Incentives in Experiments: A Theoretical Analysis

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What is an Experiment?

What is an Experiment?

The subject

- Walks into the lab
- Asked to make several choices
- Rewarded based on her choices

The researcher

- Observes subject's choices
- Learns about her preferences

The Problem

How to reward the subject such that observed choices are 'truthful'?

- Truthful = rightly represent underlying preferences
- No problem if only one choice (give her what she chose)
- Less obvious with several decision problems – the researcher analyzes the data as if each problem is isolated

Example

Experiment: Testing "rationality" in a game

1. Play the following game:

	L	R
U	$1, 1$	$0, 0$
D	$0, 0$	$1, 1$

2. Guess which strategy your opponent will pick.
   - Paid \$1 if right, \$0 if wrong.

Paying for both decisions creates a hedging problem:

Truth: \$2 if right, \$0 if wrong
Hedge: \$1 for sure

Another Example

Experiment: Correlate dictator-game giving with risk preferences

1. High-Stakes Dictator Game
   - Each subject given \$100
   - Paired with another subject (anonymously)
   - Asked how much he wants to give to the other subject (Dollar increments)
2. Holt-Laury (2002) procedure for estimating risk preferences.

#	Safe Lottery	Risky Lottery
1	(0.1, \$2.00; 0.9, \$1.60)	(0.1, \$3.85; 0.9, \$0.10)
2	(0.3, \$2.00; 0.7, \$1.60)	(0.3, \$3.85; 0.7, \$0.10)
3	(0.5, \$2.00; 0.5, \$1.60)	(0.5, \$3.85; 0.5, \$0.10)
4	(0.7, \$2.00; 0.3, \$1.60)	(0.7, \$3.85; 0.3, \$0.10)
5	(0.9, \$2.00; 0.1, \$1.60)	(0.9, \$3.85; 0.1, \$0.10)

Another Example (cont.)

Suppose paying for all 6 decisions:

- Wealth effect: Earning $90 in dictator game may reduce risk aversion
- Portfolio effect: The 5 risky lotteries as a portfolio aren't that risky

Complementarities

More generally, complementarities between decision problems may distort choices if all are paid

Example:

1. Cookie or Hot Dog?
2. Milk or Beer?

A Proposed Solution

Certainly not the first to notice this problem

A commonly used solution: Pay for one randomly-selected decision

- Known at least since Allais (1953)
- Used by Yaari (1965)
- Discussed by Holt (1986)
- Definitely not a comprehensive list..

Our name: 'Random Problem Selection' (RPS) mechanism (but other names appear in the literature).

A Problematic Example (Holt 1986, Cox et al 2011)

Let $L = (0.5, \$0; 0.5, \$3)$.

- Decision 1: L vs. \$1 for sure
- Decision 2: L vs. \$2 for sure
- Each decision chosen for payment w/ 50% probability

- Suppose $\$2 \succ L \succ \$1$
- Picking $\{L, \$2\}$ gives lottery $(0.25, \$0; 0.5, \$2; 0.25, \$3)$ (TRUTH)
- Picking $\{\$1, \$2\}$ gives lottery $(0.5, \$1; 0.5, \$2)$ (LIE)
- $\exists$ rank-dependent utility preferences where $\$2 \succ L \succ \$1$ and LIE $\succ$ TRUTH

$$U(f) = \sum_{s=1}^{n} u(x_s) \left[q(\sum_{r=1}^{s} p_r) - q(\sum_{r=1}^{s-1} p_r) \right]$$

Current Practice

Mechanism:	Only 1 Task	None Paid	One Random	Some Random	All Paid	Rank-Based	Total
	Individual Choice Experiments						
'Top 5'	7	0	3	1	3	0	14
ExpEcon	3	0	1	0	2	0	6
	Muti-Person (Game) Experiments						
'Top 5'	9	0	1	0	8	0	18
ExpEcon	8	1	3	3	5	1	21
Totals	27	1	8	4	18	1	59

1. Experimenters lack a convention.
2. Theory is unclear. Is expected utility needed for RPS??

In this paper

1. Describe an abstract model of experiment
2. Define a notion of incentive compatibility of the payment mechanism ("each decision is made as if in isolation")
3. Understand under what conditions the RPS mechanism is incentive compatible (answer: 'monotonicity')
4. Characterize the set of incentive compatible payment mechanisms (assuming monotonicity)

Also, analyze when is it OK to pay for all (or several) decisions (but not in this talk).

