Epistemic Conditions for the Failure of Nash Equilibrium

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SITE

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Motivation

Question:
Do people play Nash equilibrium? If not, why not?

- Utilities? (NE with Fehr-Schmidt preferences)
- Beliefs? (Level-K)
- Rationality? (QRE)

Rationality is *not* primitive.
'Imposed by the solution concept?'... not so obvious.
We need theory framework in which to discuss these things!

Aumann & Brandenburger (1995), bastardized

The lab environment:

- Outcome space: X ($$ payoffs to each player)
- Game form: (N, S, g) $(g : S \to X)$

Example: 1-Shot P.D. in the Lab

	L	R
U	\$2,\$2	\$0,\$3
D	\$3,\$0	\$1,\$1

A Game Form

	L	R
U	$2,2$	$0,3$
D	$3,0$	$1,1$

A Game

	L	R
U	$4,4$	$0,0$
D	$0,0$	$2,2$

A Game

	L	R
U	$0,0$	$0,0$
D	$0,0$	$0,0$

A Game

Actual game is determined by players' *types*.

Adding Epistemology

- Outcome space: X
- Game form: (N, S, g) $(g : S \to X)$
- Type space: Θ_i for each $i \in N$
- Player's type θ_i determines:
  - Payoff function: $\boldsymbol{u_i}(x; \theta_i)$
  - Beliefs about θ_{-i} : $\boldsymbol{p_i}(\theta_i)(\theta_{-i})$
  - Pure strategy choice: $\boldsymbol{s_i}(\theta_i) \in S_i \leftarrow$ **Cool!**

Given this, we can define at each θ_i:
i's 'conjecture' about s_{-i}:

$$\boldsymbol{\phi_i}(\theta_i)(s_{-i}) = \boldsymbol{p_i}(\theta_i)(\{\theta_{-i} : \boldsymbol{s_{-i}}(\theta_{-i}) = s_{-i}\})$$

i's (subjective) expected utility:

$$\boldsymbol{Eu_i}(s_i; \theta_i) = \sum_{s_{-i}} [\boldsymbol{\phi_i}(\theta_i)(s_{-i})] \, \boldsymbol{u_i}(g(s_i, s_{-i}); \theta_i)$$

Rationality & Equilibrium

A player is **rational at** θ_i if:

$$\boldsymbol{s_i}(\theta_i) \in \arg\max_{s_i} \boldsymbol{Eu_i}(s_i; \theta_i).$$

Standard definitions of **known**, **mutually known**, and **common knowledge**.

Interpretations

- Everyone comes to the lab with a θ_i
- Preferences over outcomes (inequality aversion, selfishness, etc.) captured in $\boldsymbol{u_i}(x; \theta_i)$
- Nobody mixes: I'm uncertain about your action only because I'm uncertain about your type.
- Thus, mixed-strategy equilibrium only exists in conjectures. 'Equilibrium' is a property of beliefs, not actions!

AB95's Theorem: 2 Players

Theorem

Suppose $n = 2$. *If*

1. $\boldsymbol{u}(\theta)$ *is mutually known,*
2. $\boldsymbol{\phi}(\theta)$ *is mutually known, and*
3. *rationality is mutually known.*

Then $(\phi_2(\theta_2), \phi_1(\theta_1))$ *is a MSNE of* $(N, S, u \circ g)$.

This Paper

- Subjects play five 2×2 one-shot games. Strangers, no feedback.
- For each game, elicit:
  1. Chosen action $(\boldsymbol{s_i}(\theta_i))$
     - ★ Play game
  2. Preferences over outcomes $(\boldsymbol{u_i}(\cdot;\theta_i))$
     - ★ 'Probability equivalent' of each cell. $u(x) \in [0,100]$
  3. i's beliefs about $\boldsymbol{u_j}$
     - ★ Point estimate, paid on abs. deviation
  4. Conjectures about s_j $(\boldsymbol{\phi_i}(\theta_i))$
     - ★ Grether/Karni mechanism (probability BDM)
  5. i's beliefs about $\boldsymbol{\phi_j}$.
     - ★ Point estimate, paid on abs. deviation
  6. i's beliefs about j's rationality
     - ★ Grether/Karni mechanism

If (ϕ_2, ϕ_1) is not NE, then ≥ 1 of these 3 conditions fails. WHICH??

Example Observation

	L	R
U	$\$10, 5$	$\$15, 15$
D	$\$5, 10$	$\$1, 1$

Game Form

	0%	100%
>0%	50, 50	90, 90
100%	10, 10	1, 1

Obs 180: Row's Game

	$\vee 55\%$	45%
85%	40, 10	50, 70
15%	10, 20	5, 5

Obs 180: Column's Game

	0%	100%
>0%	3, 3	4, 4
100%	2, 2	1, 1

Row's Ordinal Game

	$\vee 55\%$	45%
85%	3, 2	4, 4
15%	2, 3	1, 1

Column's Ordinal Game

Data

- 2010 Data:
  - 78 subjects
  - Very negative results. Confusing interface? (Note: blame RA)
- 2011 Data:
  - More intuitive interface & instructions
  - 72 subjects
- 2013 Data:
  - Simple pencil & paper
  - Ordinal preferences, guess s_j, no rationality. Can't test AB95...
  - 26 subjects so far
- $\sim$60 min, \$5–\$20 payout

Game Form 1: Dominance Solvable

Game Form:

	L	R
U	$\$10, 5$	$\$15, 15$
D	$\$5, 10$	$\$1, 1$

Dom. Solvable \$NE: (U,R)

Game Form 2: Symmetric Coordination

Game Form:

	L	R
U	$\$15, 15$	$\$1, 1$
D	$\$1, 1$	$\$5, 5$

Three \$NE: (U,L) $\geq$ (D,R) $\geq$ ((2/9,7/9),(2/9,7/9))

Game Form 3: Prisoners' Dilemma

Game Form:

	L	R
U	$\$10, 10$	$\$1, 15$
D	$\$15, 1$	$\$5, 5$

Dominant Strategy Equil ($): (D,R)

Game Form 4: Asymmetric Matching Pennies

Game Form:

	L	R
U	$\$15, 5$	$\$5, 10$
D	$\$5, 10$	$\$10, 5$

Unique Mixed-Strategy $NE: ((1/2,1/2),(1/3,2/3))

Game Form 5: Asymmetric Coordination

Game Form:

	L	R
U	$\$15, 5$	$\$1, 1$
D	$\$1, 1$	$\$5, 10$

Three \$NE: (U,L) $\geq$ (D,R) $\geq$ ((9/13,4/13),(2/9,7/9))

Hand-Waving Summary of 2011 Data

	Game Form			
G1:DomSolv	$^{\$}10,^{\$}5	$^{\$}15,^{\$}15	Util: Decent **Blfs: Bad**	RowRtnl: ✓ ColRtnl: OK
	$^{\$}5,^{\$}10	$^{\$}1,^{\$}1		
G2:SymCoord	$^{\$}15,^{\$}15	$^{\$}1,^{\$}1	Util: ✓ Blfs: ✓	RowRtnl: ✓ ColRtnl: ✓
	$^{\$}1,^{\$}1	$^{\$}5,^{\$}5		
G3:PD	$^{\$}10,^{\$}10	$^{\$}1,^{\$}15	**Util: V.Bad** Blfs: Bad	RowRtnl: OK ColRtnl: OK
	$^{\$}15,^{\$}1	$^{\$}5,^{\$}5		
G4:AsymMP	$^{\$}15,^{\$}5	$^{\$}5,^{\$}10	Util: Weak Blfs: ✓	**RowRtnl: Bad** **ColRtnl: Weak**
	$^{\$}5,^{\$}10	$^{\$}10,^{\$}5		
G5:AsymCoord	$^{\$}15,^{\$}5	$^{\$}1,^{\$}1	Util: OK **Blfs: Bad**	RowRtnl: Bad ColRtnl: Bad
	$^{\$}1,^{\$}1	$^{\$}5,^{\$}10		

Playing the Same Ordinal Game?

