Incentive Compatible Experiments: An Overview

P.J. Healy

Cast

Cast of characters:

- Yaron Azrieli (OSU)
- Chris Chambers (Georgetown)
- Nicolas Lambert (MIT)
- John Kagel (OSU)
- Kirby Nielsen (Caltech)
- Marina Agranov (Caltech)
- Alex Brown (Texas A&M)
- Greg Leo (Vanderbilt)
- Sam(antha) Stelnicki (OSU student)

Outline

Part 1: General experiments

Part 2: Belief elicitation

Elicitation

Goal of any experiment: elicit (coarse) information about $\succeq$

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Requirement: Incentive compatibility

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Classic mechanism design problem, except:

1. Don't have any particular SCF in mind
    - Any IC payment is fine
2. Allow random mechanisms
3. *Strict* incentive compatibility

▸ Why Pay?

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J. Political Economy (2018)

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  - Complementarities: $\{$Left shoe, Apple$\}$, $\{$Right shoe, Banana$\}$

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  - Literature: "RPS requires Expected Utility"

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  - Literature: "RPS requires Expected Utility"
    - Hadn't been proven either way

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- Pay every decision?
- Pay one random decision?
- Which do researchers use?
  - Survey from 2011:

Pay all:	56%
RPS:	25%
Pay some:	13%
Other:	6%

Incentives in Experiments

Framework for Analyzing IC:

- Choice objects: $x, y, z \in X$
- (Strict complete) preference: $\succeq \in \mathcal{O}$
- Decision problems: $D = (D_1, \dots, D_k)$, each $D_i \subseteq X$
- "True" choices: $\mu_i(\succeq) \in D_i$
    - $\mu_i(\succeq) \succeq x \;\; \forall x \in D_i$
- Stated choices (messages): $m_i \in D_i \quad m = (m_1, \dots, m_k)$
- Payment mechanism: $\phi(m) \in \mathcal{P}(X)$
    - Payment objects: $\mathcal{P}(X)$
- Experiment: (D, ϕ)

Incentives in Experiments

Definition
An experiment (D, ϕ) is **incentive compatible** if,
for every $\succeq$ and every $m \neq \mu(\succeq)$,

$$\phi(\mu(\succeq)) \text{ is strictly preferred to } \phi(m).$$

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- Pay all: bundles
  - $\phi(m) = \{$Left shoe,Right shoe$\}$

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- RPS: acts
    - $\Omega = \{\omega_1, \omega_2\}$
    - $\phi(m)(\omega_1) = \{\text{Left shoe}\}$
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$\succeq$ says *nothing* about how these objects are ranked!

Incentives in Experiments

- Preference Extension: $\succeq$ on X, $\succeq^*$ on $\mathcal{P}(X)$.
  - Example: $\succeq$ over money, $\succeq^*$ EU over lotteries

Definition
An experiment (D, ϕ) is **incentive compatible** if,
for every $\succeq$ and every $m \neq \mu(\succeq)$,

$$\phi(\mu(\succeq)) \;\succ^*\; \phi(m).$$

Theorem
If no restrictions are placed on $\succeq^*$ then an experiment is IC if and only if there is <u>one</u> decision problem and $\phi(m_1) = m_1$.

Corollary
If $k > 1$ we <u>must</u> talk about $\succeq^*$ and how it relates to $\succeq$.

Incentives in Experiments

When is the Pay-All mechanism incentive compatible?

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*Assume $D = (D_1, \ldots, D_k)$ is non-redundant ($\bigcap_i D_i = \emptyset$). If $\succeq^*$ satisfies NCaT (and nothing else is assumed) then Pay-All is the **only** IC mechanism.*

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*Pay-All is the **only** IC mechanism.*

*Redundant case just adds flexibility on "intransitive" messages.

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When is the RPS mechanism incentive compatible?

- Need an assumption about $\succeq^*$ over acts
- The RPS mechanism has the "truth dominates lies" property

	States of the World					
Payment Object	1	2	3	4	$\cdots$	k
$\phi(m_1, m_2, m_3, \ldots, m_k)$	m_1	m_2	m_3	m_4	$\cdots$	m_k
$\phi(m_1, m'_2, m_3, \ldots, m_k)$	m_1	m'_2	m_3	m_4	$\cdots$	m_k
$\phi(m_1, m'_2, m'_3, \ldots, m_k)$	m_1	m'_2	m'_3	m_4	$\cdots$	m_k

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$\phi(m_1, m_2', m_3, \dots, m_k)$	m_1	m_2'	m_3	m_4	$\cdots$	m_k
$\phi(m_1, m_2', m_3', \dots, m_k)$	m_1	m_2'	m_3'	m_4	$\cdots$	m_k

- **Monotonicity:** $\succeq^*$ respects statewise dominance (w.r.t. $\succeq$)

$$f(\omega) \succeq g(\omega) \; \forall \omega \implies f \succeq^* g$$

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*Redundant case adds flexibility on "surely-identified" sets.
**Can also add states that pay a fixed prize.