An Abstract Model of Experiment

- X: A finite set of 'objects' (no structure).
- $D = (D_1, \ldots, D_k)$: A finite list of decision problems, where each $D_i \subseteq X$. Assume $D_i \neq D_j$ and $|D_i| > 1$ for every i (can be easily relaxed).

- $\succeq$ over X (complete & transitive)
- $\mu_i(\succeq) = \{x \in D_i : (\forall y \in D_i)\ x \succeq y\}$
- $\mu(\succeq) = \times_i \mu_i(\succeq)$ ('optimal choices in isolation')

- Messages: $M = \times_i D_i$ ('announced choice')
- Payment mechanism: Maps M to 'payments'

The Example

- First decision: dictator game
  $D_1 = \{(\$100, \$0), (\$99, \$1), \ldots, (\$0, \$100)\}$. $m_1 = (\$90, \$10)$
- Next: 5-question Holt-Laury elicitation
  $D_2 = \{(0.1, \$2; \$1.60), (0.1, \$3.85; \$0.10)\}$.
  $m_2 = (0.1, \$2; \$1.60)$
  $D_3 = \{(0.3, \$2; \$1.60), (0.3, \$3.85; \$0.10)\}$.
  $m_3 = (0.3, \$2; \$1.60)$
  $D_4 = \{(0.5, \$2; \$1.60), (0.5, \$3.85; \$0.10)\}$.
  $m_4 = (0.5, \$2; \$1.60)$
  $D_5 = \{(0.7, \$2; \$1.60), (0.7, \$3.85; \$0.10)\}$.
  $m_5 = (0.7, \$3.85; \$0.10)$
  $D_6 = \{(0.9, \$2; \$1.60), (0.9, \$3.85; \$0.10)\}$.
  $m_6 = (0.9, \$3.85; \$0.10)$
- Payment: RPS Mechanism
  - Roll a 6-sided die.
  - Roll a 1: pay m_1
  - Roll a 2: pay m_2

Payments: Acts vs Lotteries

The researcher may use a randomization device (say, roll a die) to determine which element of X is chosen for payment

Two possible approaches regarding how the subject views this uncertainty:

1. Savage (1954): Payment based on a die roll is an *act*
   - Finite state space $\Omega = \{\omega_1, \ldots, \omega_n\}$
   - A payment $f(\omega) \in X$ for each $\omega \in \Omega$
   - The set of all acts is $\mathcal{F} = X^\Omega$
   - Each $m \in M$ is mapped to some act $\phi(m) \in \mathcal{F}$

2. Payment based on a die roll is an *objective lottery*
   - $\Delta(X)$ – the set of lotteries on X
   - Each $m \in M$ is mapped to some lottery $\varphi(m) \in \Delta(X)$

Incentive Compatibility (Acts)

- Each $\succeq$ over X extends to $\succeq^*$ over $\mathcal{F}$
- $\succeq^*$ agrees with $\succeq$ on constant acts
- Let $\mathcal{E}(\succeq)$ be the set of admissible extensions of $\succeq$

Definition

An experiment (D, ϕ) is **incentive compatible** with respect to $\mathcal{E}$ if, for every $\succeq$ and extension $\succeq^* \in \mathcal{E}(\succeq)$, every $m^* \in \mu(\succeq)$ and every $m \in M$,

$$\phi(m^*) \succeq^* \phi(m)$$

and

$$\phi(m^*) \succ^* \phi(m)$$

whenever $m \notin \mu(\succeq)$.

Strict incentive compatibility.

