Bayesian Overconfidence

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Overconfidence

Observations from the literature:

- Overconfidence is robust

  > *"Perhaps the most robust finding in the psychology of judgement is that people are overconfident." (DeBondt & Thaler, 1995)*

  > *"No problem in judgement and decision making is more prevalent and more potentially catastrophic than overconfidence." (Plous, 1993)*

- Overconfidence can explain many phenomena
  - Negotiation delays, excessive litigation, excessive market entry (& failure), excessive stock trading (& volatility), overinvestment by CEOs, initiation & prolonging of wars.

Problems with Overconfidence

1. Vague, subjective survey questions
2. Varying definitions of overconfidence
   - **Overplacement:** You think you're better than everyone else.
   - **Overestimation:** You think you're better than you really are.
   - **Overprecision:** You underestimate the noise in your info.
3. *Under*confidence can be observed
   - Underestimation: Lichtenstein & Fischhoff, 1977; Erev, Wallsten & Budescu, 1994; Griffn & Tversky, 1992.
   - Underplacement: Kruger, 1999; Moore & Small, 2007; Windschitl, Kruger & Simms, 2003.
4. Results apparently sensitive to task difficulty and the definition of overconfidence
   - Moore & Kim, 2003; Moore & Small, 2007.

   "The difficulty effect is one of the most consistent findings in the calibration literature..."
   (Griffin & Tversky, 1992)

This Paper

Three contributions:

1. Clearly define three distinct notions of overconfidence
2. Directly & cleanly test the difficulty ('hard-easy') effect
3. Show that over/underconfidence can be 'rationalized'

Be careful:

- Over/underconfidence is a *statistical* bias
- We don't *need* a behavioral bias to generate it
- Over/underconfidence still exists & has consequences!!

Preview of Experimental Results

- Before a task:
  - Slight overplacement by men, underplacement by women
  - No over/underestimation
- After an easy task:
  - Overplacement (ranking self higher than others)
  - Underestimation (under-guessing own score)
- After a difficult task:
  - Underplacement
  - Overestimation

Overplacement Model: Example

- Several firms building a new type of product
- Each has prior expected per-unit cost of \$10
- Production begins, actual cost of firm j is \$6
- Firm j believes \$10 was wrong for 2 reasons:
  1. \$10 was an overestimate of the *industry average*
  2. Firm j is better (cheaper) than the average
- Thus, $E_j[C_k | c_j = \$6] = \$8$, e.g.
- If $c_j < \$10$ for all j, all exhibit overplacement
- If $c_j > \$10$, j exhibits *under*placement
- Easy $\Rightarrow$ overplacement, Difficult $\Rightarrow$ underplacement

Overestimation Model: Example

- Now firms get private, unbiased signal first (prototype)
- j's prototype cost is \$6
- j believes the prior (\$10) was wrong because:
  1. \$10 was too high for the industry average
  2. Firm j is better (cheaper) than average
  3. Firm j's signal error was favorable (negative)
- $E_j[C_j|s_j = \$6] = \$7 < E_j[C_k|s_j = \$6] = \$8$
- For econometrician, same ranking *in expectation*
- Easy $\Rightarrow$ underestimation, Difficult $\Rightarrow$ overestimation

The Experiment

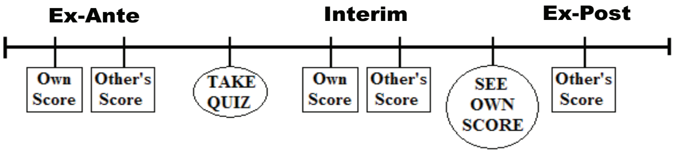
- 59 subjects from CMU & Pittsburgh area
- Each subject takes 18 trivia quizzes with 10 questions each
- 6 topics: Geography, history, movies, music, science, sports
- 3 difficulty levels: Easy, Medium, Hard
- Difficulty in randomized blocks: [EHM] [MEH] [MHE] ...
- Topics randomized uniformly

Example

What is the capital of Australia? Who painted the Sistine Chapel? Who was the first MVP of the NBA?