Summary

Pay All: No Complementarities

RPS: Monotonicity w.r.t. statewise dominance

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"Incentives in Experiments with Objective Lotteries"
Azrieli, Chambers & Healy
Experimental Economics (2020)

- RPS with lotteries instead of acts
  - Assume an objective $p \in \Delta(\Omega)$
- More restrictive setting $\Rightarrow$ more IC mechanisms??

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- RPS with lotteries instead of acts
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Theorem
Assume Monotonicity w.r.t. FOSD (and nothing else).
1. Non-redundant: Same as before (only RPS)
2. Redundant: Added flexibility on "surely-identified" sets; not useful

When Can We Use RPS?

Things we should worry about with Monotonicity/RPS:

Things I don't think we need to worry much about:

On Monotonicity

Suppose X are multi-agent payments. $\mathcal{P}(X)$ are lotteries over X.
Ex-ante fairness $\Rightarrow$ monotonicity violation

Example: Machina's mom

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What about the RDU examples where RPS wasn't IC??

Suppose X are lotteries, $\mathcal{P}(X)$ are compound lotteries.
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Monotonicity + reduction $\Rightarrow \succeq$ satisfies independence (EUT)!

Reduction + Non-EU $\Rightarrow$ ~~Monotonicity~~ $\Rightarrow$ RPS *may* not be IC

The counter-examples all assume Reduction + Non-EU

Halevy (2007): those who reduce are EU maximizers! ✓

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- Non-expected utility + reduction

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Suppose X are acts, $\mathcal{P}(X)$ are lotteries over acts (AA).
Monotonicity + order-reversal $\Rightarrow \succeq$ is ambiguity-neutral!

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Ability to "hedge" away ambiguity...

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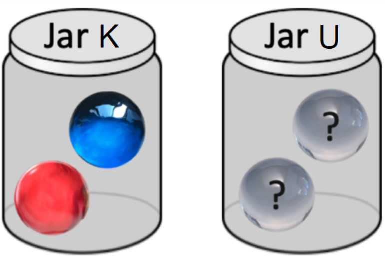
Ability to "hedge" away ambiguity...

Should we add ambiguity hedging to the "worry" list??

On Hedging

"A Direct Test of Hedging"
Healy & Stelnicki
Work in Progress

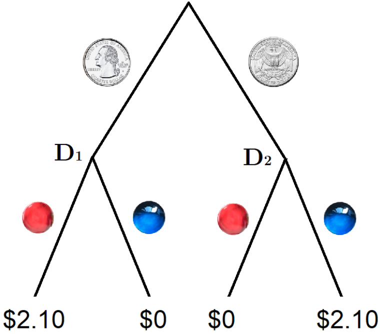


$$D_1 = \{\$2.00 \text{ if Red from K, } \$2.10 \text{ if Red from U}\}$$
$$D_2 = \{\$2.00 \text{ if Blue from K, } \$2.10 \text{ if Blue from U}\}$$

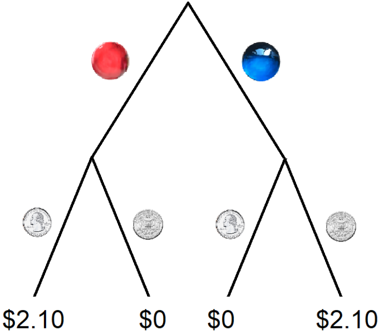
On Hedging

Picking *UU*:

Original:



Order-Reversed:



Ambiguous

$KK \succ^* UU$

50-50 Lottery For Sure

$UU \succ^* KK$

On Hedging

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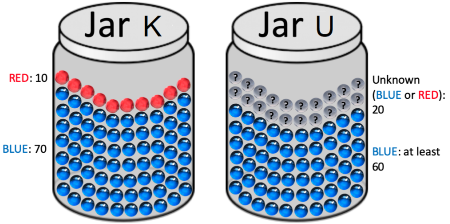
- **Hedgers:** Pick UU, say "between 50% and 50%."
- True even if the jars aren't 50-50



On Hedging

Results:

	Ask One Red	Ask One Blue		Ask Both (RPS)
K	58%	60%	KK	19%
			KU	23%
U	42%	40%	UK	44%
			UU	15%

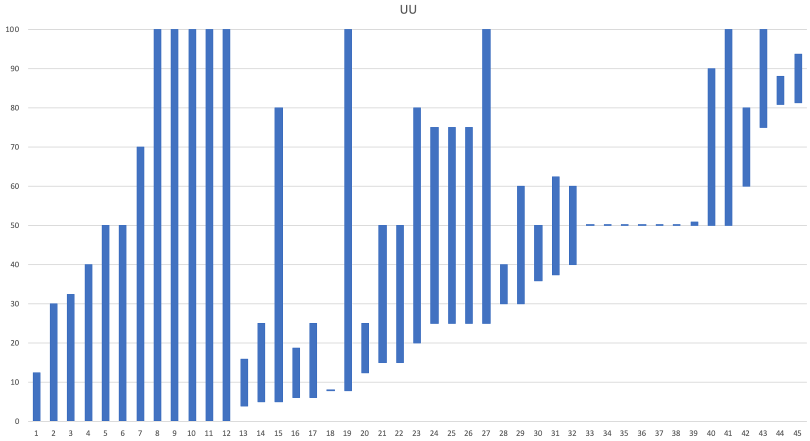
15% *UU* contains:

