Ternary Price Lists for Belief Elicitation

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Greg Leo (Loyola Marymount)

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True belief (subjective): $p = Pr(X = 1)$ Reported belief: q

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Fixed prize ($8 *e.g.*)

($\Rightarrow$ no risk-neutrality)

Prob. you win, given X:

$$[1 - (X - q)^2]\%$$

(Subj.) Overall prob.:

$p\,[1 - (1 - q)^2] +$

$(1 - p)\,[1 - (0 - q)^2]$

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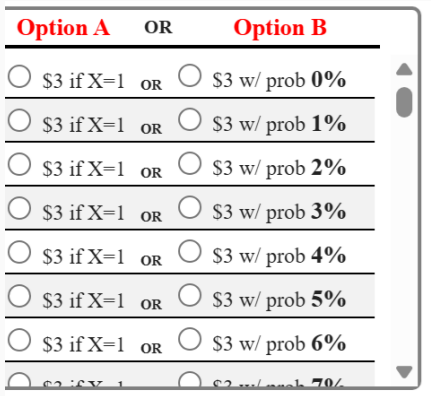

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○ $3 if X=1	OR	○ $3 w/ prob **6%**
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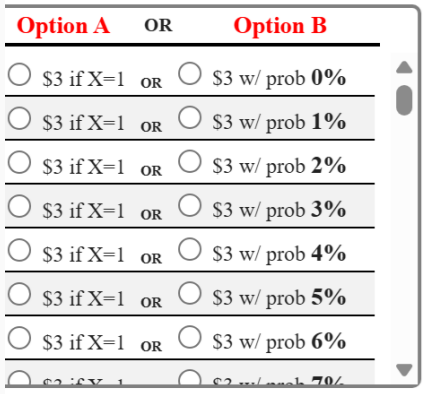

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○ $3 if X=1	OR	○ $3 w/ prob **7%**

IC: ROCL× Dominance✓

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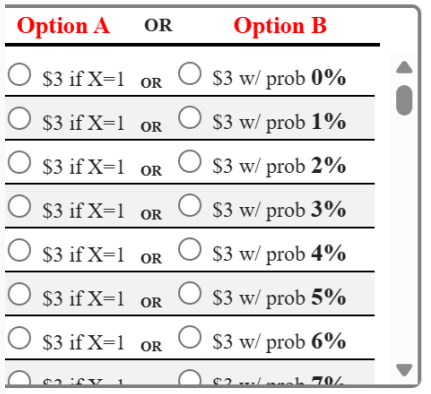

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Strength:	100% (w/ 1 prize)✓	50%×

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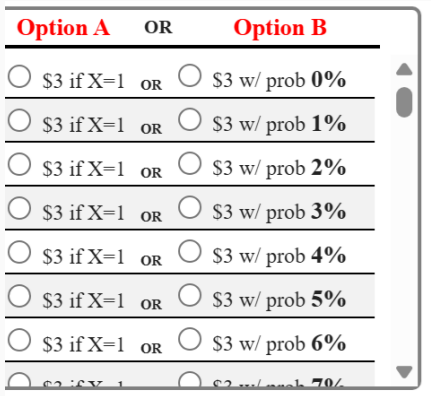

3. TPL

IC:	ROCL×	Dominance✓
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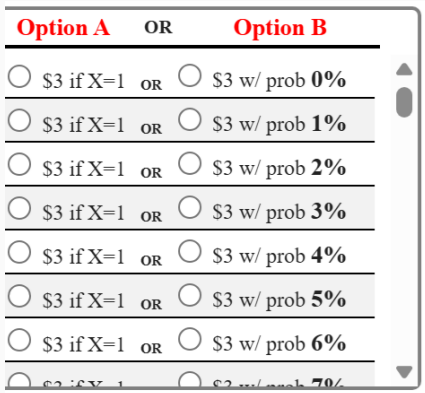

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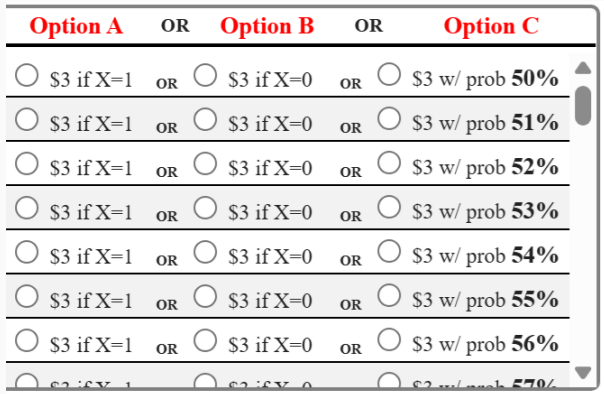
3. TPL



Option A	OR	Option B	OR	Option C
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○ $3 if X=1	OR	○ $3 if X=0	OR	○ $3 w/ prob **55%**
○ $3 if X=1	OR	○ $3 if X=0	OR	○ $3 w/ prob **56%**

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True belief (subjective): $p = Pr(X = 1)$ Reported belief: q

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Fixed prize ($8 *e.g.*)

($\Rightarrow$ no risk-neutrality)

Prob. you win, given X:

$$[1 - (X - q)^2]\%$$

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$$p\,[1 - (1 - q)^2] + (1 - p)\,[1 - (0 - q)^2]$$

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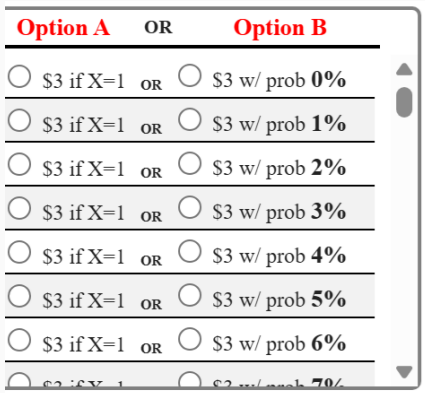

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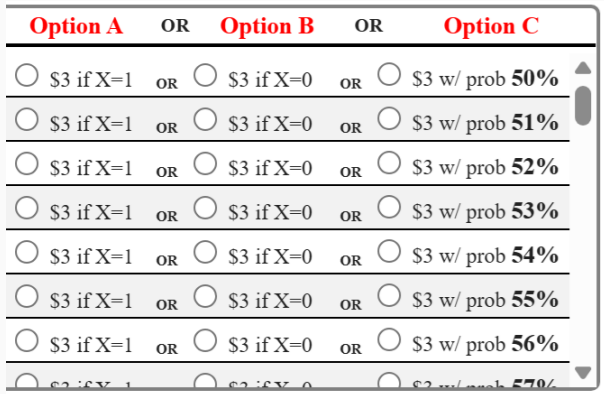