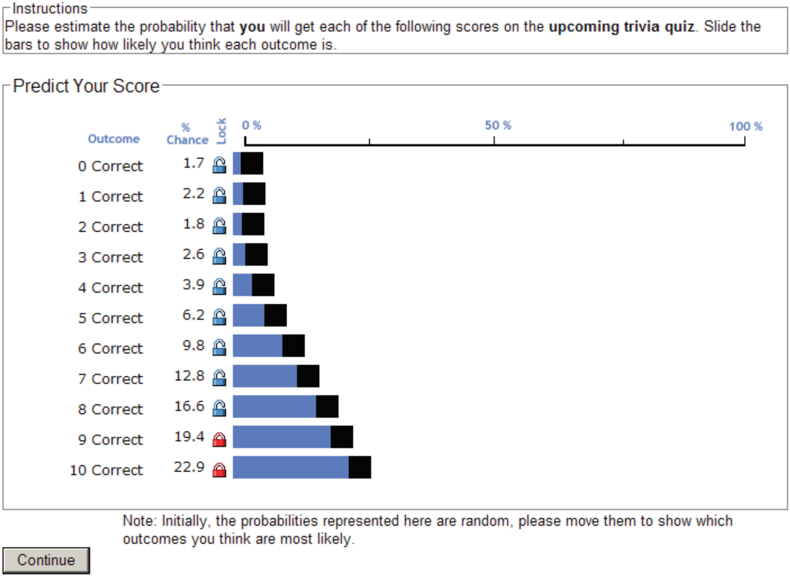
Reports & Incentives

- $r\%$ on the quiz pays $\$25\frac{r}{100}$
- 3 time stages/period: Ex-Ante, Interim, & Ex-Post
- 5 belief elicitations per period:



- Elicit *entire* belief distribution
  - $p(0)$, $p(1)$, ..., $p(10)$
  - Subjects drag slider bars
  - 'Other' is a randomly selected previous participant
- Paid via quadratic scoring rule: $1 + 2\,p(x) - \sum_{k=0}^{10} p(k)^2$

Reports & Incentives



Manipulations

Do subjects manipulate their predictions & performance?

- Avg \$ on Quiz: \$12.18, Two self-predictions: \$2.39
- Best manipulation: \$2.54 and \$4.00, resp.
- Easy quiz avg. score: 8.86/10
  - 11/492 (2.2%) scores in $\{0, 1\}$ from 9 different subjects
- All quizzes:
  - 23/1476 (1.6%) predict in $\{0, 1, 9, 10\}$ & are correct

Smaller manipulations??

Result 1: Test Differences

Result

Scores are (1) low on hard quizzes, (2) above average on medium quizzes, and (3) high on easy quizzes. Subjects perceive these differences.

Dependant Variable	Score	$E(\text{Self}\mid\text{Interim})$
Easy	**8.864**	**8.644**
	(83.48)	(79.68)
Medium	**5.925**	**5.930**
	(55.80)	(54.67)
Difficult	**0.693**	**1.503**
	(6.53)	(13.86)

Shifts in interim beliefs are smaller than shifts in scores

Result 2: Ex-Ante Overconfidence

Result

No ex-ante over/underestimation.

- $E[\text{Self}|\text{Ex-Ante}] - \text{Score} \approx 0$ (p-val 0.581)
- No effect by gender

Result

Men exhibit slight overplacement, women slight underplacement

- Period 1 median: Men: 0.436 Women: -0.148 (p-vals 0.008, 0.090)
- All periods: Men: 0.126 Women: -0.058 (p-vals < 0.001, 0.006)
- Pooled across genders: 0.020 (p-val 0.022)
- Magnitudes are ~~small~~ tiny ($n = 1,476$)
- (Niederle & Vesterlund 2007)

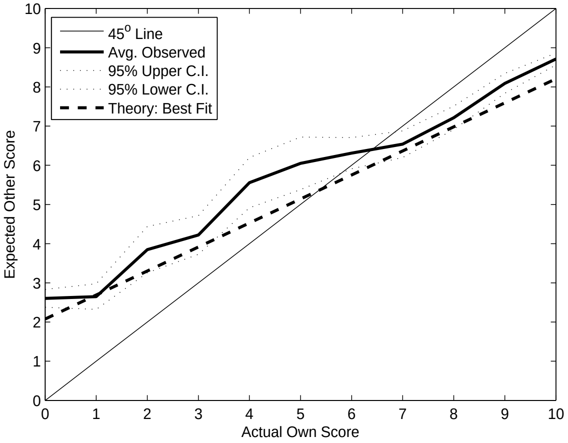
Result 3: Over/underplacement

Result

Subjects exhibit overplacement after easy quizzes and underplacement after difficult quizzes.

Dependant Variable	$E(\text{Self}\|\text{Interim}) - E(\text{Other}\|\text{Interim})$	$\text{Score} - E(\text{Other}\|\text{Ex-Post})$
Easy	**0.385**	**0.369**
	(4.02)	(3.58)
Medium	**-0.221**	**-0.284**
	(-2.30)	(-2.76)
Difficult	**-1.448**	**-1.660**
	(-15.12)	(-16.14)

Result 3: Over/underplacement



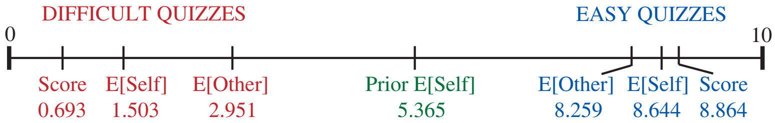
Result 4: Over/underestimation

Result

Subjects exhibit underestimation after easy quizzes and overestimation after difficult quizzes.