Game Form			Row=Col	=GameForm
G1:DomSolv	$^{\$}10,^{\$}5$	$^{\$}15,^{\$}15$	69.4%	100%
	$^{\$}5,^{\$}10$	$^{\$}1,^{\$}1$		
G2:SymCoord	$^{\$}15,^{\$}15$	$^{\$}1,^{\$}1$	88.9%	100%
	$^{\$}1,^{\$}1$	$^{\$}5,^{\$}5$		
G3:PD	$^{\$}10,^{\$}10$	$^{\$}1,^{\$}15$	36.1%	92.3%
	$^{\$}15,^{\$}1$	$^{\$}5,^{\$}5$		
G4:AsymMP	$^{\$}15,^{\$}5$	$^{\$}5,^{\$}10$	52.8%	100%
	$^{\$}5,^{\$}10$	$^{\$}10,^{\$}5$		
G5:AsymCoord	$^{\$}15,^{\$}5$	$^{\$}1,^{\$}1$	75.0%	100%
	$^{\$}1,^{\$}1$	$^{\$}5,^{\$}10$		
		Overall:	64.4%	99.1%
H_O: Random Response:			6.25%	6.25%

Testing AB95 w/ 2011 Data

- u mutual knowledge: If same ordinal game
- ϕ mutual knowledge: If max $\pm 10\%$ error
- Ratn'l mutual knowledge: If true, $>= 75\%$ prob

- **10**/180 observations satisfy these 3 conditions.
- 9: Game 2 (SymCoord). 1: Game 1 (DomSolv)

The 2013 Data

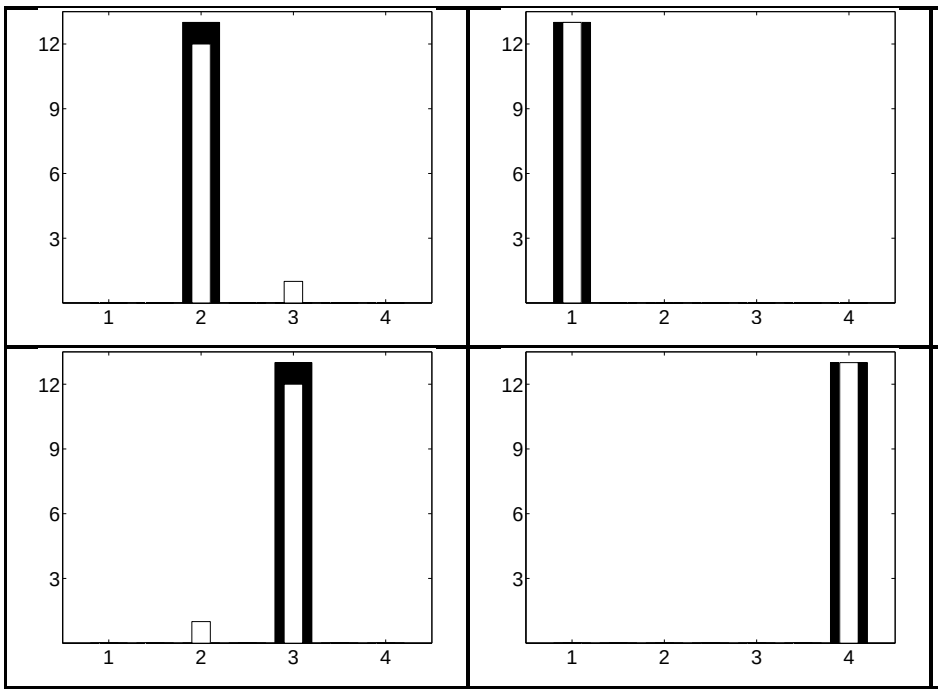
- Simple enough that I trust the data
- Not rich enough to test at the individual (pair) level

2013 Data: Preference Ranking Histograms

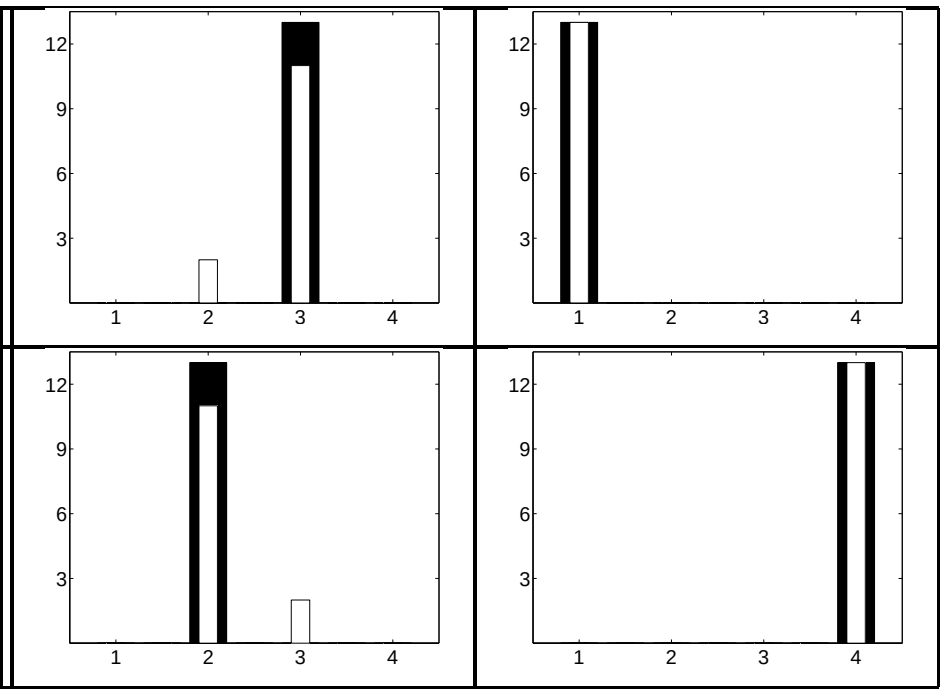
GAME 1: DOMINANCE SOLVABLE

	L	R
U	$\$10, 5$	$\$15, 15$
D	$\$5, 10$	$\$1, 1$

Row Prefs



Col Prefs



Play, Conjectures & Beliefs)

GAME 1: DOMINANCE SOLVABLE

			Play	15%	85%
			1stBlf	8%	92%
Play	1stBlf	2ndBlf	15%	85%	
100%	92%	100%	\$10,\$5	\$15,\$15	
0%	8%	0%	\$5,\$10	\$1,\$1	

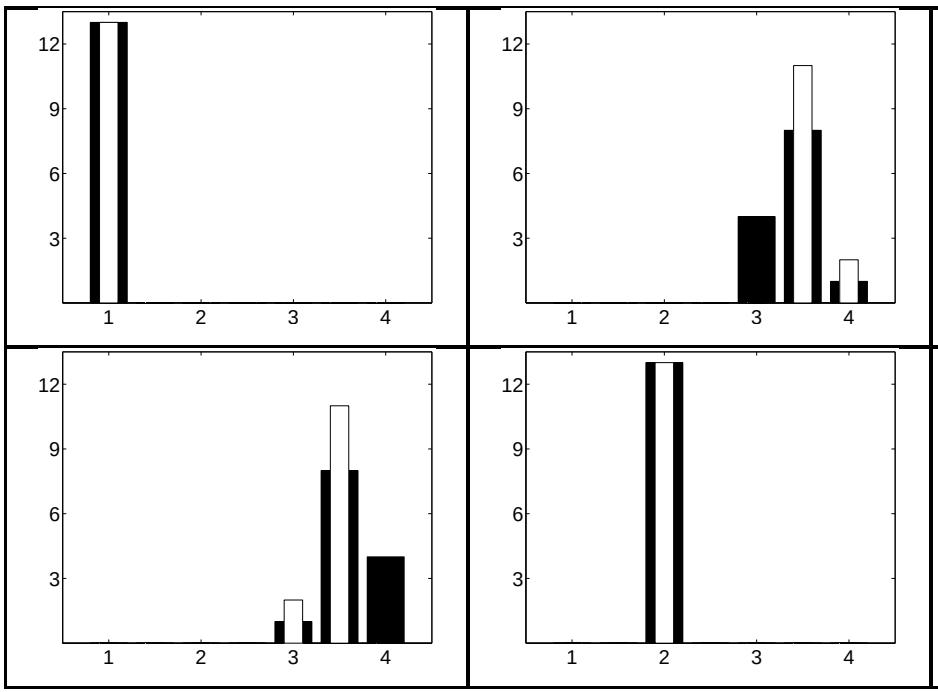
Utility ✓
Beliefs OK
Rationality *seems* to fail, but maybe not at indiv. level