- Ambiguity Loving & Monotonicity
- Ambiguity Neutral & ∼50-50 beliefs & Monotonicity
- Ambiguity Averse & Hedging

UK>KU ⇒ red more likely ⇒ Ask One should differ

On Hedging

Belief ranges of the 15% who choose *UU* in Ask Both:

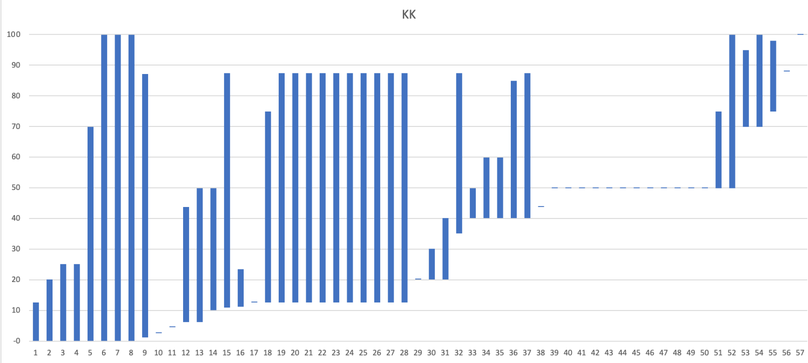


$\sim$15% are consistent with hedging. Or, $\sim$2% overall.

On Hedging

Belief ranges of the 19% who choose *KK*.

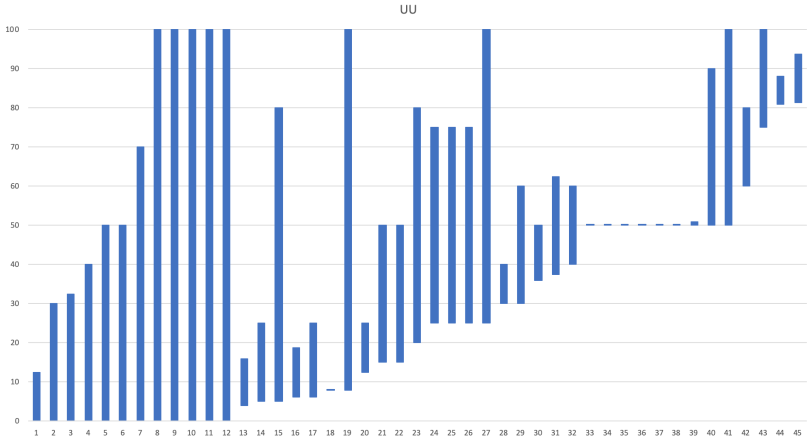
$(1/2)(1/8) + (1/2)(7/8) = 1/2$



21% say $[50, 50]$. 17% say $[1/8, 7/8]$.

On Hedging

Back to *UU*:



Could be some non-reducers here, but Order Reversal fails

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Things we should worry about with Monotonicity/RPS:

- Ex-ante fairness

Things I don't think we need to worry much about:

- Non-expected utility + reduction
- Ambiguity hedging

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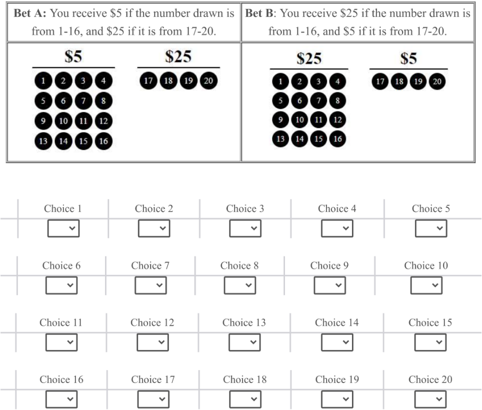
Our conjecture: Preference for randomization (violates Monotonicity)

Randomization

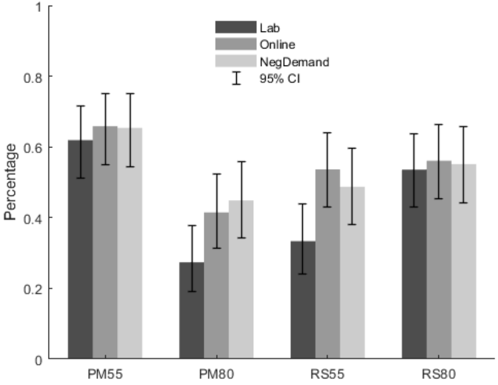
“Stable Randomization”
Agranov, Healy & Nielsen
Working Paper



Randomization

- "PM" Questions: dominance
- "RS" Questions: risky-safe

Percentage of people who mix:



Mixing highly correlated across decisions and games. "Mixing types"

When Can We Use RPS?