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	1. BQSR	2. MPL	3. TPL
IC:	ROCL ✗	Dominance ✓	Dominance ✓
Strength:	100% (w/ 1 prize) ✓	50% ✗	

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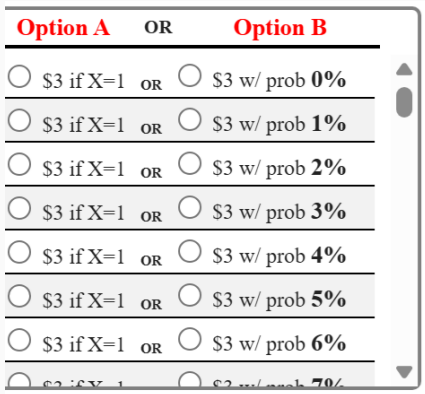

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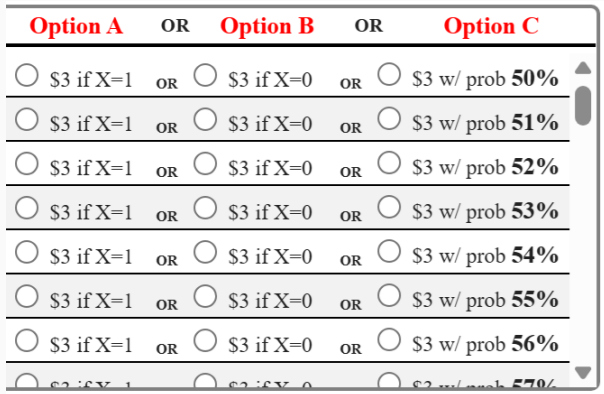

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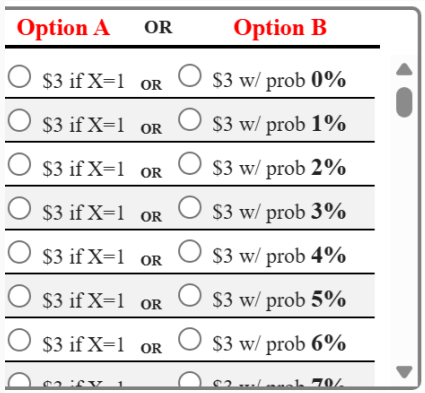

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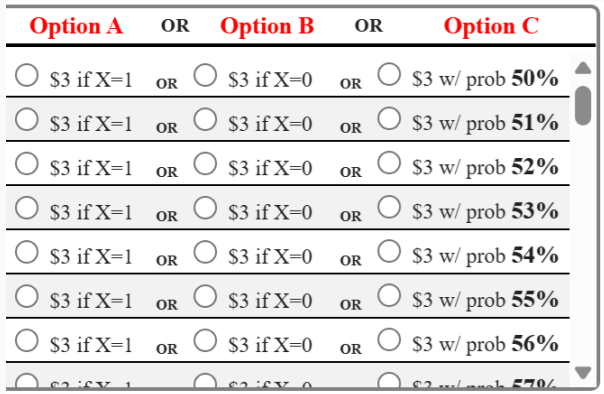

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	BQSR	MPL	TPL
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Strength:	100% (w/ 1 prize)✓	50%×	100%✓
Expmt:	Good Enough*✓	Good Enough✓	

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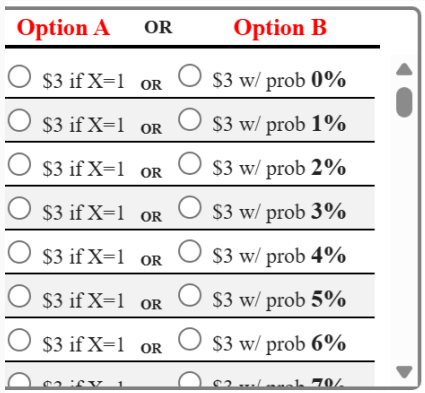

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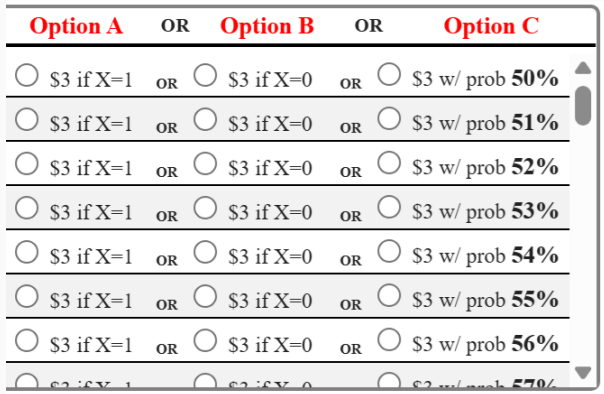

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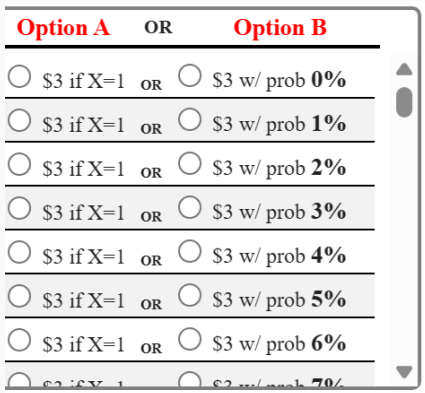

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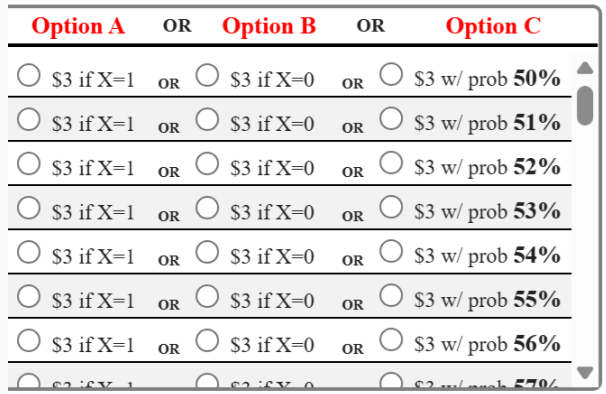