Dependant Variable	$E(\text{Self}\|\text{Interim}) - \text{Score}$
Easy	**-0.219**
	(-3.98)
Medium	0.006
	(0.10)
Difficult	**0.810**
	(14.69)

The Basic Pattern



Difficult Quiz:	Overestimation & Underplacement
Easy Quiz:	Underestimation & Overplacement

Overprecision

- Actual scores are highly bimodal: $Pr\{0, 10\} = 0.498$
- Ex-ante beliefs are not:
  - Avg $p_i\{0, 10\} = 0.164$ ($<$ uniform)
  - Last period: Avg $p_i\{0, 10\} = 0.247$ ($>$ uniform)
- Thus, overprecision

Other Results

- Time Trend/Learning:
  - No block effects $\implies$ no (obvious) time trend
- Subjects' over/underplacement is usually wrong
  - 44.5% exhibit interim overplacement after easy quizzes... of those, 35.6% were correct.
  - 63.2% exhibit interim underplacement after hard quizzes... of those, 39.5% were correct.
- Underplacement is larger in magnitude

The Basic Model

- Players simultaneously perform a task (quiz)
- Player i's score is x_i, a realization of X_i
- Players believe $E[X_i] = E[X_j] = S$
- Example: $X_i = S + L_i$
  - 'Simplicity' (S) has mean μ
  - 'Luck' (L_i) has mean zero
- Players see x_i, report beliefs about X_j

Definition

Overplacement: $E[X_j|x_i] < x_i$
Underplacement: $E[X_j|x_i] > x_i$

The Basic Intuition

- Suppose $X_i = S + L_i$ with $S \sim \mathcal{N}(\mu, \sigma_S^2)$ and $L_i \sim \mathcal{N}(0, \sigma_L^2)$.
- In this case Bayes's rule implies

$$E[S|x_i] = \underbrace{\left[\frac{\sigma_L^2}{\sigma_L^2 + \sigma_S^2}\right]}_{\alpha} \mu + \underbrace{\left[\frac{\sigma_S^2}{\sigma_L^2 + \sigma_S^2}\right]}_{1-\alpha} x_i$$

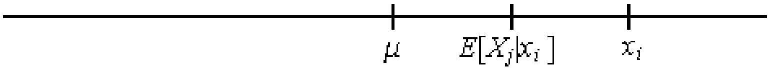
- But this means

$$\begin{aligned} E[X_j|x_i] &= E[S + L_j|x_i] \\ &= E[S|x_i] + \underbrace{E[L_j|x_i]}_{0} \\ &= (\alpha)\,\mu + (1-\alpha)\,x_i \end{aligned}$$

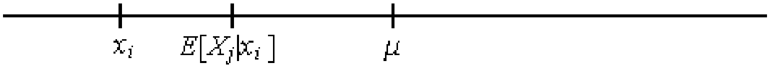
- Thus, $E[X_j|x_i]$ is between μ and x_i.

The Basic Intuition

- Whenever $E[X_j|x_i]$ is between μ and x_i we have:
- Higher-than-expected score $\implies$ overplacement



- Lower-than-expected score $\implies$ underplacement



Discrete Scores

- Suppose agents take a 10-question quiz
- $x_i \in \{0, 1, \ldots, 10\}$
- Subjects believe $X_i \sim binom(10, p)$ with $p \sim beta(\beta_1, \beta_2)$
- Then $\mu = 10\frac{\beta_1}{\beta_1+\beta_2}$ and

$$E[X_j|x_i] = \underbrace{\left[\frac{\beta_1+\beta_2}{\beta_1+\beta_2+10}\right]}_{\alpha}\mu + \underbrace{\left[1 - \frac{\beta_1+\beta_2}{\beta_1+\beta_2+10}\right]}_{1-\alpha} x_i$$

- $x_i > \mu \implies E[X_j|x_i] < x_i$ (overplacement)
  $x_i < \mu \implies E[X_j|x_i] > x_i$ (underplacement)

Robustness

Does Bayes's rule *always* imply 'betweenness':

$$E[S|X_i = x_i] = \alpha\,\mu + (1-\alpha)\,x_i?$$

Chambers & Healy (2008):

- Bayes's rule implies nothing in general
- If $f(S)$ and $f(L_i)$ are symmetric & quasiconcave then yes
- Counter-examples with highly bimodal beliefs (compare: data)

