y \geqslant p \cdot \hat{x}$, as $\hat{x}$ minimizes expenditure over $U(\hat{x})$. But y is in the budget β, so we conclude $p \cdot y = p \cdot \hat{x}$.

For λ satisfying $0 < \lambda < 1$, define $x(\lambda) = (1-\lambda)y + \lambda\tilde{x}$. Then $p \cdot \tilde{x} < p \cdot x(\lambda) < p \cdot \hat{x}$. Since X is convex, $x(\lambda) \in \beta$ for all $0 < \lambda \leqslant 1$.

But $x(\lambda) \to y$ as $\lambda \to 0$, and y belongs to the open set $P(\hat{x})$ (lower semicontinuity), so for some $\varepsilon > 0$, for every $\lambda < \varepsilon$ we have $x(\lambda) \in P(\hat{x}) \subset U(\hat{x})$. See Figure 11.7.1. But for such λ we have $p \cdot x(\lambda) < p \cdot \hat{x}$, which contradicts the hypothesis that $\hat{x}$ minimizes $p \cdot x$ over $U(\hat{x})$.

Therefore $\hat{x}$ maximizes u over $\beta = \beta(p, p \cdot \hat{x})$. ∎

To see what may happen if the cheaper-point assumption is violated, consider the following example.

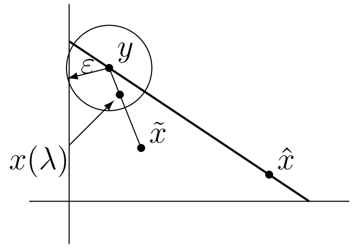

Figure 11.7.1. Expenditure minimization implies preference maximization.

11.7.3 Example (Why the cheaper point is needed) Let $X = \boldsymbol{R}^2_+$. Let $u(x_1, x_2) = x_1 + x_2$. Let $\hat{x} = (1, 0)$ and $p = (0, 1)$. Then $\hat{x}$ minimizes $p \cdot x$ over $U(\hat{x})$. But $\beta(p, p \cdot \hat{x}) = \beta(p, 0)$, which is just the x_1-axis. This budget set is unbounded and no utility maximizer exists. See Figure 11.7.2.

If you don't like the fact that I resorted to using a zero price, consider the case where $X = \{x \in \boldsymbol{R}^2_+ : x_1 + x_2 \geqslant 2\}$. Let $u(x_1, x_2) = 2x_1 + x_2$, $p = (1, 1)$, and $\hat{x} = (1, 1)$. Again $\hat{x}$ minimizes expenditure over $U(\hat{x})$, but $\bar{x} = (2, 0)$ is $\succcurlyeq$-greatest in the budget set $\beta(p, p \cdot \hat{x}) = \beta(p, 2)$, which is the southwest boundary of the consumption set. See Figure 11.7.3. □

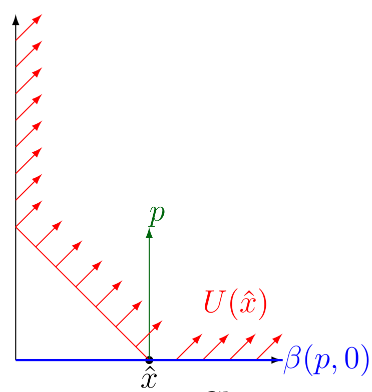

Figure 11.7.2. Cheaper-point violation 1.

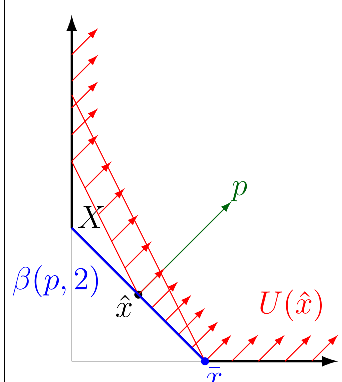

Figure 11.7.3. Cheaper-point violation 2.

11.7.4 Corollary *Assume X is convex, and u is lower semicontinuous and monotonic. Let p be given and set $w = p \cdot x^*$. Assume there is a point $\tilde{x} \in X$ satisfying $p \cdot \tilde{x} < w$.*

Then x^* maximizes u over $\beta(p, w)$ if and only if x^* minimizes $p \cdot x$ over $U(x^*)$.

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