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IC:	ROCL×	Dominance✓	Dominance✓
Strength:	100% (w/ 1 prize)✓	50%×	100%✓
Expmt:	Good Enough*✓	Good Enough✓	Good Enough✓

Our Recommendation: Use MPL, in scroll box, in instructions only

The End

Appendix

The Three Prongs

1. Incentive compatibility requirements
2. Strength of incentives
3. Performance on a Prolific Experiment

Incentive Compatibility Requirements

The Binarized Quadratic Scoring Rule (BSR)

Event: $X \in \{0, 1\}$
Subjective belief: p = $\Pr(X = 1)$

Binarized Quadratic Scoring Rule (Savage 1971; Hossai & Okui 2013):

- Fixed prize ($8)
- Announce subjective belief q
- Paid objective lotteries:
  - \$8 w/ prob $s_1(q) = 1 - (1 - q)^2$ if $X = 1$
  - \$8 w/ prob $s_0(q) = 1 - (0 - q)^2$ if $X = 0$

Conditions for Incentive Compatibility

Incentive Compatibility $\Leftarrow$ reduction:

$$U(q|p) = \underbrace{p \cdot [1-(1-q)^2] + (1-p) \cdot [1-(0-q)^2]}_{\text{Overall } Pr(\$8) \text{ when announcing } q}$$

"Subjective-Objective Reduction"

- "Probabilistic sophistication" (Machina & Schmeidler 1995)
- Weakening of ROCL (binary lotteries only)
- But, requires integration of subj. & obj. uncertainty

IC under S-O reduction because $U(q|p)$ is max'd at $q^* = p$

Multiple Price Lists (MPL)

Row#	Option A	OR	Option B
1	\$8 if $X = 1$	or	\$8 w/ prob 1%
2	\$8 if $X = 1$	or	\$8 w/ prob 2%
⋮	⋮	⋮	⋮
q	\$8 if $X = 1$	or	\$8 w/ prob q%
$q+1$	\$8 if $X = 1$	or	\$8 w/ prob $q+1$%
$q+2$	\$8 if $X = 1$	or	\$8 w/ prob $q+2$%
$q+3$	\$8 if $X = 1$	or	\$8 w/ prob $q+3$%
⋮	⋮	⋮	⋮
99	\$8 if $X = 1$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 w/ prob 100%

Choose Option A or Option B (single switch point q)

One row randomly selected for payment

Multiple Price Lists (MPL)

Row#	Option A	OR	Option B
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2	\$8 if $X = 1$	or	\$8 w/ prob 2%
⋮	⋮	⋮	⋮
q	\$8 if $X = 1$	or	\$8 w/ prob q%
$q + 1$	\$8 if $X = 1$	or	\$8 w/ prob $q + 1$%
$q + 2$	\$8 if $X = 1$	or	\$8 w/ prob $q + 2$%
$q + 3$	\$8 if $X = 1$	or	\$8 w/ prob $q + 3$%
⋮	⋮	⋮	⋮
99	\$8 if $X = 1$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 w/ prob 100%

Just have them type in their belief

"I think $Pr(X = 1)$ is _____"

Multiple Price Lists (MPL)

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⋮	⋮	⋮	⋮
q	\$8 if $X = 1$	or	\$8 w/ prob q%
$q + 1$	\$8 if $X = 1$	or	\$8 w/ prob $q + 1$%
$q + 2$	\$8 if $X = 1$	or	\$8 w/ prob $q + 2$%
$q + 3$	\$8 if $X = 1$	or	\$8 w/ prob $q + 3$%
⋮	⋮	⋮	⋮
99	\$8 if $X = 1$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 w/ prob 100%

If you lie, you get the less-preferred option on some rows

I.C. as long as subject respects **statewise dominance** in rows

Ternary Price Lists (TPL)

Row#	Option A	OR	Option B	OR	Option C
50	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 50%
51	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 51%
⋮	⋮	⋮	⋮	⋮	⋮
q	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob q%
$q + 1$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 1$%
$q + 2$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 2$%
$q + 3$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 3$%
⋮	⋮	⋮	⋮	⋮	⋮
99	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 100%

Choose Option A or Option B or Option C (single switch point q)

Again, just have them type in their belief: _____

Ternary Price Lists (TPL)

Row#	Option A	OR	Option B	OR	Option C
50	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 50%
51	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 51%
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
q	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob q%
$q + 1$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 1$%
$q + 2$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 2$%
$q + 3$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 3$%
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
99	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 100%

This example: $p > 0.5$

Ternary Price Lists (TPL)

Row#	Option A	OR	Option B	OR	Option C
50	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 50%
51	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 51%
⋮	⋮	⋮	⋮	⋮	⋮
q	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob q%
$q + 1$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$ w/ prob $q + 1$%
$q + 2$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$ w/ prob $q + 2$%
$q + 3$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 3$%
⋮	⋮	⋮	⋮	⋮	⋮
99	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 100%

If you lie, you get the less-preferred option on some rows

I.C. as long as subject respects **statewise dominance** in rows

Ternary Price Lists (TPL)

Row#	Option A	OR	Option B	OR	Option C
50	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 50%
51	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 51%
⋮	⋮	⋮	⋮	⋮	⋮
q	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob q%
$q + 1$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 1$%
$q + 2$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 2$%
$q + 3$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 3$%
⋮	⋮	⋮	⋮	⋮	⋮
99	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 100%

This example: $p < 0.5$

Ternary Price Lists (TPL)

Row#	Option A	OR	Option B	OR	Option C
50	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 50%
51	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 51%
⋮	⋮	⋮	⋮	⋮	⋮
q	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob q%
$q + 1$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 1$%
$q + 2$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 2$%
$q + 3$	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob $q + 3$%
⋮	⋮	⋮	⋮	⋮	⋮
99	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 if $X = 0$	or	\$8 w/ prob 100%

If you lie, you get the less-preferred option on some rows

I.C. as long as subject respects **statewise dominance** in rows

MPL and TPL vs BQSR

Theorem (Healy & Kagel):

All BQSRs are I.C.

$\Updownarrow$

Subjective-Objective Reduction

$\Downarrow$

Statewise Dominance

$\Downarrow$

MPL and TPL are I.C.