Overestimation

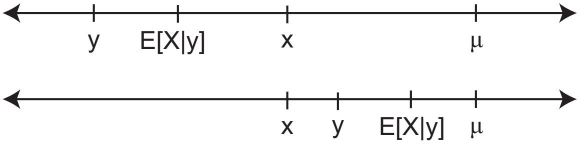
- Add an 'intermediate' stage
- Agents have performed the task, but don't know their score
- Agents receive a noisy signal (Y_i) of their score, with

$$E[Y_i|x_i] = x_i$$

- Example: $Y_i = X_i + u_i$

Overestimation

- Define overestimation as $x_i < E\left[X_i | y_i\right]$?
- Problem: Results depend on realization of y_i



Overestimation

- Solution: *expected* overestimation
- Compare x_i to researcher's expectation of subject's expectation of X_i

$$\underbrace{E_{Y_i}[\ \underbrace{E[X_i|y_i]}_{i\text{'s expectation}}\ |x_i]}_{\text{researcher's expectation}}$$

Definition

Overestimation in expectation: $x_i < E_{Y_i}\left[E\left[X_i|y_i\right]|x_i\right]$
Underestimation in expectation: $E_{Y_i}\left[E\left[X_i|y_i\right]|x_i\right] < x_i$

Normal Priors

- $Y_i = X_i + U_i$, all normally distributed with $E[U_i] = 0$
- $E[X_i|y_i] = \bar{\alpha}\mu + (1-\bar{\alpha})y_i$

$$\begin{aligned} E_{Y_i}[E[X_i|y_i]|x_i] &= E_{Y_i}[\bar{\alpha}\mu + (1-\bar{\alpha})y_i|x_i] \\ &= \bar{\alpha}\mu + (1-\bar{\alpha})E[Y_i|x_i] \\ &= \bar{\alpha}\mu + (1-\bar{\alpha})x_i \end{aligned}$$

- On average, i's expectation of his score is between μ and x_i
  - Lower-than-expected score $\implies$ overestimation in expectation
    Higher-than-expected score $\implies$ underestimation in expectation

Summary of Predictions

Task Difficulty	Relative Performance	Absolute Performance
Unexpectedly Easy	Overplacement	Underestimation
Unexpectedly Difficult	Underplacement	Overestimation

Model Extensions

Multi-Dimensional Signals

- The *contents* of the test might provide 2^{nd} signal
- Let $R_i = S + Q_i$ with $E[Q_i] = 0$.
- Then $E[R_i] = \mu$
- A very extreme R_i could reverse the results
  "I did well, but the final was just a copy of the midterm!"
- Like Y_i, econometrician doesn't observe R_i
- On average, 2^{nd} signal reinforces μ
- Magnitude of shifts is smaller
  - (See paper for normal example)

Model Extensions

Ability and Prior Overconfidence

- Agnostic about its source (learned, bias, . . . ?)
- $X_i = S + L_i + A_i$ with $E[A_i] \neq 0$
- $X_i = S + \hat{L}_i + E[A_i]$, with $E[\hat{L}_i] = 0$
- Easy tasks *increase* overplacement
- Difficult tasks *decrease* overplacement
- 'Luck' has higher variance, increasing shift magnitudes

Non-Bayesian Updating

People aren't perfect Bayesians (Grether, etc.)

- Only need betweenness, not Bayes's rule
- Non-Bayesians can exhibit betweenness
- Predictions are 'robust' in this respect

Model vs. Data

- Data are consistent with basic predictions
- Betweenness satisfied 64.8% of the time
- Betweenness or reversing (also sufficient): 80.1%
- Interim expected score closer to mean than actual score
- Learning/experience should reduce the effect
  - Each quiz is different
  - Data for last 3 quizzes has same pattern, smaller magnitudes
  - Subjects *do* get slightly better at predicting scores
  - No overconfidence with repetitive tasks (Kahneman & Riepe 1992) or expertise (bridge: Keren 1987; horses: Hausch et al 1981; weather: Murphy & Winkler 1984)

Other Models

Van den Steen (*AER* 2004) and Santos-Pinto & Sobel (*AER* 2005)

- Set of choices $X = \{x_1, x_2, \ldots, x_N\}$
- Different objective functions/beliefs of success $f_i(x)$
- $x_i^* \neq x_j^*$, but both think they're right & other is wrong
- No inference from others' choices

- Predicts overplacement, not underplacement or over/underestimation
- Task difficulty not relevant

Other Models

Zabojnik (*ET* 2004)

- Can either produce & consume or test your ability
- Payoff is convex in ability
- High test results $\Rightarrow$ high opportunity cost to testing
- Asymmetric testing $\Rightarrow$ overestimation on average
- Predicts overplacement, not underplacement or over/underestimation
- Task difficulty not relevant

Other Models

Dubra & Krishna (2008)

- Takes our idea to the extreme
- "Given population-level overconfidence data, is there *any* signaling model that could rationalize the data?"
- Our paper: uncertainty about difficulty, score serves as the signal
- Their paper: Signal could come from anywhere, observed or not
- Mostly negative results: "almost everything can be rationalized"