Things we should worry about with Monotonicity/RPS:

- Ex-ante fairness
- Repeated choices (same or similar)

Things I don't think we need to worry much about:

- Non-expected utility + reduction
- Ambiguity hedging

Separated Decisions

"Separated Decisions"
Brown & Healy
EER (2018)

Row #	Option A		or	Option B	
1	Balls 1-10 pay $10 (50% chance of $10)	Balls 11-20 pay $5 (50% chance of $5)	or	Ball 1 pays $15 (5% chance of $15)	Balls 2-20 pay $0 (95% chance of $0)
2	Balls 1-10 pay $10 (50% chance of $10)	Balls 11-20 pay $5 (50% chance of $5)	or	Balls 1-2 pay $15 (10% chance of $15)	Balls 3-20 pay $0 (90% chance of $0)
3	Balls 1-10 pay $10 (50% chance of $10)	Balls 11-20 pay $5 (50% chance of $5)	or	Balls 1-3 pay $15 (15% chance of $15)	Balls 4-20 pay $0 (85% chance of $0)
4	Balls 1-10 pay $10 (50% chance of $10)	Balls 11-20 pay $5 (50% chance of $5)	or	Balls 1-4 pay $15 (20% chance of $15)	Balls 5-20 pay $0 (80% chance of $0)
	Balls 1-10 pay $10	Balls 11-20 pay $5		Balls 1-5 pay $15	Balls 6-20 pay $0
⋮	⋮			⋮	⋮
18	(50% chance of $10)	(50% chance of $5)	or	(90% chance of $15)	(10% chance of $0)
19	Balls 1-10 pay $10 (50% chance of $10)	Balls 11-20 pay $5 (50% chance of $5)	or	Balls 1-19 pay $15 (95% chance of $15)	Ball 20 pays $0 (5% chance of $0)
20	Balls 1-10 pay $10 (50% chance of $10)	Balls 11-20 pay $5 (50% chance of $5)	or	All Balls pay $15 (100% chance of $15)	(0% chance of $0)

Separated Decisions

Direct test of Monotonicity:

- List-RPS: See all rows, RPS payment
- List-R14: See all rows, only paid for row 14

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	% Risky on Row 14
List-RPS	52%
List-14	70%

List formatting violates monotonicity.

Separated Decisions

Direct test of Monotonicity:

- Separated-RPS:
  See all rows on separate screens in random order, RPS payment
- Separated-R14:
  See all rows on separate screens in random order, pay row 14

Separated Decisions

Direct test of Monotonicity:

- Separated-RPS:
  See all rows on separate screens in random order, RPS payment
- Separated-R14:
  See all rows on separate screens in random order, pay row 14

	% Risky on Row 14
Sep-RPS	59%
Sep-14	56%

Separated formatting restores monotonicity.
Multiple switching: 5% $\rightarrow$ 33%, but usually very minor
Recommendation: Separate your decisions!

When Can We Use RPS?

Things we should worry about with Monotonicity/RPS:

- Ex-ante fairness
- Repeated choices (same or similar)
- Showing choices all together

Things I don't think we need to worry much about:

- Non-expected utility + reduction
- Ambiguity hedging

When Can We Use RPS?

Things we should worry about with Monotonicity/RPS:

- Ex-ante fairness
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Things I don't think we need to worry much about:

- Non-expected utility + reduction
- Ambiguity hedging

That's it!

What Can We Learn?

"Constrained Preference Elicitation"
Azrieli, Chambers & Healy
Theoretical Economics (2021)

Structure theorems on what we can learn about $\succeq$ from any experiment.

How Can We Learn It?

"Minimal Experiments"
Healy & Leo
Work in Progress

Given: Something you want to learn about $\succeq$.

- Example: is $p(E)$ in $[0, \frac{1}{3})$, $[\frac{1}{3}, \frac{2}{3})$, or $[\frac{2}{3}, 1]$?

Step 1: Which experiments would elicit that?

Step 2: Which experiment is the "simplest"?

- $D_1 = \{\$10\ \mathit{if}\ E, \$10\ \mathit{if}\ E^C, \$10\ w/\ 66\%\}$

Outline

Part 1: General experiments

Part 2: Belief elicitation

Testing Elicitation Mechanisms

"Testing Elicitation Mechanisms Via Team Chat"
Healy & Kagel
Work in Progress

Belief Elicitation Mechanisms:

- Quadratic scoring rule (QSR; Brier 1950)
    - Logarithmic, spherical...
    - QSR corrected for risk aversion (Harrison et al. 2014)
- Binarized scoring rules (BSR; Savage 1971, Hossain & Okui 2013)
- BDM for probabilities (Marschak 1963, Grether 1981)
    - Clock BDM (Karni 2009)
- Multiple Price List (MPL; Holt & Smith 2016)

What Do The Data Say?