So, from IC perspective, MPL & TPL $\succ$ BQSR

Strength of Incentives

The Measure of the Strength of Preferences

Define $U(q|p) =$ overall $Pr(\$8)$ when announcing q w/ belief p.

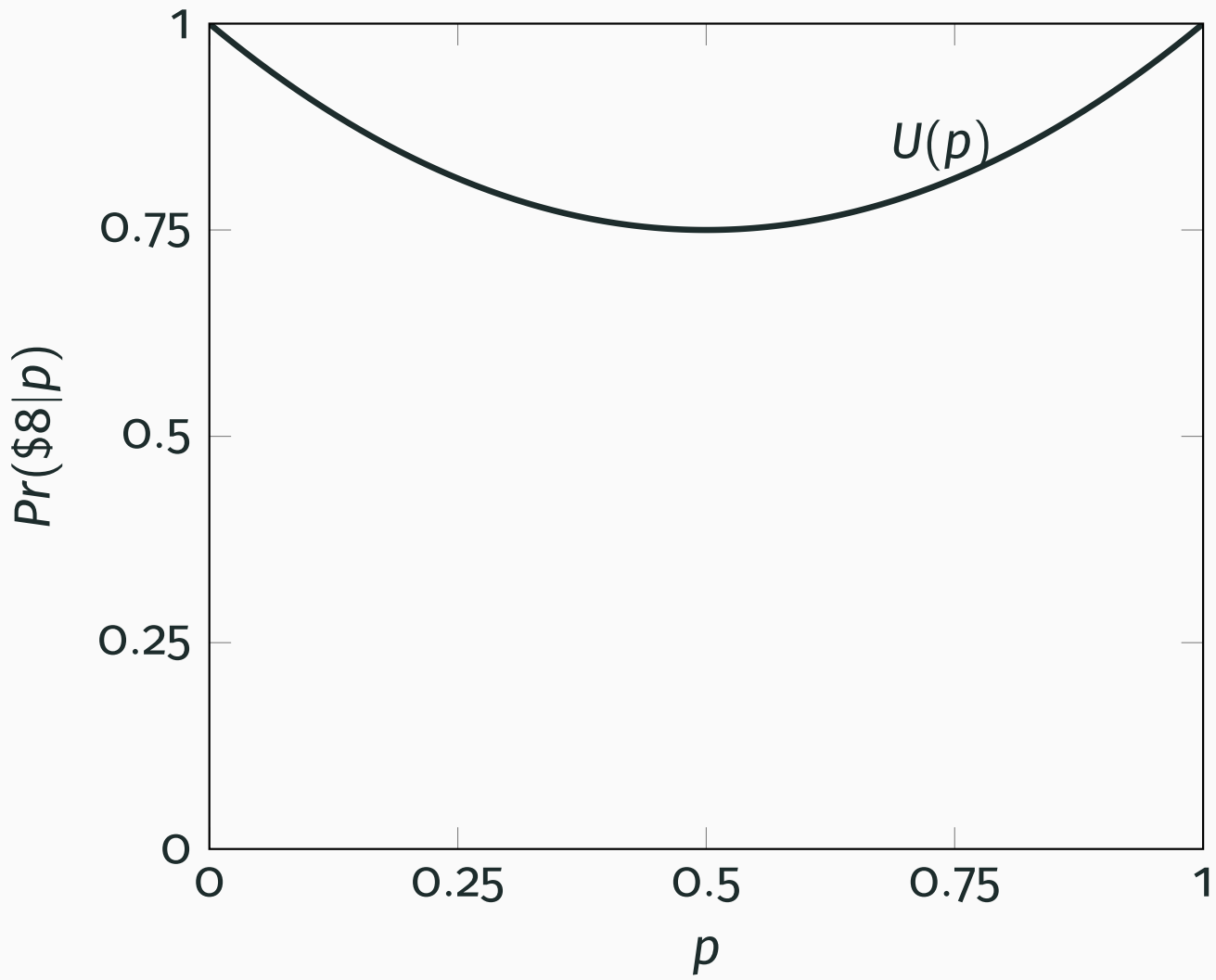
How to measure the cost of a "local" deviation away from $q = p$?

- Our measure: Concavity. ($-U''(q|p)$ at $q = p$)
  - Caveat: Objective measure. May differ from actual prefs if subject isn't maximizing overall $Pr(\$8)$.

Envelope theorem result:

- Define $U(p) = U(q|p)$
  - Classic result: IC $\iff$ $U(p)$ is convex
- Envelope theorem $\Rightarrow -U''(q|p) = U''(p) \geq 0$
- So, our measure is just $U''(p)$

