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**ECON 8877**
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# Hypothesis Test Assignment

**Due by Monday, November 6th 2023 at 4:00 PM**

## Problem 1

Use MATLAB, R, or Stata to write a program that does the following:

1. Generate two random samples $(X_1, \ldots, X_m)$ and $(Y_1, \ldots, Y_n)$ where $X_i \sim N(0,1)$ and $Y_i \sim N(0,1)$. Set $n = 10$ and $m = \rho n$ where for now we set $\rho = 1$.
2. Run a $t$-test, Wilcoxon test, uncorrected permutation test, and corrected permutation test on the difference in means. Use $\alpha = 0.05$, as usual.
3. Repeat these steps 1,000 times and count the fraction of times each test rejects the null. Save this number.
4. Repeat this for various values of $n$ ranging from 10 to 200. Keep $\rho = 1$. Plot the rejection frequencies of each test as a function of $n$. Compare this to 0.05.
5. Now set $Y_i \sim N(0.2, 1)$ but keep $X_i \sim N(0,1)$. Repeat the above exercise. For each test, what $n$ is needed to (correctly) reject the null 80% of the time? The plots are called power curves.
6. Now plot power curves for a larger difference in means, such as $Y_i \sim N(1,1)$.
7. Plot power curves for the same means but different variances. For example $X_i \sim N(0,1)$ and $Y_i \sim N(0,2)$. Keep $\rho = 1$ for now.
8. Repeat the last exercise, but pick a different value of $\rho$. Is the corrected permutation test asymptotically valid?
9. Finally, come up with two crazy distributions that have the same mean, but are not normal. For example, you can mix normal distributions by generating half of the $X_i$ sample from $N(-1,1)$ and half from $N(1,1)$, while $Y_i$ is a mixture of $N(-3,1)$ and $N(3,1)$. Or mix together other distributions such as beta distributions. Just make sure $X_i$ and $Y_i$ have the same true mean. Then plot power curves.
10. In the end, which test would you use for your own research?

For the permutation test correction, see Chung and Romano [2013].

## References

EunYi Chung and Joseph P. Romano. Exact and asymptotically robust permutation tests. *The Annals of Statistics*, 41(2):484–507, April 2013. ISSN 0090-5364, 2168-8966. doi: 10.1214/13-AOS1090.