- Offerman & Sonnemans (2004): QSR$\sim$None
- Armantier & Treich (2013): QSR$\succ$None
- Huck & Weizsacker (2002): QSR$\succ$BDM
- Hollars et al. (2010): BDM$\succ$QSR
- Hao & Houser (2012): BDM-Clock$\succ$BDM
- Hossain & Okui (2013): BSR$\succ$QSR
- Harrison et al. (2014): BSR$\sim$QSR-Corr$\succ$QSR
- Holt & Smith (2016); MPL$\succ$BDM

Best performers: BSR and MPL

Our Motivations

- Offerman & Sonnemans (2004): QSR$\sim$None
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- Holt & Smith (2016); MPL$\succ$BDM

Motivation: Compare MPL to BSR in theory and in the lab

Quadratic Scoring Rule

Suppose $X \in \{0, 1\}$.
Want to elicit $p = Pr(X = 1)$.
Subject announces q, gets paid:

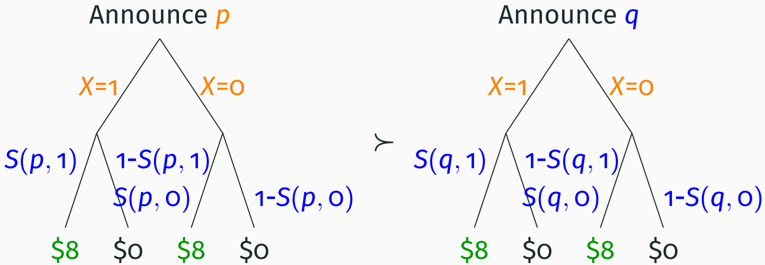
$$S(q, X) = 1 - (X - q)^2$$

IC requires risk neutrality.

Solution: pay in probabilities
Savage (1971) $\rightarrow$ C. Smith (1961) $\rightarrow$ Savage (1954)

Conditions for Incentive Compatibility

Proof of Incentive Compatibility:



$$\Updownarrow$$

$$\underbrace{p \cdot S(p,1) + (1-p) \cdot S(p,0)}_{Pr(\$8)\ \text{if truth}} > \underbrace{p \cdot S(q,1) + (1-p) \cdot S(q,0)}_{Pr(\$8)\ \text{if lie}}$$

This requires "Subjective-Objective Reduction"

- Weakening of ROCL: Applies only to two-prize lotteries

Multiple Price Lists (MPL)

Row#	Option A	OR	Option B
1	\$8 if $X = 1$	or	\$8 w/ prob 1%
2	\$8 if $X = 1$	or	\$8 w/ prob 2%
$\vdots$	$\vdots$	$\vdots$	$\vdots$
q	\$8 if $X = 1$	or	\$8 w/ prob q%
$q+1$	\$8 if $X = 1$	or	\$8 w/ prob $q+1$%
$q+2$	\$8 if $X = 1$	or	\$8 w/ prob $q+2$%
$q+3$	\$8 if $X = 1$	or	\$8 w/ prob $q+3$%
$\vdots$	$\vdots$	$\vdots$	$\vdots$
99	\$8 if $X = 1$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 w/ prob 100%

Choose Option A or Option B (single switch point q)

One row randomly selected for payment

Multiple Price Lists (MPL)

Row#	Option A	OR	Option B
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"Multiple Price List" (MPL) version of BDM for probabilities
Holt & Smith (2016)

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$\vdots$	$\vdots$	$\vdots$	$\vdots$
99	\$8 if $X = 1$	or	\$8 w/ prob 99%
100	\$8 if $X = 1$	or	\$8 w/ prob 100%

If you lie, you get the less-preferred option on some rows

I.C. as long as subject respects **statewise dominance** in rows

MPL vs BSR

Proposition:

All BSRs are I.C.

$$\Updownarrow$$

Subjective-Objective Reduction

$$\Downarrow$$

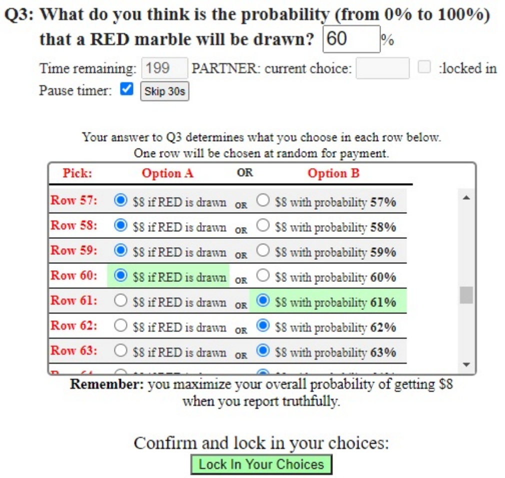
Statewise Dominance

$$\Downarrow$$

Any MPL is I.C.

Our Experiment

- Compare **BSR** to **MPL**
- Put subjects in teams of two, working together via chat
    - Cooper & Kagel (2005,2009,2020)
- Scan chat transcripts for (1) true beliefs, (2) manipulation
- Variety of questions (objective, subjective)
    - Focus here on objective questions



Q3: What do you think is the probability (from 0% to 100%) that a RED marble will be drawn? 60 %

Time remaining: 199 PARTNER: current choice: :locked in

Pause timer: Skip 30s

Your answer to Q3 determines what you choose in each row below.
One row will be chosen at random for payment.