$U''(p)$



Example $U(p)$ for the BQSR

Binarization Constraint

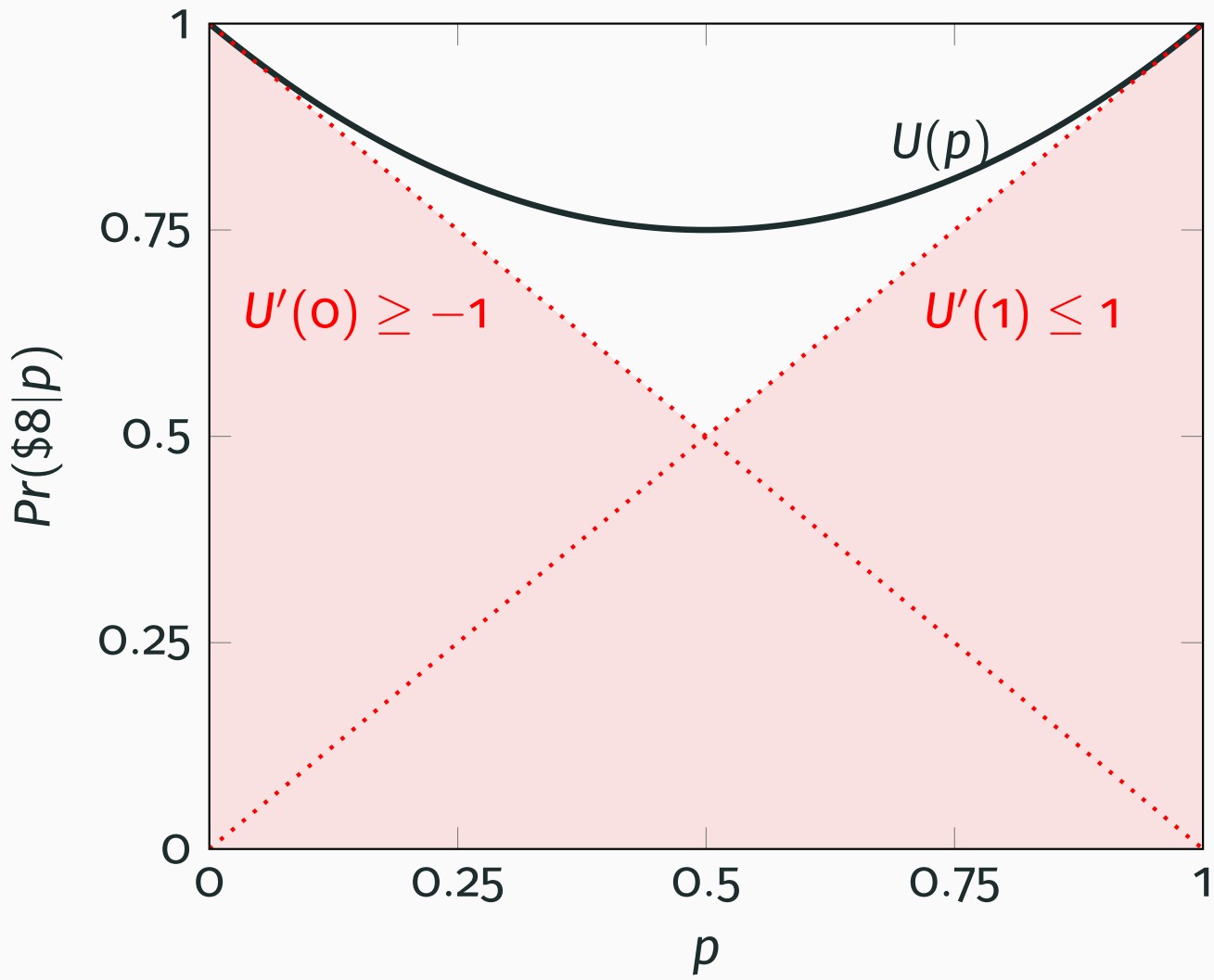
An extra constraint due to probability payments (1 fixed prize):

From Savage (1971):

$$U'(p) = \underbrace{Pr(\$8|X=1)}_{\in[0,1]} - \underbrace{Pr(\$8|X=0)}_{\in[0,1]}$$

Thus, $U'(p) \in [-1, 1]$ for all p

$U''(p)$



Fixed prize $\Rightarrow |U'(p)| \leq 1$

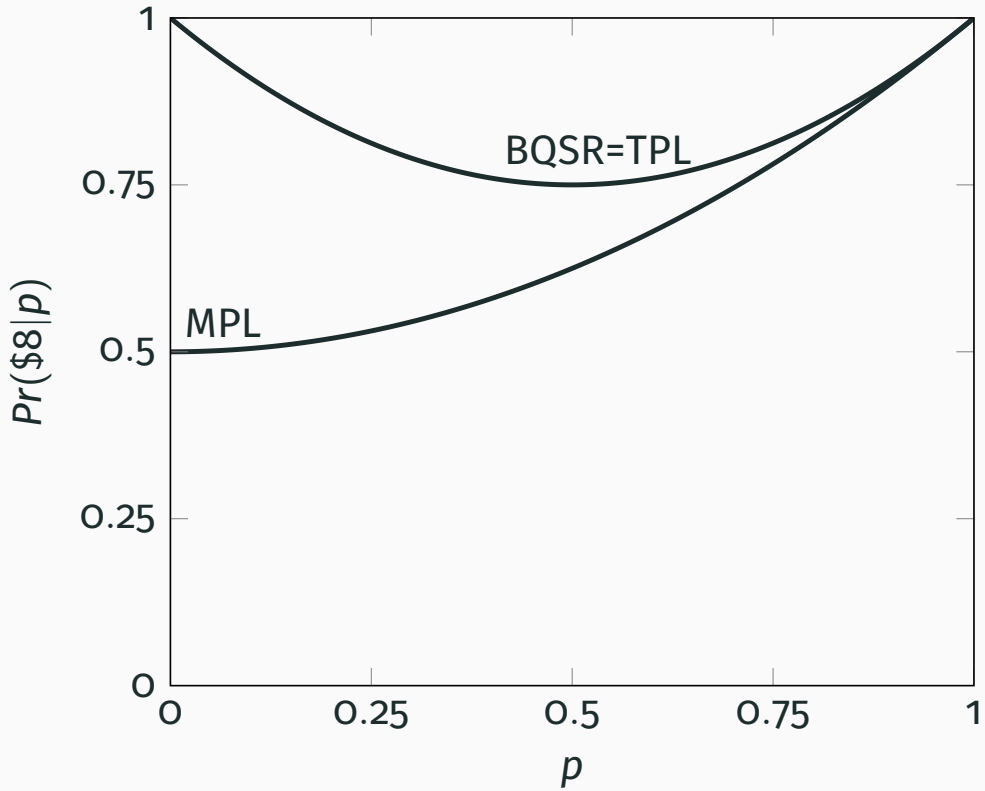
This bounds "average" convexity: $\int_0^1 U''(p)dp = U'(1) - U'(0) \leq 2$

$U''(p)$ for Price Lists

Does this math hold for MPL/TPL?

- Problem 1: finite # rows
  - Here: assume uncountable rows
  - Now can announce any $q \in [0, 1]$
  - Paper: limit as # rows $\to \infty$
- Problem 2: depends on Pr(each row chosen) & then reducing
  - Here: assume uniform
  - Paper: more general

Comparing the Mechanisms



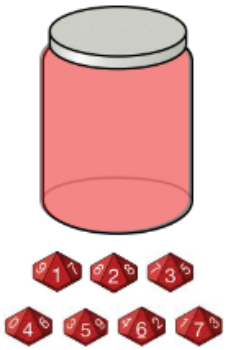
Mechanism	Truth-Telling Incentive Measure	Strength of Incentives	Uniform Incentives?
BQSR	$U''(p) = 2 \ \forall p$	100%	Yes
MPL*	$U''(p) = 1 \ \forall p$	50%	Yes
TPL*	$U''(p) = 2 \ \forall p$	100%	Yes

*Assuming each row is equally likely to be paid, and then calculating overall $Pr(\$8)$.