2013 Data: Preference Ranking Histograms

GAME 2: SYMMETRIC COORDINATION

	L	R
U	$15, 15	$1, 1
D	$1, 1	$5, 5

Row Prefs



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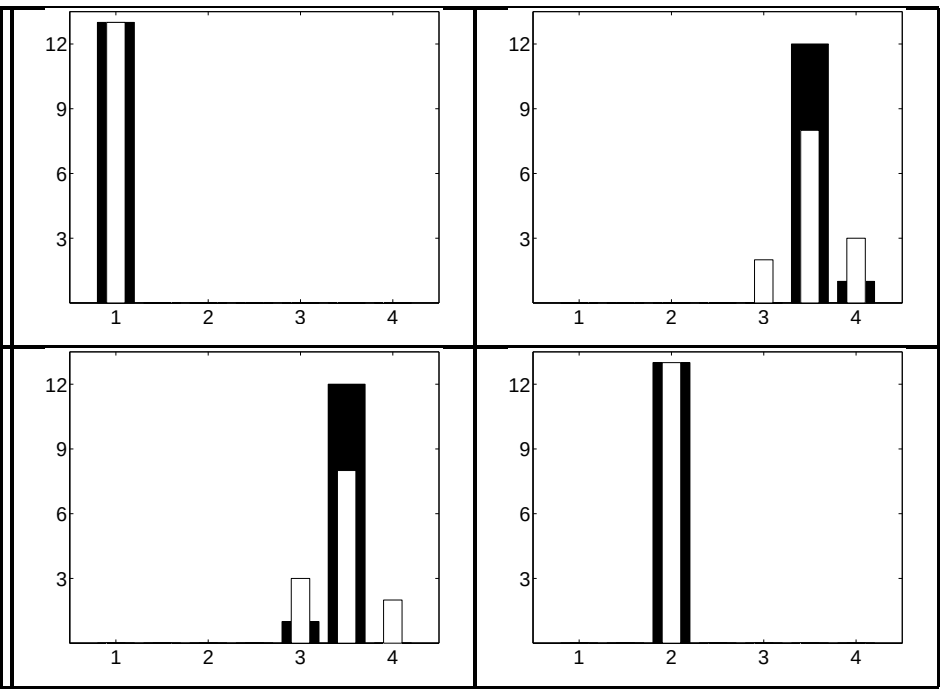


Play, Conjectures & Beliefs)

GAME 2: SYMMETRIC COORDINATION

		Play	92%	8%
		1stBlf	100%	0%
Play	1stBlf	2ndBlf	100%	0%
100%	100%	100%	\$15,\$15	\$1,\$1
0%	0%	0%	\$1,\$1	\$5,\$5

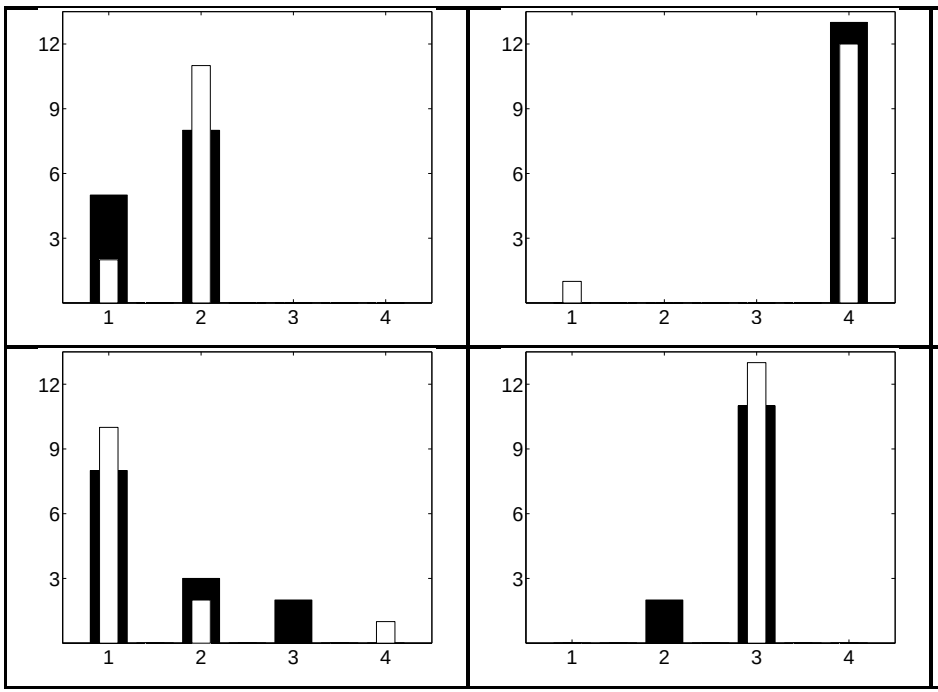
Utility ✓
Beliefs ✓
A game theory home run!!

2013 Data: Preference Ranking Histograms

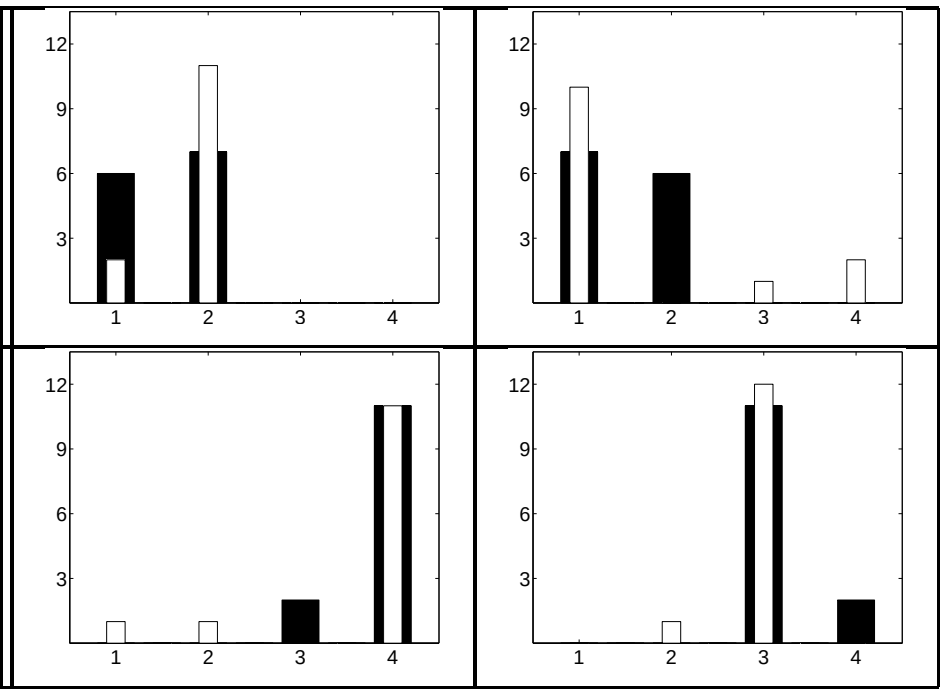
GAME 3: PRISONERS' DILEMMA

	L	R
U	$\$10, 10$	$\$1, 15$
D	$\$15, 1$	$\$5, 5$

Row Prefs



Col Prefs



Play, Conjectures & Beliefs)

GAME 3: PRISONERS' DILEMMA

		Play	54%	46%
		1stBlf	31%	69%
Play	1stBlf	2ndBlf	46%	54%
54%	31%	38%	\$10,\$10	\$1,\$15
46%	69%	62%	\$15,\$1	\$5,\$5

Utility **X**
Beliefs **X**
Not complete-info game.