Conclusion

What we have accomplished:

- Clear definitions of ‘overconfidence’
- Experiment that compiles disperse results
- Simple explanation of the source of overconfidence

The End

Updating Toward the Signal

with Christopher P. Chambers

The Setting

- X = some random variable of interest
- $Z = X + \tilde{\varepsilon}$ = noisy signal of X
- $\tilde{\varepsilon}$ may depend on X
- $E[\tilde{\varepsilon}|X = x] = 0 \ \forall x$
- Care about $E[X|Z = z]$
- Often assumed that $E[X|z] = \alpha z + (1 - \alpha)E[X]$
- When is this appropriate? Is it robust?

Assumptions

- Assume all r.v.'s are real-valued and have cts densities & finite means
- Consider *families* of error terms $\mathcal{E}$
- **Questions:** What conditions on X and $\mathcal{E}$ guarantee

$$E[X|z] = \alpha z + (1-\alpha)E[X] \text{ ?} \tag{1}$$

  Does (1) imply anything about X or $\mathcal{E}$?
- Relevant properties of r.v.'s:
  - *Symmetric:* symmetric density about the mean
  - *Quasiconcave:* quasiconcave density (unimodal)

Definitions

Definition

X **updates toward the signal w.r.t** $\mathcal{E}$ (**UTS-**$\mathcal{E}$) if $\forall \tilde{\varepsilon} \in \mathcal{E}$, $\forall z$ $\exists \alpha \in [0, 1]$ s.t.

$$E[X|Z = z] = \alpha z + (1 - \alpha)E[X]. \tag{2}$$

Definition

X **updates in the direction of the signal w.r.t** $\mathcal{E}$ (**UDS-**$\mathcal{E}$) if equation (2) holds with $\alpha \geq 0$ $\forall \tilde{\varepsilon} \in \mathcal{E}$.

Definition

X satisfies **mean reinforcement with respect to** $\mathcal{E}$ (**MR-**$\mathcal{E}$) if $\forall \tilde{\varepsilon} \in \mathcal{E}$

$$E[X|z = E[X]] = E[X] \tag{3}$$

Definitions

UTS:

UDS:

MR:

Error Terms

All error terms are continuous, mean-zero, and satisfy sym. dep.:

Definition

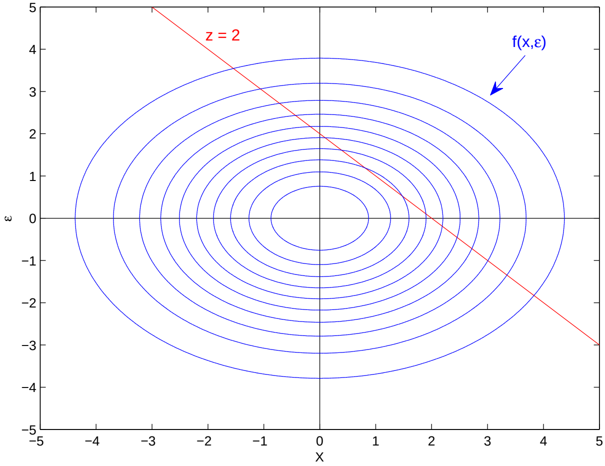
$\tilde{\varepsilon}$ satisfies **symmetric dependence** if, for almost every $\varepsilon, a \in \mathbb{R}$,

$$f_{\tilde{\varepsilon}}(\varepsilon | X = E[X] + a) = f_{\tilde{\varepsilon}}(\varepsilon | X = E[X] - a).$$

If X is symmetric, sym.dep. gives a symmetric joint distribution:
(Wlog, assume throughout that $E[X] = 0$)

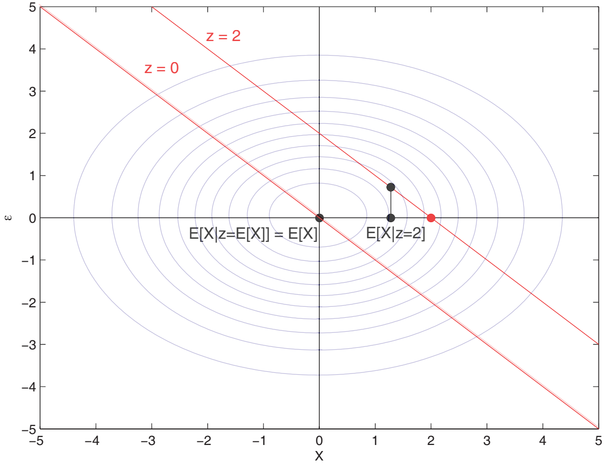
$$\begin{aligned} f(x, \varepsilon) &= f_X(x)\, f_{\tilde{\varepsilon}}(\varepsilon | x) \\ &= f_X(-x)\, f_{\tilde{\varepsilon}}(\varepsilon | -x) \\ &= f(-x, \varepsilon) \end{aligned}$$