Pick:	Option A	OR	Option B
Row 57:	$8 if RED is drawn	OR	$8 with probability 57%
Row 58:	$8 if RED is drawn	OR	$8 with probability 58%
Row 59:	$8 if RED is drawn	OR	$8 with probability 59%
Row 60:	$8 if RED is drawn	OR	$8 with probability 60%
Row 61:	$8 if RED is drawn	OR	$8 with probability 61%
Row 62:	$8 if RED is drawn	OR	$8 with probability 62%
Row 63:	$8 if RED is drawn	OR	$8 with probability 63%

Remember: you maximize your overall probability of getting $8 when you report truthfully.

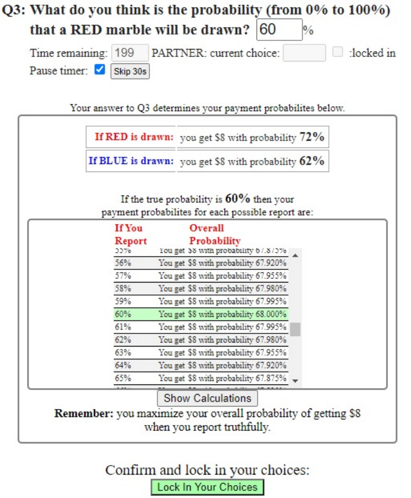
Confirm and lock in your choices:

Lock In Your Choices

Link

Note: subjects saw the same phrase in all three treatments

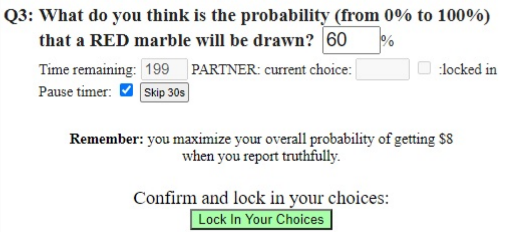
The Mechanism Interfaces: BSR



Link

Note: subjects saw the same phrase in all three treatments

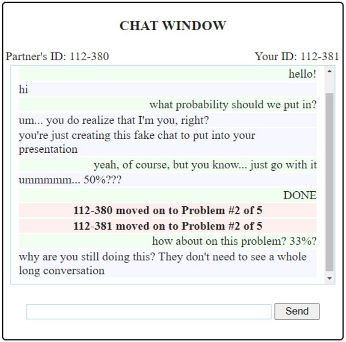
The Mechanism Interfaces: NoInfo

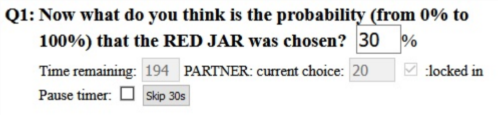


Link

Note: subjects saw the same phrase in all three treatments

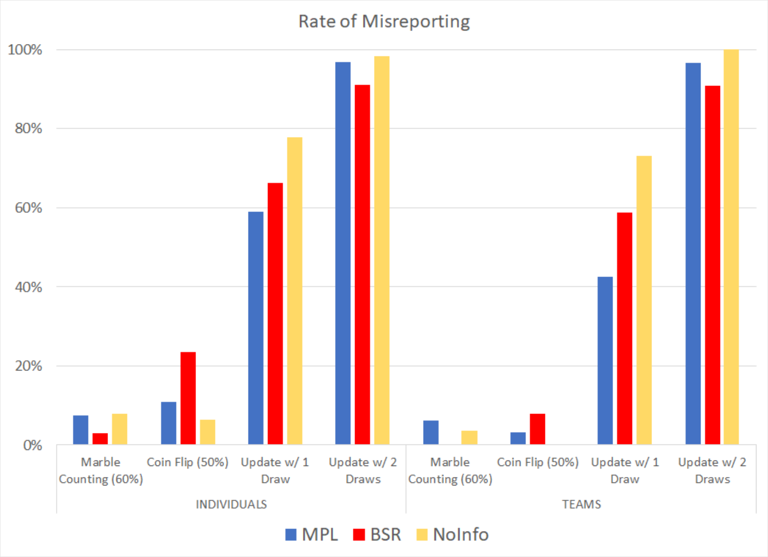
Teams Interface





- Use chat window to communicate
- Must lock in the same number to proceed
- If time runs out, one choice is randomly used

Misreporting Rate: Objective Probabilities



Chat Encoding

Two Types of Evidence of IC Failures:

Deviate Discuss deviating from their belief

- May not specify *why* they're deviating

Manipulate Discuss manipulation of payoffs

- May not end up deviating from their belief

Warning: So far, only encoded by me

Chat Encoding

Two Types of Evidence of IC Failures:

Deviate Discuss deviating from their belief

- May not specify *why* they're deviating

Manipulate Discuss manipulation of payoffs

- May not end up deviating from their belief

Mechanism	MPL	BSR	NoInfo
Deviate	2/33	2/34	0/27
Manipulate	1/33	5/34	0/27

Marble Counting Chat

ID#181	MPL	ID#187
i have 12 for red		
and 8 for blue		
		12, 20, and 75%?
		yes
75 sounds good with me		
12|20|75%		**12|20|75%**