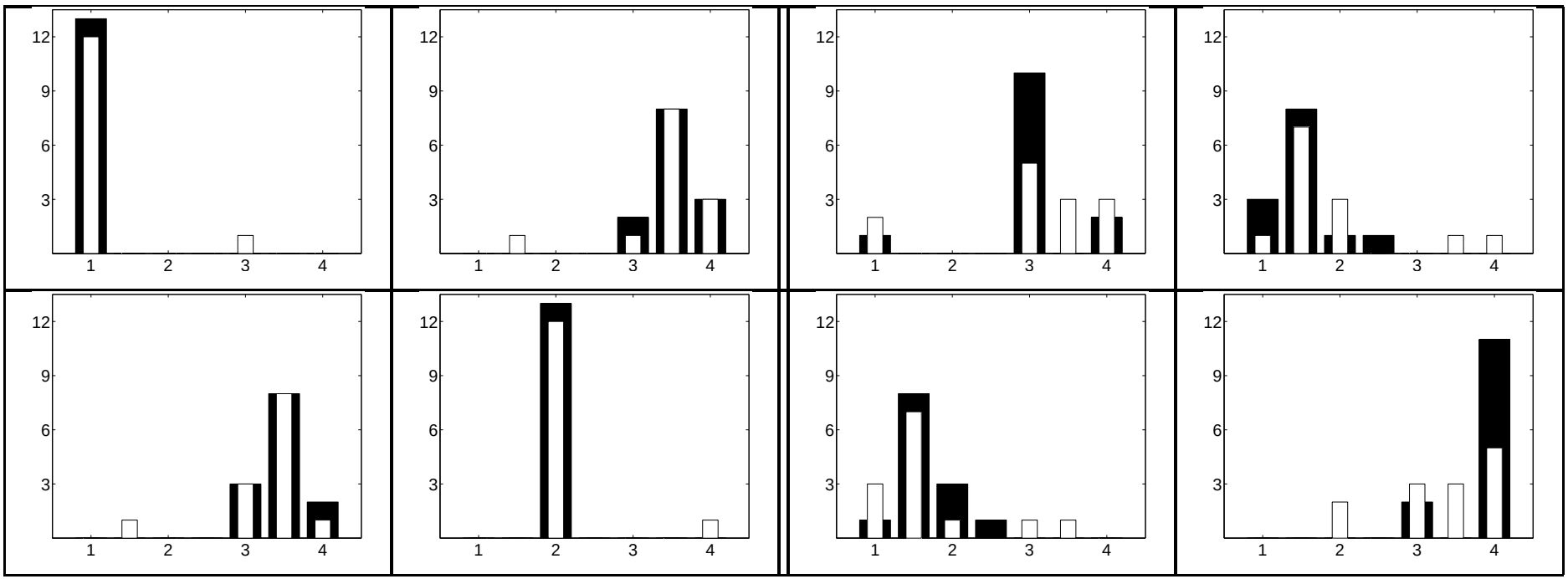
2013 Data: Preference Ranking Histograms

GAME 4: ASYMMETRIC MATCHING PENNIES

	L	R
U	$\$15, 5$	$\$5, 10$
D	$\$5, 10$	$\$10, 5$

Row Prefs

Col Prefs



Play, Conjectures & Beliefs)

GAME 4: ASYMMETRIC MATCHING PENNIES

		Play	54%	46%
		1stBlf	69%	31%
Play	1stBlf	2ndBlf	62%	38%
85%	92%	100%	\$15,\$5	\$5,\$10
15%	8%	0%	\$5,\$10	\$10,\$5

Utility OK, but a little shaky
Beliefs ✓
Rationality seems to fail for Column

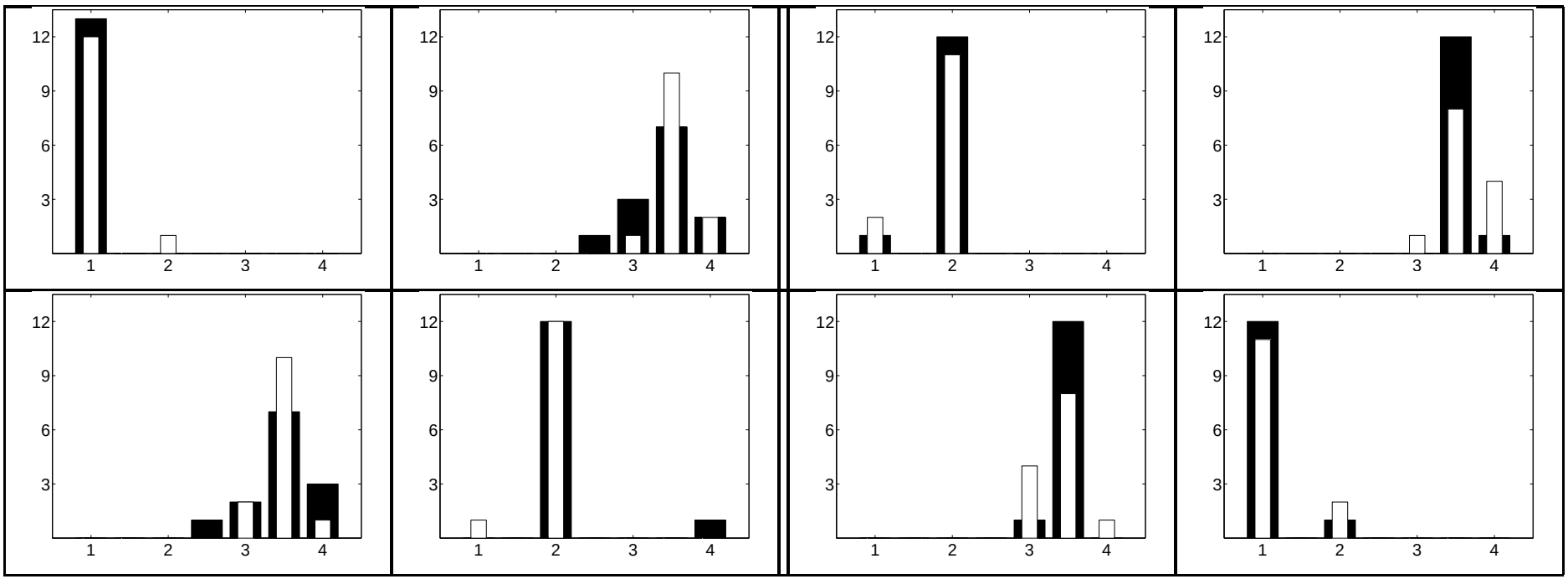
2013 Data: Preference Ranking Histograms

GAME 5: ASYMMETRIC COORDINATION

	L	R
U	$\$15, 5$	$\$1, 1$
D	$\$1, 1$	$\$5, 10$

Row Prefs

Col Prefs



Play, Conjectures & Beliefs)

GAME 5: ASYMMETRIC COORDINATION

		Play	54%	46%
		1stBlf	31%	69%
Play	1stBlf	2ndBlf	23%	77%
92%	100%	92%	\$15,\$5	\$1,\$1
8%	0%	8%	\$1,\$1	\$5,\$10

Utility ✓
Beliefs OK
Rationality seems to fail for Column

Summary

- Sometimes not even playing same game!
  NE not defined
- Subjects are pretty bad at 2nd order beliefs.
- Beliefs about rationality are reasonably good.

- When are utilities mutual knowledge??

- Respect for Bayesian games... but beliefs?

- WARNING: Confound with reliability of elicitation procedure.
  See: Old data vs. New data

The End.

Your payoffs are in red. The other player's payoffs are in blue.

Game 5	L	R
U	\$15, \$5	\$1, \$1
D	\$1, \$1	\$5, \$10

You are the row player. Choose an action (row) for game 5 ○ U ○ D

Game 1	L	R
U	$10, $5	$15, $15
D	$5, $10	$1, $1

Your action in Game 1: U

Game 2	L	R
U	$15,$15	$1,$1
D	$1,$1	$5,$5

Your action in Game 2: L

Game 3	L	R
U	$10, $10	$1, $15
D	$15, $1	$5, $5

Your action in Game 3: U

Game 4	L	R
U	$15, $5	$5, $10
D	$5, $10	$10, $5

Your action in Game 4: L

Game 5	L	R
U	$15, $5	$1, $1
D	$1, $1	$5, $10

Your action in Game 5: U

What would you like to do now?

- I'm happy with these choices. Proceed with the experiment.
- I'd like to go back and revise some of these choices.