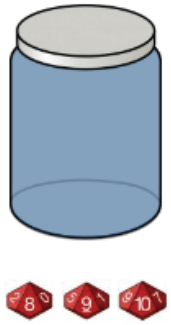
Performance on Prolific

Experiment Interface: BQSR

The computer will roll a 10-sided die to choose one of these two jars. The Red Jar is chosen if the die comes up 1 through 7.



7 in 10



3 in 10

If I think the probability of the Red Jar is [ ] %
then my chances of getting $3 would be:

If Red Jar: you get $3 with probability __%

If Blue Jar: you get $3 with probability __%

If the true probability is ___% then your payment probabilites for each possible report are:

If You Report	Overall Probability
0%	You get $3 with probability __%
1%	You get $3 with probability __%
2%	You get $3 with probability __%
3%	You get $3 with probability __%
4%	You get $3 with probability __%
5%	You get $3 with probability __%
6%	You get $3 with probability __%
7%	You get $3 with probability __%
8%	You get $3 with probability __%
9%	You get $3 with probability __%

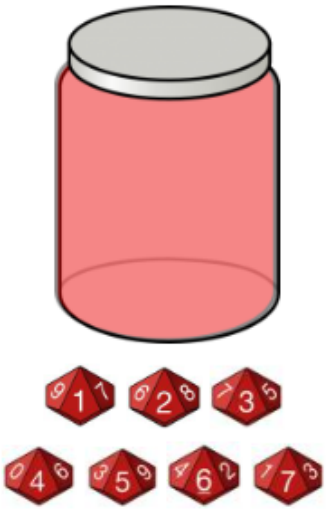
Show Calculations

Remember: you maximize your overall probability of getting $3 when you report truthfully.

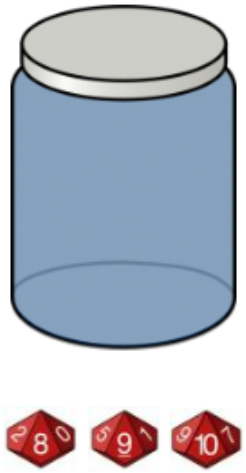
What do you think is the probability (from 0% to 100%) that the RED JAR was chosen?

[ ] %

Experiment Interface: MPL



7 in 10



3 in 10

If I think the probability of the Red Jar is [ ] %
then my choices would be:

Pick:	Option A	OR	Option B
Row 0:	○ $3 if the Red Jar is chosen	OR	○ $3 with probability 0%
Row 1:	○ $3 if the Red Jar is chosen	OR	○ $3 with probability 1%
Row 2:	○ $3 if the Red Jar is chosen	OR	○ $3 with probability 2%
Row 3:	○ $3 if the Red Jar is chosen	OR	○ $3 with probability 3%
Row 4:	○ $3 if the Red Jar is chosen	OR	○ $3 with probability 4%
Row 5:	○ $3 if the Red Jar is chosen	OR	○ $3 with probability 5%
Row 6:	○ $3 if the Red Jar is chosen	OR	○ $3 with probability 6%
Row 7:	○ $3 if the Red Jar is chosen	OR	○ $3 with probability 7%

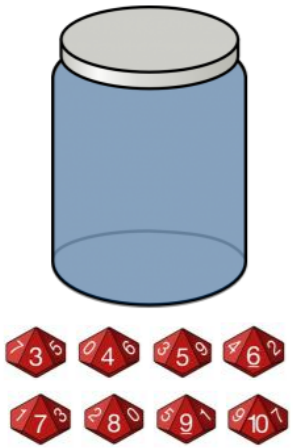
Remember: you maximize your overall probability of getting $3 when you report truthfully.

What do you think is the probability (from 0% to 100%) that the RED JAR was chosen?

Experiment Interface: TPL

The computer will roll a 10-sided die to choose one of these two jars. The Red Jar is chosen if the die comes up 1 or 2.

2 in 10



8 in 10

If I think the probability of "the RED jar was chosen" is [ ] **%**
then my probability of "the BLUE jar was chosen" is [ ] **%**
and so my choices would be:

Pick:	Option A	OR	Option B	OR	Option C
Row 50:	○ $3 if the RED jar was chosen	OR	○ $3 if the BLUE jar was chosen	OR	○ $3 with probability **50%**
Row 51:	○ $3 if the RED jar was chosen	OR	○ $3 if the BLUE jar was chosen	OR	○ $3 with probability **51%**
Row 52:	○ $3 if the RED jar was chosen	OR	○ $3 if the BLUE jar was chosen	OR	○ $3 with probability **52%**
Row 53:	○ $3 if the RED jar was chosen	OR	○ $3 if the BLUE jar was chosen	OR	○ $3 with probability **53%**
Row 54:	○ $3 if the RED jar was chosen	OR	○ $3 if the BLUE jar was chosen	OR	○ $3 with probability **54%**
Row 55:	○ $3 if the RED jar was chosen	OR	○ $3 if the BLUE jar was chosen	OR	○ $3 with probability **55%**
Row 56:	○ $3 if the RED jar was chosen	OR	○ $3 if the BLUE jar was chosen	OR	○ $3 with probability **56%**

Remember: you maximize your overall probability of getting $3 when you report truthfully.

What do you think is the probability (from 0% to 100%) that the RED JAR was chosen?

[ ] %

Experiment Interface: Two-Stage TPL

The computer will roll a 10-sided die to choose one of these two jars. The Red Jar is chosen if the die comes up 1, 2, or 3.

Which do you think is more likely? (choose one)

If I think the probability of (select one above) is ____ **%**
then my choices would be:

Pick:	Option A	OR	Option B
Row 50:	$3 if (select one above)	OR	$3 with probability **50%**
Row 51:	$3 if (select one above)	OR	$3 with probability **51%**
Row 52:	$3 if (select one above)	OR	$3 with probability **52%**
Row 53:	$3 if (select one above)	OR	$3 with probability **53%**
Row 54:	$3 if (select one above)	OR	$3 with probability **54%**
Row 55:	$3 if (select one above)	OR	$3 with probability **55%**
Row 56:	$3 if (select one above)	OR	$3 with probability **56%**

Remember: you maximize your overall probability of getting $3 when you report truthfully.

Which do you think is more likely?