Visualizing the Conditions



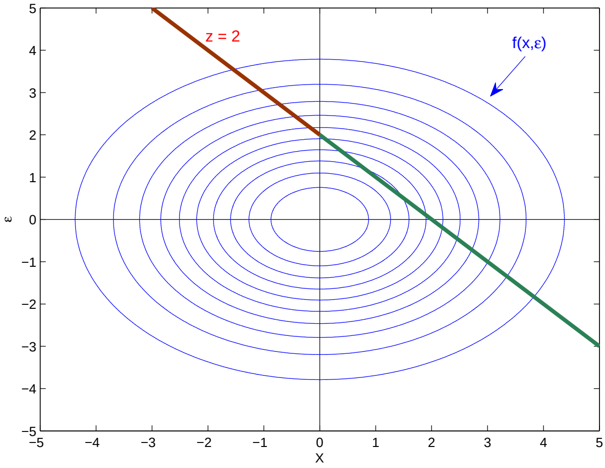
$$E[X|z] \propto \int x\, f(x, z - x)\, dx$$

The Normal-Normal Case



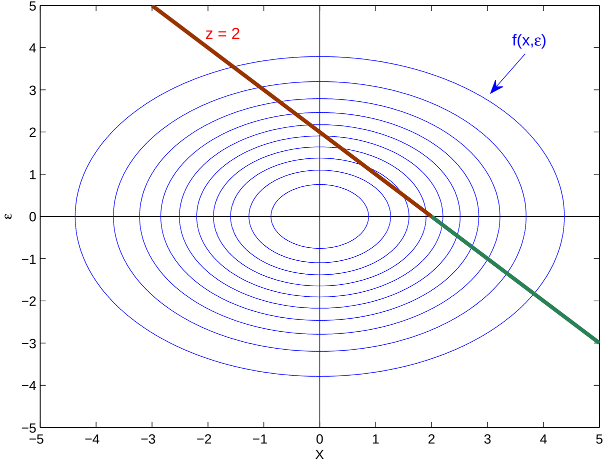
If $X \sim \mathcal{N}(0,2)$ and $\tilde{\varepsilon} \sim \mathcal{N}(0,1)$ then $E[X|z=2] = 1.6$. UTS!

Visualizing the Conditions



Here, $E[X|z = 2] > 0$

Visualizing the Conditions



Here, $E[X|z = 2] < 2$ (or $E[\tilde{\varepsilon}|z = 2] > 0$)

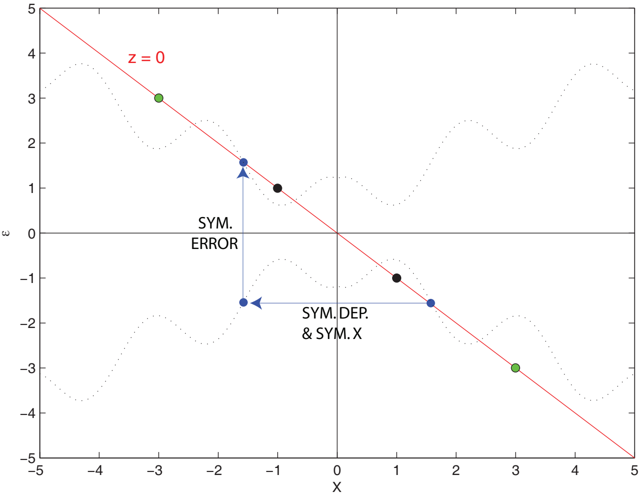
MR: Sufficient Conditions

Proposition

If X is symmetric and $\tilde{\varepsilon}$ is symmetric then X satisfies MR-$\{\tilde{\varepsilon}\}$

Proof.

See pictures... □



$$\int_{-\infty}^{0} -x \; f_{\tilde{\varepsilon}}(0 - x|x) f_X(x) dx = \int_{0}^{\infty} x \; f_{\tilde{\varepsilon}}(0 - x|x) f_X(x) dx$$

UDS: Sufficient Conditions

X symmetric, $\tilde{\varepsilon}$ symmetric $\nRightarrow X$ UDS-$\{\tilde{\varepsilon}\}$.

Example

Let $f_X(x) = \begin{cases} \frac{1}{3}\left(1 - \frac{|x|}{3}\right) & \text{if} \quad x \in [-3, 3] \\ 0 & \text{otherwise} \end{cases}$

and $\tilde{\varepsilon} \sim (-2, \frac{1}{2}; 2, \frac{1}{2})$. Then $E[X|z] = -z$, so UDS fails.