Your payoffs are in red. The other player's payoffs are in blue.

Game 1	L	R
U	$10, $5	$15, $15
D	$5, $10	$1, $1

Question A: Ranking Outcomes

Please answer the following questions as honestly as possible:

(Me:$20, Them:$20) is worth the same to me as a	100	percent chance of getting (Me:$20, Them:$20)
(Me:$5, Them:$10) is worth the same to me as a		percent chance of getting (Me:$20, Them:$20)
(Me:$15, Them:$15) is worth the same to me as a		percent chance of getting (Me:$20, Them:$20)
(Me:$10, Them:$5) is worth the same to me as a		percent chance of getting (Me:$20, Them:$20)
(Me:$1, Them:$1) is worth the same to me as a		percent chance of getting (Me:$20, Them:$20)
(Me:$0, Them:$0) is worth the same to me as a	0	percent chance of getting (Me:$20, Them:$20)

Your payoffs are in red. The other player's payoffs are in blue.

Game 2	L	R
U	$15, $15	$1, $1
D	$1, $1	$5, $5

Question B: Guessing the Other's Ranking

Please answer the following questions as honestly as possible:

I think the other player said (Them:$20, Me:$20) is worth the same to them as a	100	percent chance of getting (Them:$20, Me:$20)
I think the other player said (Them:$15, Me:$15) is worth the same to them as a		percent chance of getting (Them:$20, Me:$20)
I think the other player said (Them:$1, Me:$1) is worth the same to them as a		percent chance of getting (Them:$20, Me:$20)
I think the other player said (Them:$5, Me:$5) is worth the same to them as a		percent chance of getting (Them:$20, Me:$20)
I think the other player said (Them:$0, Me:$0) is worth the same to them as a	0	percent chance of getting (Them:$20, Me:$20)

Your payoffs are in red. The other player's payoffs are in blue.

Game 1	L	R
U	\$10, \$5	\$15, \$15
D	\$5, \$10	\$1, \$1

Question C: Guessing their Choice

Please answer the following question as honestly as possible:

I think there is a [ ] percent chance that the other player chose U in Game 1. (Please enter a number from 0-100.)

Your payoffs are in red. The other player's payoffs are in blue.

Game 1	L	R
U	$10, $5	$15, $15
D	$5, $10	$1, $1

Question D: Guessing their Guess of YOUR Choice

Please answer the following question as honestly as possible:

When the other player was asked about me, I think they said there was a [ ] percent chance that I played L in Game 1. (Please enter a number from 0-100.)

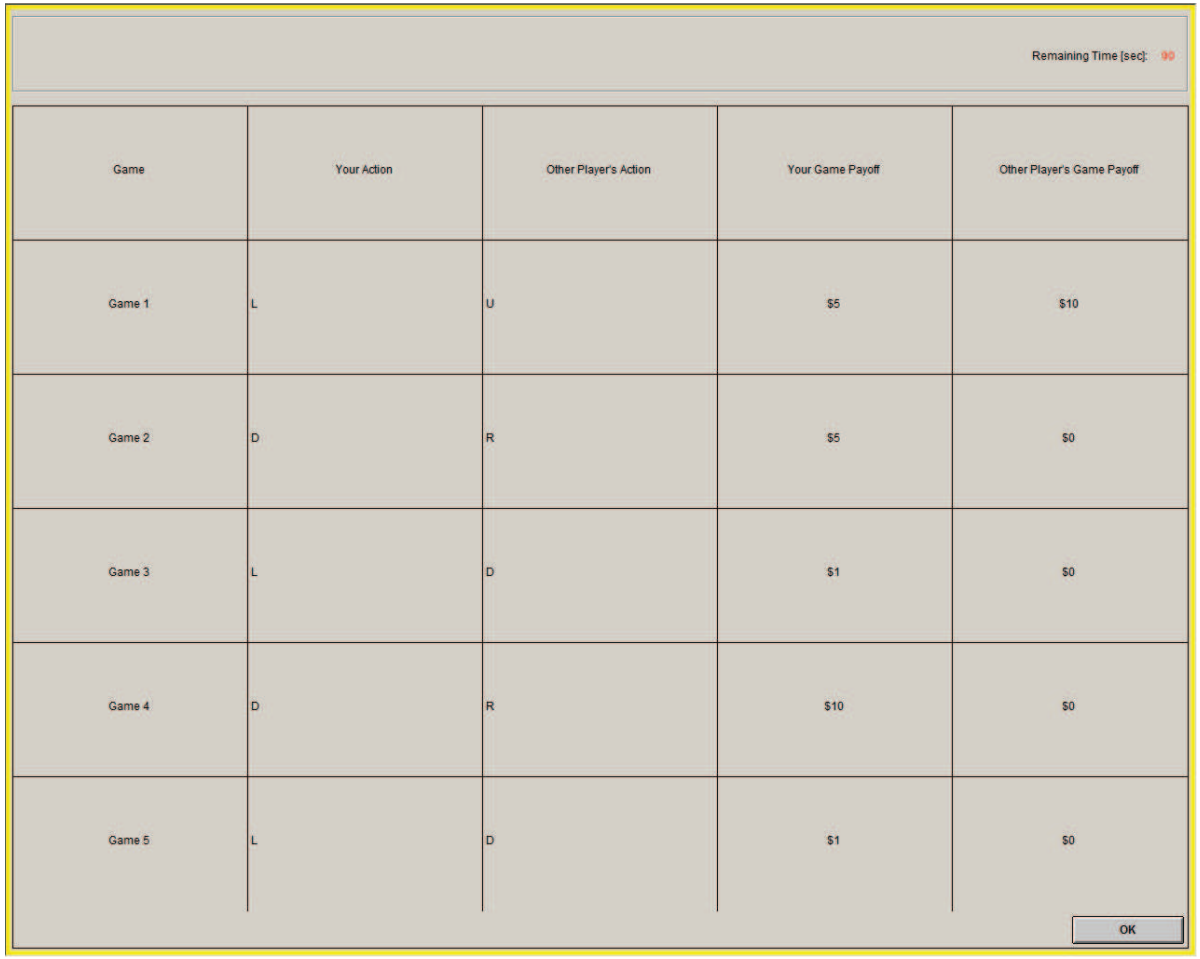
Your payoffs are in red. The other player's payoffs are in blue.