Coin Flip Chat

ID#257	BSR	ID#260
		50 ?
id say 60		
		Why
cause heads is always more likely		
		Thats just false
55 is a compromise		
		Which is also wrong but whatever
55%		**55%**

ID#357	BSR	ID#365
	(no chat)	
75%		**75%**

Deviate: MPL

$$12/20 = 60\%$$

ID#352	MPL	ID#353
		60%
12 red marbles, 20 total, so 60%		
		Yea but I am thinking should we really put the correct number for probability
I mean yeah i think		
Although its random, its the best “odds” then		
		alright
60%		**60%**

Manipulation: BSR

Capital of Australia

ID#407	BSR	ID#414
hi		
		hi
		i noticed that the higher you make their percentage,
		the higher our probability percentage gets
yeah that's true		
		but the closer to 50, the more equal the probs
i say we go for a big one		
85		**85**

The Story

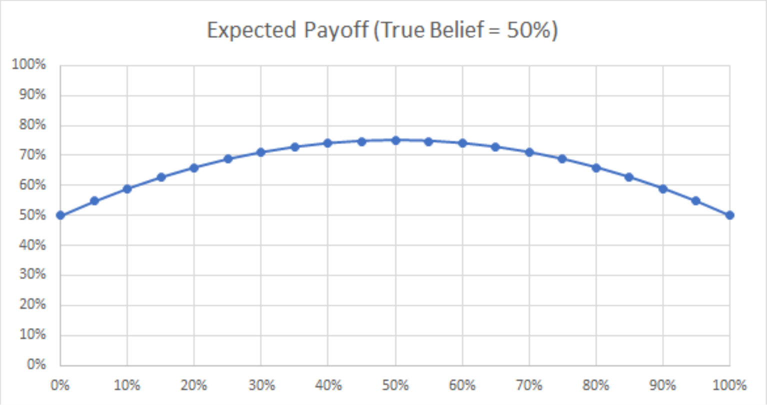
- Chats conclude they're **not** successfully manipulating
  - Maybe slightly more *attempts* in BSR?
- NoInfo performs well when easy, worst when hard
- Implication: Mechanism details can be distracting **or** useful
  - Easy problems: details get in the way, $\uparrow$ mistakes
  - Harder problems: details maybe help focus, $\downarrow$ mistakes

Summary

- Theory:
    1. MPL has superior IC properties
    2. Some scoring rules are equiv. to an MPL, but not BQSR
- Empirics:
    1. MPL and BSR perform similarly
    2. NoInfo works well when easy, not when hard
    3. Very little evidence of manipulation
        - Subjects are confused/overwhelmed, not manipulating

Coarse Elicitation

"Coarse Elicitation"
Healy & Leo
Work in Progress



Coarse Elicitation

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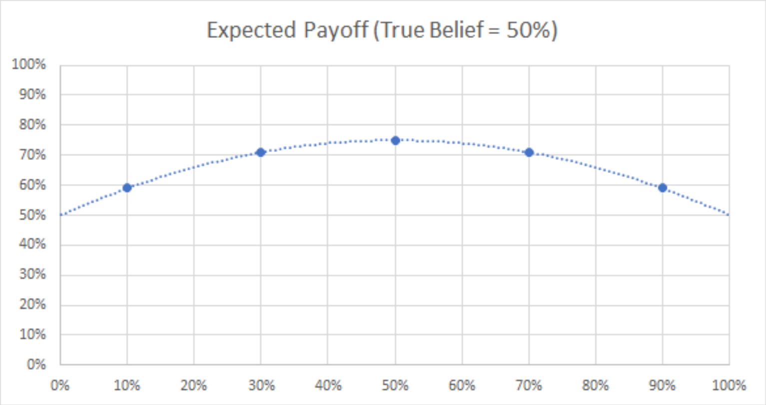
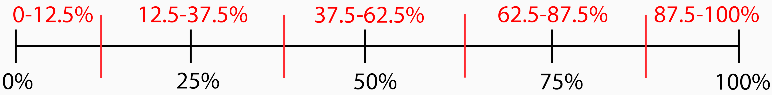
Work in Progress



Coarse Elicitation

"Midpoint Property"



Theorem: The only* differentiable scoring rule that satisfies the midpoint property for *any* grid is the quadratic scoring rule.

*Up to a rescaling.

Simple alternative: Coarse MPL

What Can Be Elicited

"Elicitability"
Azrieli, Chambers, Healy & Lambert
Work in Progress

- **Goal:** elicit subjective $p(E)$ for some event $E \subseteq \Omega$
- **Problem:** states $\omega \in \Omega$ are not observable! Only signals $y \in Y$.

Examples:

- Climate change
- Beliefs in repeated PD w/ private monitoring
- Vaccine effectiveness

Question: can we still learn beliefs over Ω using only Y?