(select one)

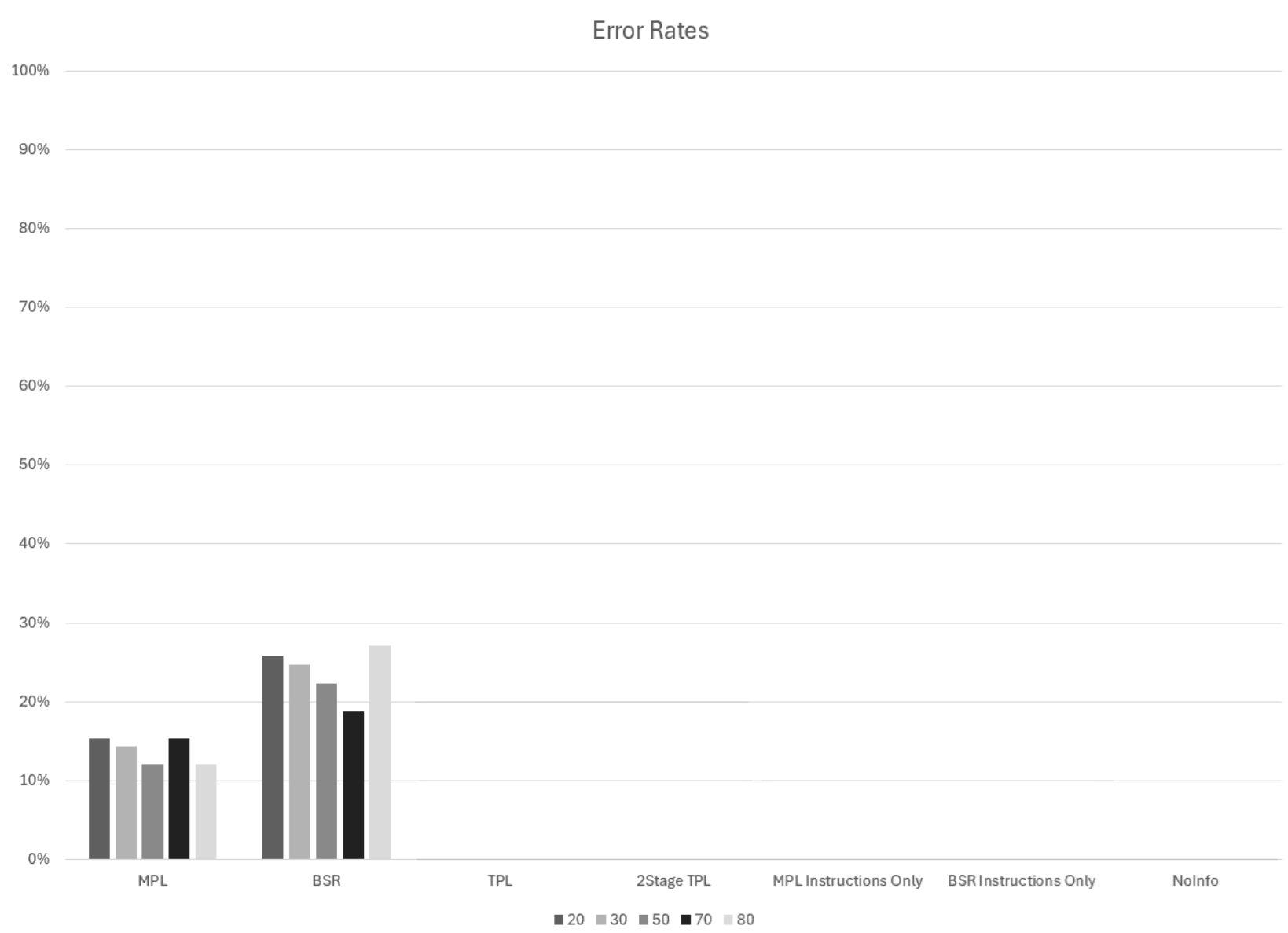
What do you think is the probability (from 50% to 100%) that (select one above) was chosen?