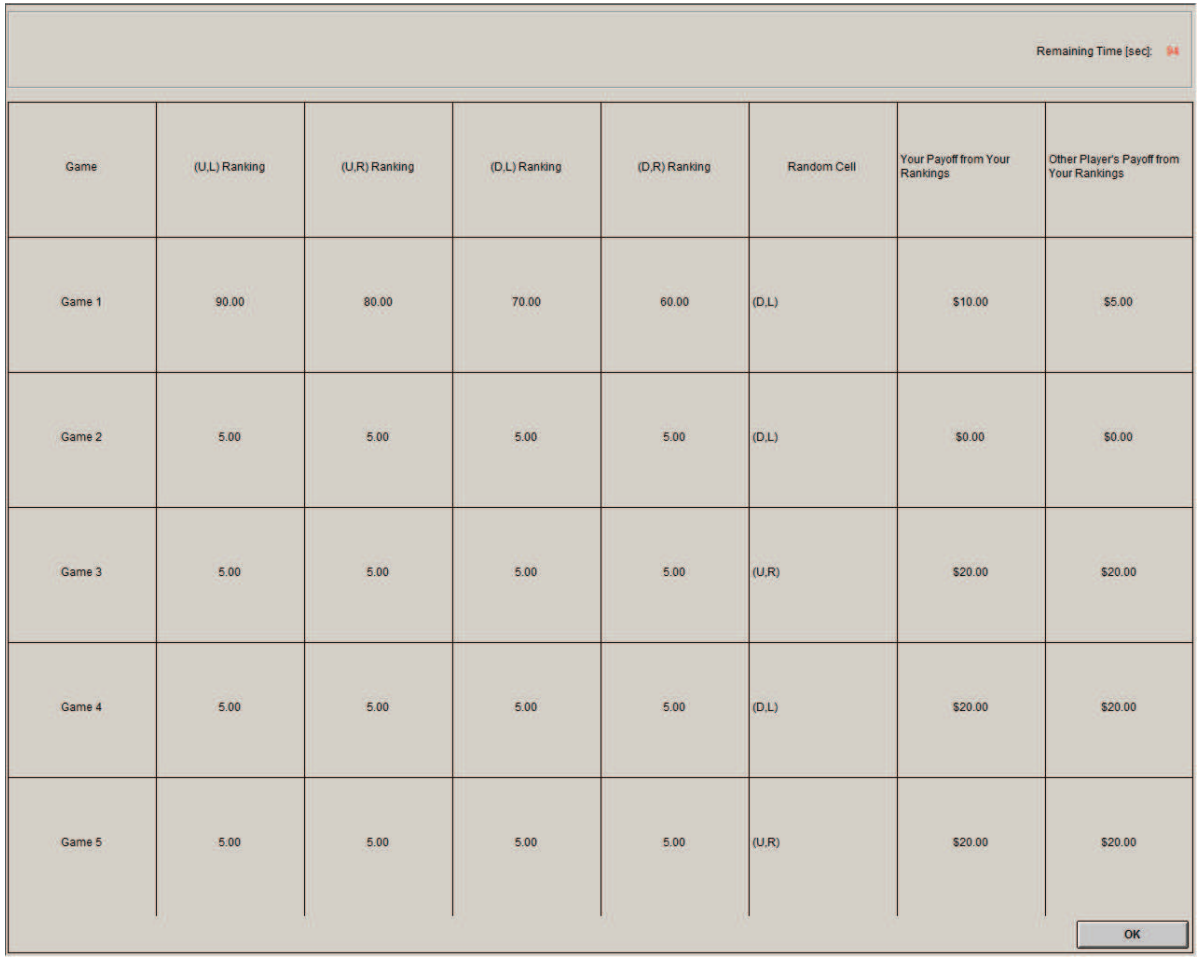
Game 1	L	R
U	\$10, \$5	\$15, \$15
D	\$5, \$10	\$1, \$1

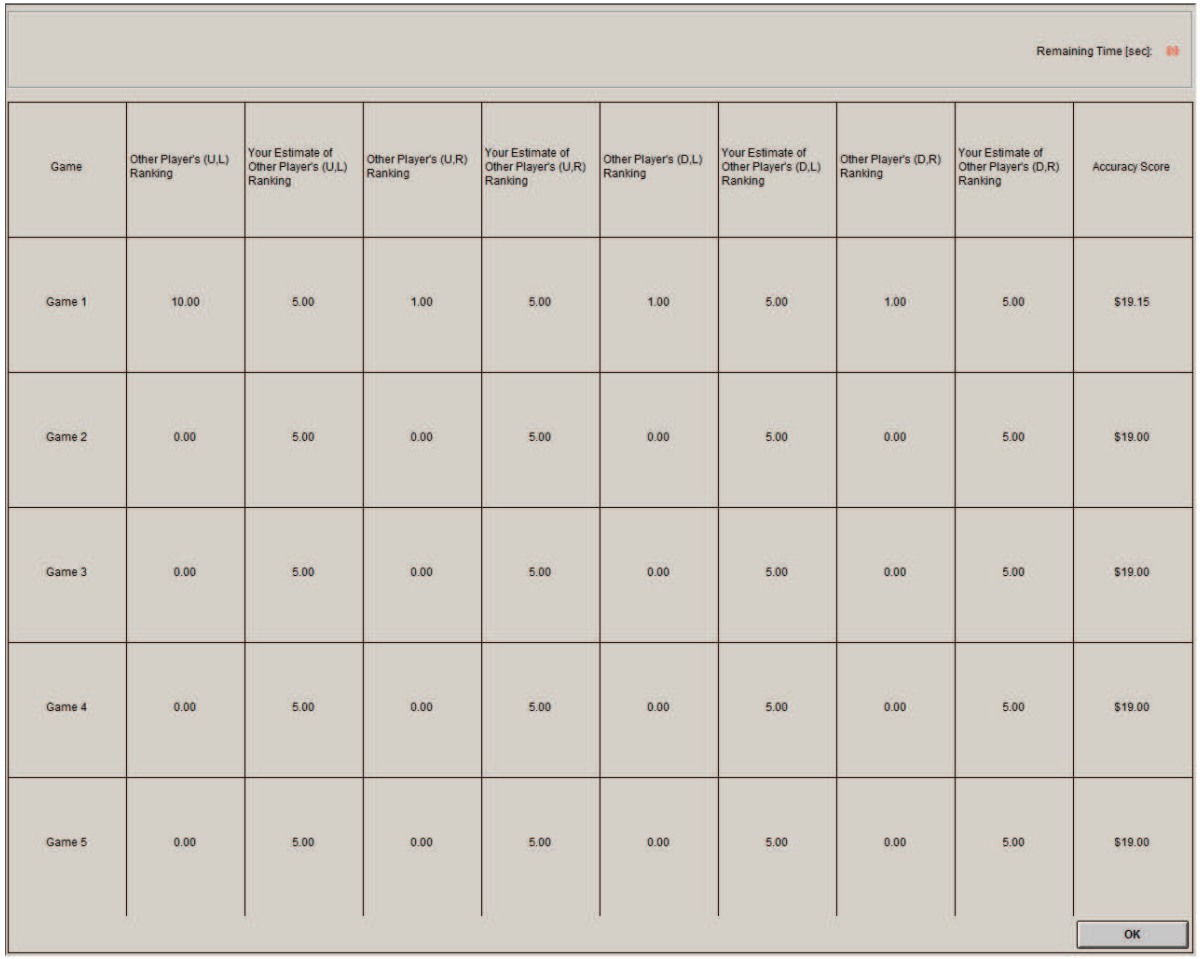
Question E: Guessing if the other person's choice was consistent

Please answer the following question as honestly as possible:

I think there is a [ ] percent chance that the other player's choice in game 1 was consistent with their guesses from Part 2. (Please enter a number from 0-100.)


Remaining Time [sec]: 90
Game
Your Action
Other Player's Action
Your Game Payoff
Other Player's Game Payoff
Game 1
L
U
$5
$10
Game 2
D
R
$5
$0
Game 3
L
D
$1
$0
Game 4
D
R
$10
$0
Game 5
L
D
$1
$0
OK