Vaccine Example (of course)

State: efficacy. $\omega \in \Omega = \{0, 1/2, 1\}$
Agent: medical researcher. Has belief $p \in \Delta(\Omega)$
Principal: management. Wants to learn about p
Signal: outcome of 1 trial. $y \in Y = \{S, H\}$
Info Structure: $\Pi(y|\omega)$

Π		Y Sick (S)	Healthy (H)
Ω	0	1	0
	1/2	0.5	0.5
	1	0	1

Induced Belief on *Y*: $p_\Pi(S) = \vec{p} \cdot \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix}$

Vaccine Example: A Tale of Three Agents

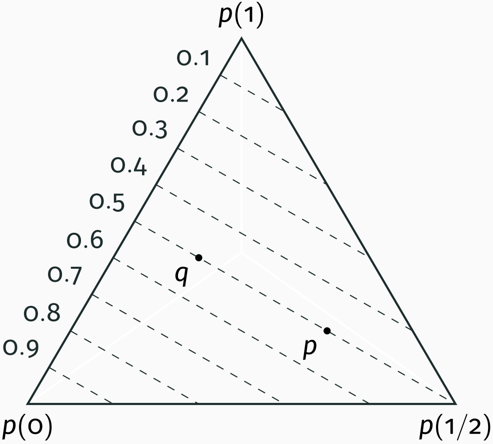
Ann's p	Sick (S) $1/2$	Healthy (H) $1/2$
0	1	0
1	0.5	0.5
0	0	1

Bob's p	Sick (S) $1/2$	Healthy (H) $1/2$
1/2	1	0
0	0.5	0.5
1/2	0	1

Charlie's p	Sick (S) $1/2$	Healthy (H) $1/2$
1/3	1	0
1/3	0.5	0.5
1/3	0	1

Vaccine Example

$p_\Pi(S) =$



The Question

Given Π, what can we learn about p?

Main Result:
Π generates a partition of $\Delta(\Omega)$ based on p_Π.
p and q can be distinguished iff $p_\Pi \neq q_\Pi$

Assumptions:

1. Π is known
2. p_Π is derived from p and Π via *reduction*
3. p_Π can be elicited (BQSR, MPL, ...)

Vaccine Example: Two Subjects

Now suppose vaccine trial has two patients (iid)
$Y = \{0, 1, 2\}$ gives # of Healthy patients

		Y: 0	1	2
Ω	0	1	0	0
	1/2	0.25	0.50	0.25
	1	0	0	1

Three linearly independent columns! Π has full rank.
$p_\Pi = \vec{p} \cdot \Pi \implies p_\Pi \cdot \Pi^{-1} = \vec{p}$!!

Full rank $\Rightarrow$ We can perfectly back out any belief!

Random Variables

In general, with k observations, you learn the first k moments of p

Three states: two moments is enough to learn p

$|\Omega| = n$: then $n - 1$ observations gives you p

Other Stuff We Know

- Can elicit *median* of $\omega \Leftrightarrow$ can elicit entire p
- Can add covariates
  - Π_{man} and Π_{woman}, $Y = (Y_{man} \times Y_{woman})$
- Infinite states & signals
  - Gaussian linear model: $y = \beta_0 + \beta_1 x + \varepsilon$
    - Full rank! One observation gives entire p
  - Non-parametric linear model: $E[y|x] = \beta_0 + \beta_1 x$
    - One obs: $E_p[\beta_0]$, $E_p[\beta_1]$.
    - Two obs: $Var_p[\beta_0]$, $Var_p[\beta_1]$.
    - $\dots$
  - Probit: $y = \mathbb{1}_{\{\beta_0+\beta_1 x+\varepsilon>0\}}$
    - Need *infinite* data to get $E_p[\beta_0]$, $E_p[\beta_1]$!!
- New ordering of Information Structures
  - "Π_2 elicits more than Π_1"
  - Blackwell Dominance $\Rightarrow$ Elicits More

Summary of Belief Elicitation

- BQSR and MPL both work fine

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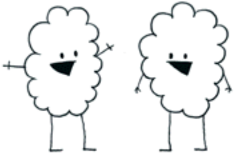
Summary of Belief Elicitation

- BQSR and MPL both work fine
- Manipulation doesn't seem to be a huge problem
- You can do coarse elicitation
- Unobservable states limits what we can learn
    - More observations helps

Sorry!!



Do Incentives Matter?

Overarching goal: Strict incentive compatibility of experiments

Why pay?

- Real payments $\uparrow$ risk aversion
  - Smith & Walker (1993), Wilcox (1993), Beattie & Loomes (1997), Camerer & Hogarth (1999)
  - Holt & Laury (2005): hypothetical stake size doesn't matter
- Real payments $\uparrow$ selfishness
  - Sefton (1992); Forsythe, Horowitz, Savin, & Sefton (1994); Clot, Grolleau & Ibanez (2018)
- Real payments $\uparrow$ correlation with Big 5
  - Lönnqvist et al. (2011)
- Hypothetical bias is real, hard to predict
  - Haghani et al. (2021); Laury & Holt (2008)
- But there are arguments not to pay...
  - Rubinstein (2001,2013); Harbi et al. (2015); Falk et al. (2016); Ben-Ner et al. (2008)

Return