How To Pay Subjects
OR
Incentive Compatibility Across Decision Problems

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What is an Experiment?

1. A list of ‘decision problems’
2. A payment mechanism
Decision Problems:

1. $D_1$: dictator game
2. $D_2$–$D_6$: 5-question Holt-Laury elicitation

Data:

1. $m_1 = (9, 1) \in D_1$
2. $m_2 = (10\% , 2.00; 90\% , 1.60) \in D_2$
   : 
3. $m_6 = (90\% , 3.85; 10\% , 0.10) \in D_6$

How to pay?
Formalization

- Decision problem: \( D_i \subseteq X \).
  \( D = (D_1, \ldots, D_k) \).
- Preference: \( \succeq \) over \( X \)
- True choices: \( \mu(\succeq) \subseteq \times_i D_i \)
- Announced choice: \( m \in \times_i D_i \)
- Notation for this talk:
  - truthful announcement: \( m^* \in \mu(\succeq) \)
  - lie: \( \hat{m} \notin \mu(\succeq) \)
  - generic announcement: \( m \in \times_i D_i \)
Payments modeled as Savage (1954) acts:

- **Finite state space** $\Omega$
  - $\{\text{Sun, Rain}\}$
  - $\{\text{Heads, Tails}\}$
  - $\{1, 2, 3, 4, 5, 6\}$

- **Outcome** $f(\omega)$ for each state $\omega$
  - $f(\text{Sun}) = \text{Warm}$, $f(\text{Rain}) = \text{Wet}$
  - $f(\text{Heads}) = m_1$, $f(\text{Tails}) = m_2$
  - $f(1) = m_1$, $f(2) = m_2$, $\ldots$, $f(6) = m_6$

- **Act:** $f : \Omega \rightarrow X$
Payment Mechanism

- Payment Mechanism: $\phi$
- Each $\phi(m)$ is an act
- Subject choosing between acts $\phi(m^*)$, $\phi(\hat{m})$, $\phi(m)$, …

- Preferences over acts: $\succeq^*$
- $\succeq^*$ over acts is an extension of $\succeq$ over $X$
- Incentive compatibility:

  $$\phi(m^*) \succeq^* \phi(m) \quad \forall m$$

  $$\phi(m^*) \succ^* \phi(\hat{m}) \quad \forall \hat{m}$$
The Random Problem Selection (RPS) Mechanism

RPS Mechanism:

- State space partitioned into $k$ events: $\{\Omega_1, \ldots, \Omega_k\}$ (one for each $D_i$)
- $\omega \in \Omega_i \Rightarrow \phi(m)(\omega) = m_i$

“Pay for one randomly-chosen decision.”
The Only Assumption

**Theorem 1:** RPS is incentive compatible if $\succeq^*$ satisfies **monotonicity**.

$\succeq^*$ over acts satisfies **monotonicity** if:

\[
f \succeq^* g \iff f(\omega) \succeq g(\omega) \quad \forall \omega \in \Omega
\]

\[
f \succ^* g \iff f(\omega) \succeq g(\omega) \quad \forall \omega \in \Omega
\]

\[
f(\omega') \succ g(\omega') \quad \text{for some } \omega' \in \Omega
\]

- All EU, Maxmin, Choquet prefs satisfy monotonicity
- **separability**
- **reduction of compound lotteries**
- Violations: *ex-ante* fairness, ‘uncertainty effect’, confusion
Other IC Mechanisms?

Example:

- $D_1 = \{x, y\}$, $D_2 = \{y, z\}$, $D_3 = \{x, z\}$
- Assume $m$ is rationalizable (doesn’t reveal cycles)
- Can always infer most-preferred choice in $E = \{x, y, z\}$
- We call $E$ a **surely identified** (SI) set

Surely identified sets:

- Each $D_i$ is SI
- Every singleton $\{x\}$ is SI
- All SI sets must be in $\bigcup_i D_i$
- Let $SI(D)$ be the family of SI sets
Random **Set** Selection (RSS) Mechanism:

- Each $\omega$ is assigned to some SI set $E_\omega$
- If rationalizable $m$ is announced and state $\omega$ obtains, pay the most-preferred item from $E_\omega$

RPS is an RSS where each $E_\omega = D_i$ for some $i$
Theorem 2: A mechanism is incentive compatible if and only if it is an RSS mechanism satisfying:

1. each \( D_i \) is in \( SI(\{E_\omega\}_{\omega \in \Omega}) \), and
2. non-rationalizable messages map to different acts than rationalizable messages.

If your payment mechanism doesn’t look like this, you may be getting bad data.

RPS is the ‘simplest’ of these IC mechanisms.
Corollary: If you pay for multiple decisions, the mechanism is not incentive compatible.

Proof:
1. Let $X = S \cup B(S)$ (singletons $\cup$ bundles)
   - Singleton: $\{(10\%, \$2.00; 90\%, \$1.60)\}$
   - Bundle: $\{(10\%, \$2.00; 90\%, \$1.60), ($9, $1)\}$
2. All $D_i \subseteq S \Rightarrow$ all SI sets are in $S$
3. Thus, can never pay from $B(S)$

Exceptions:
1. ‘Show-Up Fees’ (don’t depend on $m$)
2. You know there are no complementarities
3. Repeated games/choices over bundles
The RPS (‘pay for one’) mechanism works under very weak assumptions
   ▶ Monotonicity is way weaker than EUT
   ▶ Can still be violated: fairness, ‘certainty effect’, confusion

There are other IC mechanisms, but nothing you’d ever use.

Do NOT pay for multiple decisions/periods
   ▶ Except within a supergame or ‘bundled decisions’
“Don’t screw up your experiments by doing stupid stuff.”
-Christopher P. Chambers