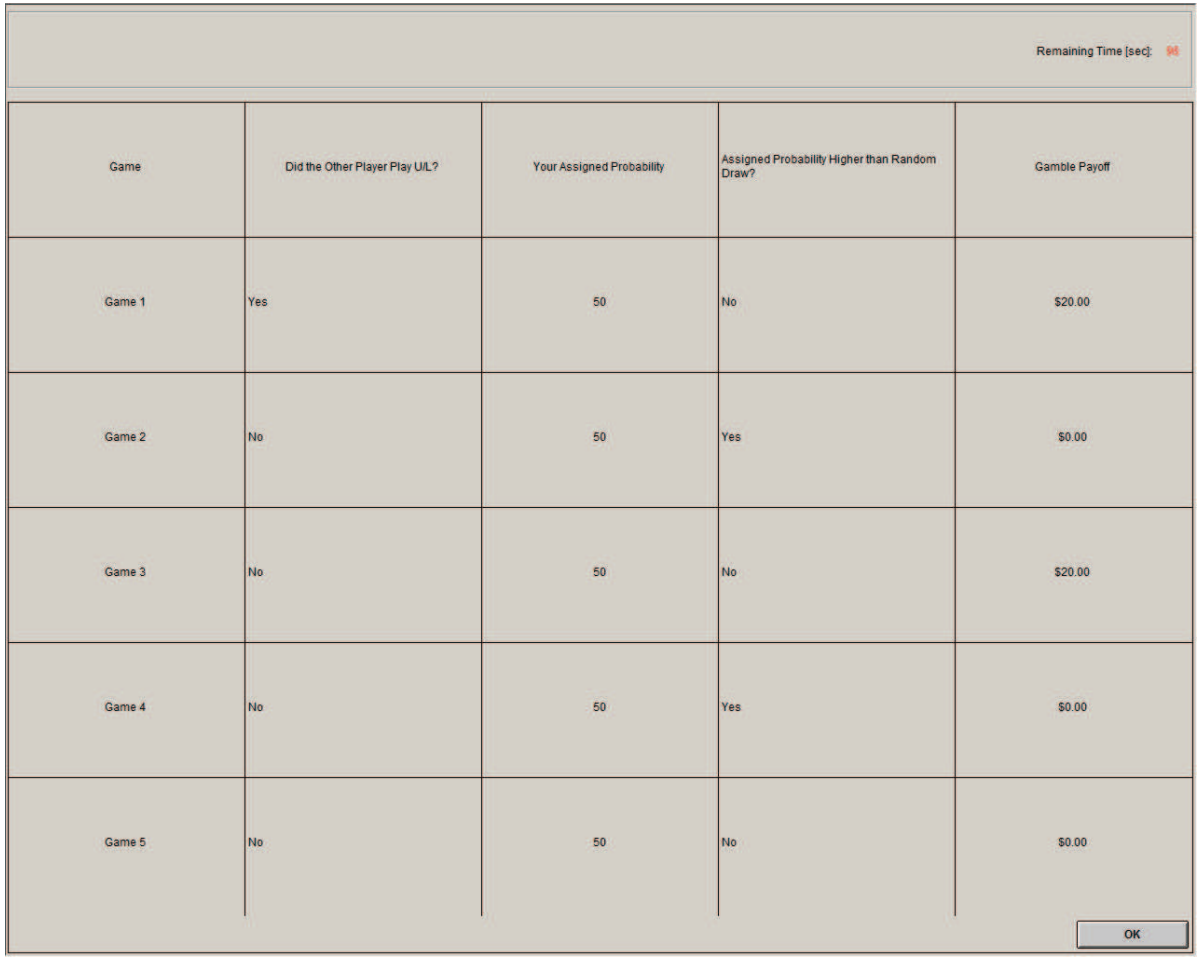

Remaining Time [sec]: 94
Game
(U,L) Ranking
(U,R) Ranking
(D,L) Ranking
(D,R) Ranking
Random Cell
Your Payoff from Your Rankings
Other Player's Payoff from Your Rankings
Game 1
90.00
80.00
70.00
60.00
(D,L)
$10.00
$5.00
Game 2
5.00
5.00
5.00
5.00
(D,L)
$0.00
$0.00
Game 3
5.00
5.00
5.00
5.00
(U,R)
$20.00
$20.00
Game 4
5.00
5.00
5.00
5.00
(D,L)
$20.00
$20.00
Game 5
5.00
5.00
5.00
5.00
(U,R)
$20.00
$20.00
OK

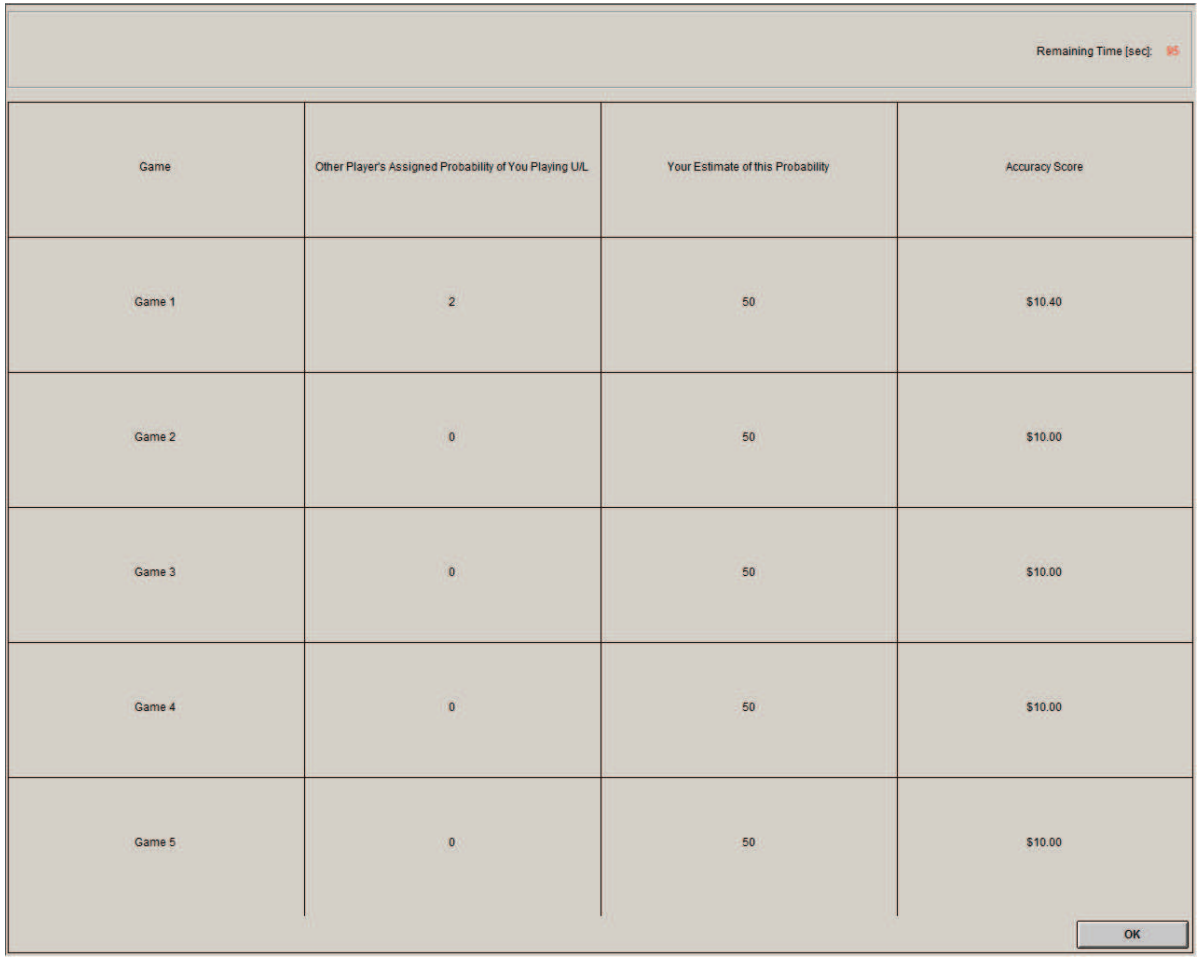


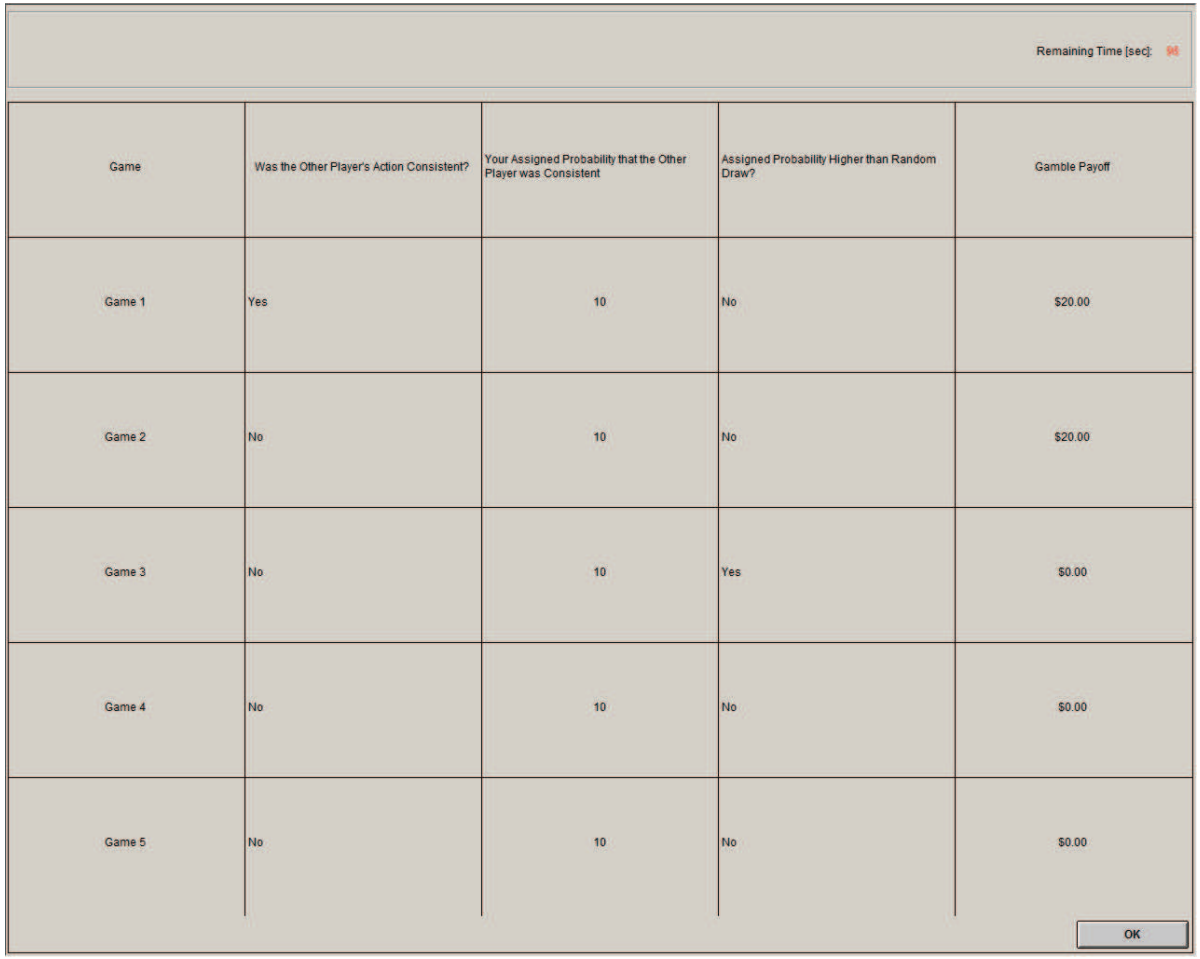
Remaining Time [sec]:

Game	Other Player's (U,L) Ranking	Your Estimate of Other Player's (U,L) Ranking	Other Player's (U,R) Ranking	Your Estimate of Other Player's (U,R) Ranking	Other Player's (D,L) Ranking	Your Estimate of Other Player's (D,L) Ranking	Other Player's (D,R) Ranking	Your Estimate of Other Player's (D,R) Ranking	Accuracy Score
Game 1	10.00	5.00	1.00	5.00	1.00	5.00	1.00	5.00	$19.15
Game 2	0.00	5.00	0.00	5.00	0.00	5.00	0.00	5.00	$19.00
Game 3	0.00	5.00	0.00	5.00	0.00	5.00	0.00	5.00	$19.00
Game 4	0.00	5.00	0.00	5.00	0.00	5.00	0.00	5.00	$19.00
Game 5	0.00	5.00	0.00	5.00	0.00	5.00	0.00	5.00	$19.00

OK


Remaining Time [sec]:
Game
Did the Other Player Play U/L?
Your Assigned Probability
Assigned Probability Higher than Random Draw?
Gamble Payoff
Game 1
Yes
50
No
$20.00
Game 2
No
50
Yes
$0.00
Game 3
No
50
No
$20.00
Game 4
No
50
Yes
$0.00
Game 5
No
50
No
$0.00
OK

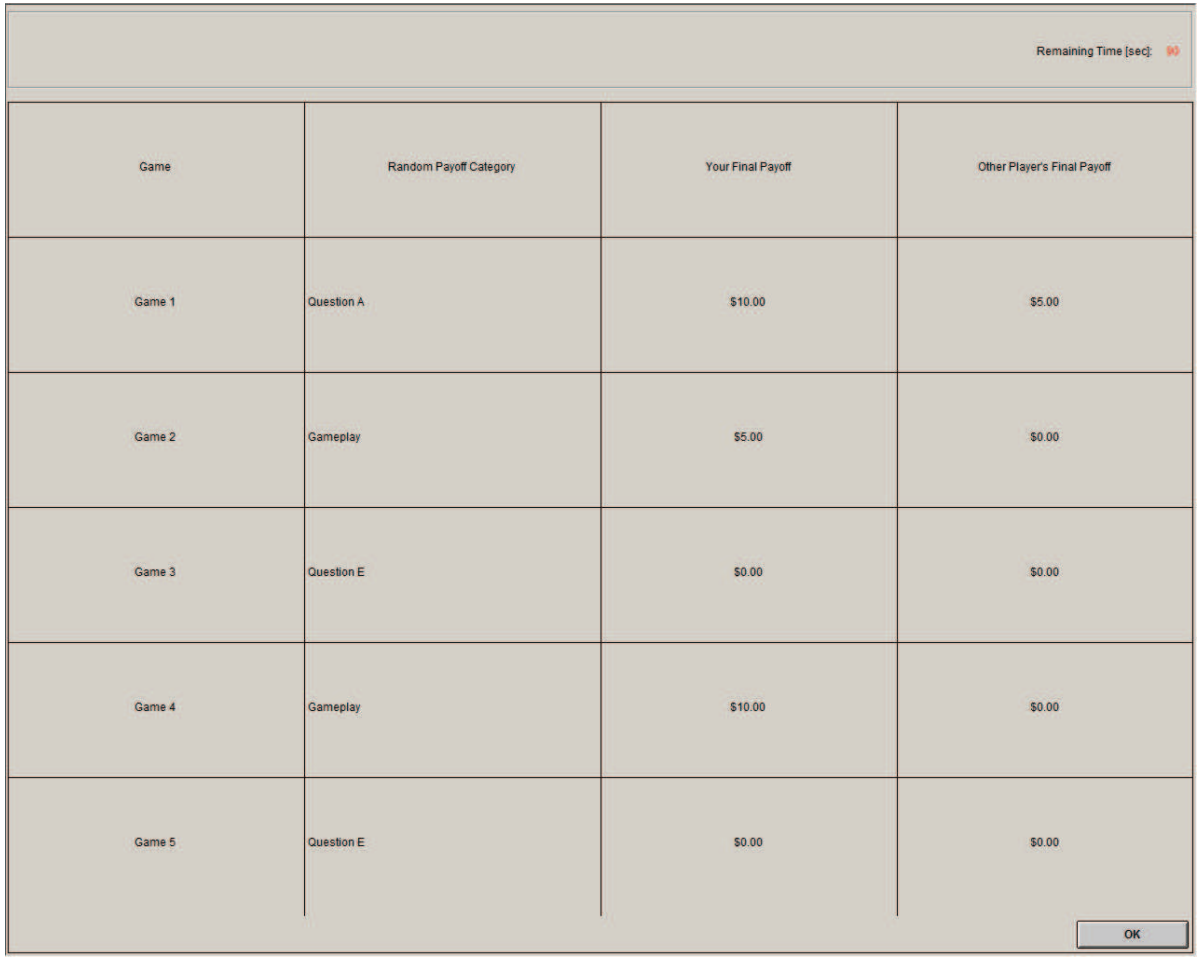

Remaining Time [sec]: 95
Game
Other Player's Assigned Probability of You Playing U/L
Your Estimate of this Probability
Accuracy Score
Game 1
2
50
$10.40
Game 2
0
50
$10.00
Game 3
0
50
$10.00
Game 4
0
50
$10.00
Game 5
0
50
$10.00
OK



Remaining Time [sec]:

Game	Was the Other Player's Action Consistent?	Your Assigned Probability that the Other Player was Consistent	Assigned Probability Higher than Random Draw?	Gamble Payoff
Game 1	Yes	10	No	$20.00
Game 2	No	10	No	$20.00
Game 3	No	10	Yes	$0.00
Game 4	No	10	No	$0.00
Game 5	No	10	No	$0.00

OK


Remaining Time [sec]:
Game
Random Payoff Category
Your Final Payoff
Other Player's Final Payoff
Game 1
Question A
$10.00
$5.00
Game 2
Gameplay
$5.00
$0.00
Game 3
Question E
$0.00
$0.00
Game 4
Gameplay
$10.00
$0.00
Game 5
Question E
$0.00
$0.00
OK

Game Randomly Chosen for Payoff	3
Your Total Profit (rounded up to nearest dollar, including show-up payment)	$10
Your Participant ID Number	2