To get UDS, need another restriction on errors:

Proposition

If X is symmetric and $\tilde{\varepsilon}$ is symmetric and quasiconcave then X satisfies UDS-$\{\tilde{\varepsilon}\}$

UTS: Sufficient Conditions

X symmetric, $\tilde{\varepsilon}$ sym & q.c. $\not\Rightarrow$ X UTS-$\{\tilde{\varepsilon}\}$.

Example

Let $f_{\tilde{\varepsilon}}(\varepsilon) = \begin{cases} \frac{1}{3}\left(1 - \frac{|\varepsilon|}{3}\right) & \text{if} \quad \varepsilon \in [-3, 3] \\ 0 & \text{otherwise} \end{cases}$

and $X = (-2, \frac{1}{2}; 2, \frac{1}{2})$. Then $E[X|z] = 2z$, so UTS fails.

Proposition

If X is symmetric and quasiconcave and $\tilde{\varepsilon}$ is symmetric, quasiconcave, and independent of X then X satisfies UTS-$\{\tilde{\varepsilon}\}$.

Summary of Results

Family of Error Terms			Prior			Condition
Sym			Sym		$\Rightarrow$	MR
Sym			Sym		$\not\Rightarrow$	UDS
Sym	QC		Sym		$\Rightarrow$	UDS
Sym	QC		Sym		$\not\Rightarrow$	UTS
Sym	QC	Ind*	Sym	QC	$\Rightarrow$	UTS

MR: Necessary Conditions 1

Let $\mathcal{E}_{2pt} = \{\tilde{\varepsilon} \sim (-y, p; y, 1-p) : y \in \mathbb{R}\}$.

Proposition

Pick any $\mathcal{E}$ with $\mathcal{E}_{2pt} \subseteq \overline{\mathcal{E}}$.
If X satisfies MR-$\mathcal{E}$ then X is symmetric.

Proof.

1. Pick any $y > 0$ and let $\tilde{\varepsilon} \sim (-y, \frac{1}{2}; y, \frac{1}{2})$.
2. $z = 0$ means $x \in \{-y, y\}$.
3. Thus, $E[X|z=0] \propto -yf_X(-y) + yf_X(y)$.
4. MR-$\mathcal{E}$ means $-y\, f_X(-y) + y\, f_X(y) = 0$ for every $y > 0$.
5. Thus, $f_X(y) = f_X(-y)$, so X is symmetric.

□

UDS: An Impossibility Result

Can we get the stronger concept of UDS?

Proposition

If $\mathcal{E}_{2pt} \subseteq \overline{\mathcal{E}}$ then there does not exist an X such that X UDS-$\mathcal{E}$.

MR: Necessary Conditions 2

Let $\mathcal{E}_U = \{\tilde{\varepsilon} \sim U[-y, y] : y \in \mathbb{R}\}$.

Proposition

Pick any $\mathcal{E}$ with $\mathcal{E}_U \subseteq \overline{\mathcal{E}}$.
If X satisfies MR-$\mathcal{E}$ then X is symmetric.

UTS: Necessary Conditions

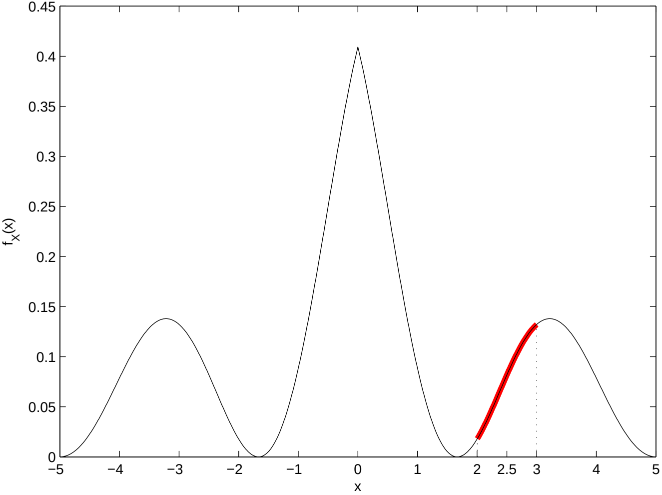
Proposition

Pick any $\mathcal{E}$ with $\mathcal{E}_U \subseteq \overline{\mathcal{E}}$.
If X satisfies UTS-$\mathcal{E}$ then X is symmetric **and quasiconcave**.