____ %

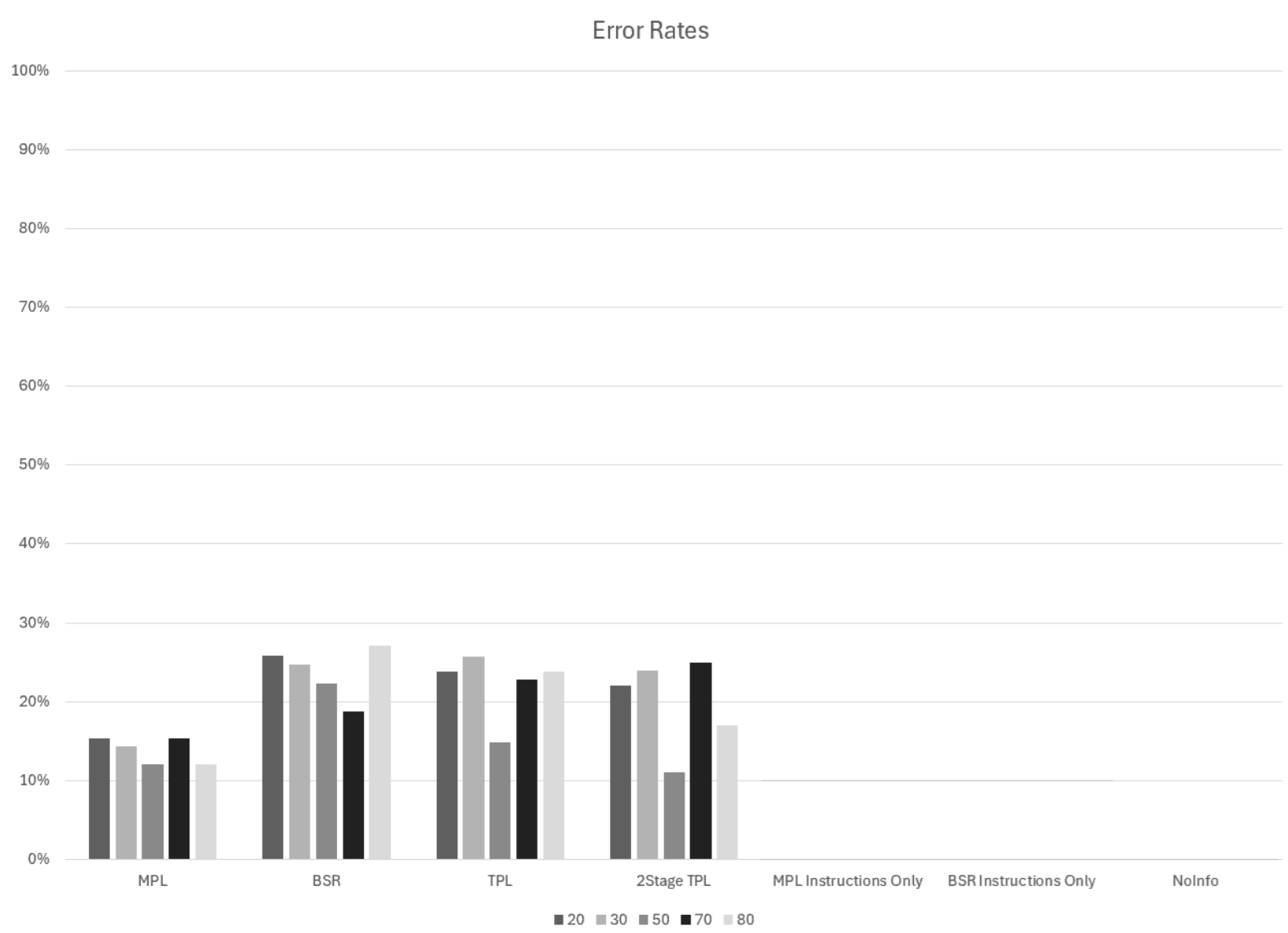
Details

- Prolific
- $n \approx 100$ per treatment
- Based on Danz et al. (2022)
- Most data from Healy & Kagel
- New treatments: TPL & 2-Stage TPL

Results


Error Rates
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
MPL
BSR
TPL
2Stage TPL
MPL Instructions Only
BSR Instructions Only
NoInfo
20
30
50
70
80

Results


Error Rates
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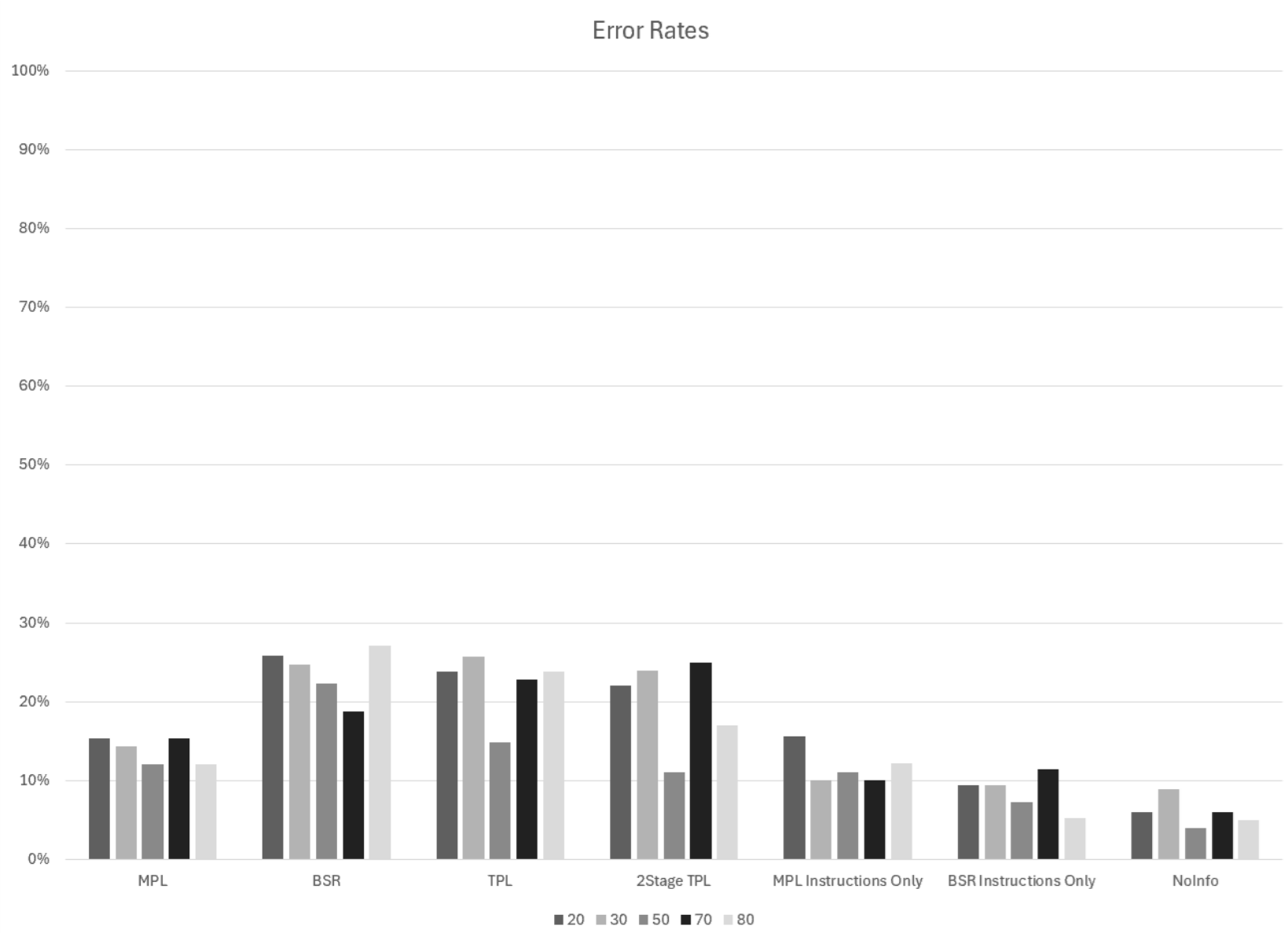
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
MPL
BSR
TPL
2Stage TPL
MPL Instructions Only
BSR Instructions Only
NoInfo
20
30
50
70
80

Results


Error Rates
100%
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80%
70%
60%
50%
40%
30%
20%
10%
0%
MPL
BSR
TPL
2Stage TPL
MPL Instructions Only
BSR Instructions Only
NoInfo
20
30
50
70
80

Our Recommendations

- Lit review (not here) $\Rightarrow$ Incentives do matter
  - Avoids “default” answers like 50%
- But: The exact mechanism doesn’t matter much
- Our preference: MPL, shown in instructions only
  - Can easily explain the dominance nature of IC

The End
(for real)