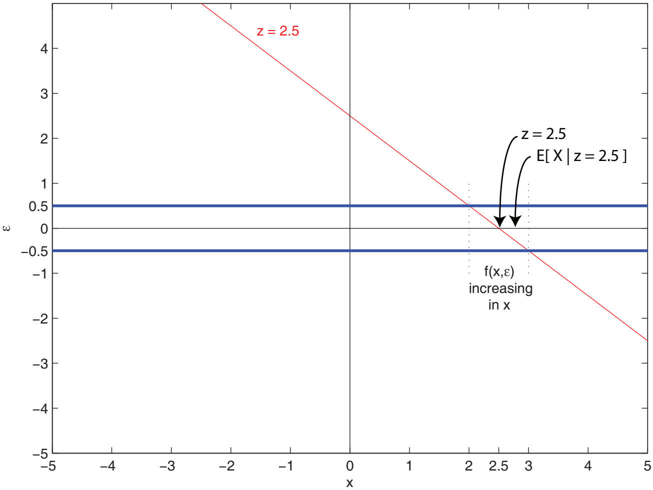
Proof (Sketch).

- UTS-$\mathcal{E}$ $\Rightarrow$ MR-$\mathcal{E}$ $\Rightarrow$ X symmetric. ✓
- For quasiconcavity, see pictures...

□



Not quasiconcave: f_X is increasing on $[2,3]$



If $\tilde{\varepsilon} \sim U[-\frac{1}{2}, \frac{1}{2}]$ then $E[X|z = 2.5] > z = 2.5$. UTS-$\mathcal{E}$ fails.

Summary of Results

Family of Error Terms			Prior			Condition
Sym			Sym		$\Rightarrow$	MR
Sym			Sym		$\not\Rightarrow$	UDS
Sym	QC		Sym		$\Rightarrow$	UDS
Sym	QC		Sym		$\not\Rightarrow$	UTS
Sym	QC	Ind*	Sym	QC	$\Rightarrow$	UTS
$\mathcal{E}_{2pt} \subseteq \overline{\mathcal{E}}$			Sym		$\Leftarrow$	MR
$\mathcal{E}_{2pt} \subseteq \overline{\mathcal{E}}$			$\nexists$		$\Leftarrow$	UDS
$\mathcal{E}_{U} \subseteq \overline{\mathcal{E}}$			Sym		$\Leftarrow$	MR
$\mathcal{E}_{U} \subseteq \overline{\mathcal{E}}$			Sym	QC	$\Leftarrow$	UTS

Characterizations

Can form various 'iff' statements:

1. If $\mathcal{E}_{2pt} \subseteq \overline{\mathcal{E}}$ and all $\tilde{\varepsilon} \in \mathcal{E}$ are symmetric then

$$X \text{ Symmetric} \Leftrightarrow \text{MR}-\mathcal{E}$$

2. If $\mathcal{E}_U \subseteq \overline{\mathcal{E}}$ and all $\tilde{\varepsilon} \in \mathcal{E}$ are symmetric & q.c. then

$$X \text{ Symmetric} \Leftrightarrow \text{MR}-\mathcal{E} \Leftrightarrow \text{UDS}-\mathcal{E}$$

3. If $\mathcal{E}_U \subseteq \overline{\mathcal{E}}$ and all $\tilde{\varepsilon} \in \mathcal{E}$ are symmetric, q.c., & indep. then

$$X \text{ Sym \& Q.C.} \Leftrightarrow \text{UTS}-\mathcal{E}$$

Interpretations

- If you want UTS, assume symmetry & q.c. of X & $\tilde{\varepsilon}$.
    - Don't need normal distributions...
- If you have a normal prior, UTS is fairly robust to changes in $\tilde{\varepsilon}$.
- If you have a normal prior, e.g., assuming UTS means assuming sym. & q.c. of $\tilde{\varepsilon}$.
- If you *don't* use a sym. & q.c. prior then there is some uniformly distributed $\tilde{\varepsilon}$ and some z such that UTS fails.
- Econometrics: posterior mean = estimate of X. UTS $\Rightarrow$ a 'well behaved' estimate.

On the Robustness of Good News and Bad News

with Christopher P. Chambers

Milgrom 1981

- Milgrom (1981) "Good News and Bad News" *Bell Journal*
- $Z = X + \tilde{\varepsilon}$
- Consider two signals $z' > z$
- Want monotonicity of the posterior distributions

Theorem (Milgrom 1981)

$f(z|x)$ satisfies strict MLRP iff $z' > z$ implies that $F(\cdot|z') >_{\mathrm{FOSD}} F(\cdot|z)$ for all non-degenerate priors on X

- Note: allows *all* priors (X) but fixes an error term ($\tilde{\varepsilon}$).

Our Result

- Suppose we fix a prior and allow the error to vary.

Theorem

Let X be a non-degenerate bounded random variable. There exists a noise term $\tilde{\varepsilon}$ that is symmetric, quasiconcave, and independent of X and two real numbers $z' > z$ for which $F(\cdot|z) >_{\text{FOSD}} F(\cdot|z')$.

- With freedom in the error term you can always *reverse* the Milgrom result!
- Where should our models have more freedom: in the prior or in the error?

THE END