Preliminary Observation

Proposition

If no restrictions are placed on $\mathcal{E}(\succeq)$*, then there is an IC payment mechanism if and only if there is only one decision problem* $(k = 1)$*.*

Monotonicity

What restrictions on $\succeq^*$?

- (Subjective) expected utility representation
- Probabilistic sophistication
- Uncertainty aversion (say, maxmin expected utility)

  $\vdots$

(Statewise) Monotonicity:

$$f(\omega) \succeq g(\omega)\ \forall \omega \Rightarrow f \succeq^* g$$
$$\text{and } f(\omega) \succ g(\omega) \text{ for some } \omega \Rightarrow f \succ^* g$$

$\mathcal{E}^{\mathrm{mon}}(\succeq)$ = set of all monotonic extensions of $\succeq$

Monotonicity

	States of the World					
Act	1	2	3	4	5	6
f	\$1	\$25	pizza	\$0	\$1	Twix
g	\$1	\$24	pizza	\$0	\$1	Mars

\$25 $\succ$ \$24 and Twix$\succ$Mars $\Rightarrow f \succ^* g$

Monotonicity and Dominance

Lemma

*An experiment (D, ϕ) is incentive compatible w.r.t. $\mathcal{E}^{\text{mon}}$ if and only if it has the "***Truth Dominates Lies***" property:*

For every $\succeq$, $m^* \in \mu(\succeq)$, $m \in M$ and $\omega \in \Omega$,

$$\phi(m^*)(\omega) \succeq \phi(m)(\omega).$$

If $m \notin \mu(\succeq)$ then there is $\omega \in \Omega$ such that

$$\phi(m^*)(\omega) \succ \phi(m)(\omega).$$

The RPS Mechanism

Definition

ϕ is an RPS mechanism if $\exists$ a partition $\{\Omega_1, \ldots, \Omega_k\}$ of Ω into non-empty sets such that

$$\omega \in \Omega_i \Rightarrow \phi(m)(\omega) = m_i.$$

(Assume each Ω_i is non-null.)

RPS and Monotonicity

Proposition

If only monotonic extensions are admissible ($\mathcal{E} \subseteq \mathcal{E}^{\text{mon}}$) then any RPS mechanism is incentive compatible.

Sketch of Proof:

Suppose each $D_i = \{x_i, y_i, z_i, \ldots\}$
Suppose $x_i = \mu_i(\succeq)$ for each i

	States of the World					
Act	1	2	3	4	$\cdots$	k
$\phi(x_1, x_2, x_3, \ldots, x_k)$	x_1	x_2	x_3	x_4	$\cdots$	x_k
$\phi(x_1, y_2, x_3, \ldots, x_k)$	x_1	y_2	x_3	x_4	$\cdots$	x_k
$\phi(x_1, y_2, z_3, \ldots, x_k)$	x_1	y_2	z_3	x_4	$\cdots$	x_k

Now apply previous lemma.

Monotonicity (on a restricted domain) is also necessary for incentive compatibility of the RPS mechanism.

Monotonicity

Is monotonicity strong?

Suppose X is a space of lotteries.
Monotonicity + reduction $\Rightarrow$ independence (EUT)

Suppose X is a space of acts.
Monotonicity + order-reversal $\Rightarrow$ ambiguity neutrality

Other IC Mechanisms?

Maintaining the monotonicity assumption ($\mathcal{E} = \mathcal{E}^{\text{mon}}$), what is the class of all incentive compatible mechanisms?

From now on, assume only strict $\succeq$ are admissible:

- A unique maximal element in each decision problem ($\mu(\succeq)$ is a singleton).
- There may be $m \in M$ that cannot be rationalized:
  $D_1 = \{x, y\}$, $D_2 = \{y, z\}$, $D_3 = \{x, z\}$
  $m = (x, y, z)$ is not *rationalizable*

  M_R=rationalizable messages
  M_{NR}=non-rationalizable messages

Surely Identified Sets

Example: $D_1 = \{x, y\}$, $D_2 = \{y, z\}$, $D_3 = \{x, z\}$
Consider $E = \{x, y, z\}$
If $m \in M_R$, then we know your favorite thing in E.

Definition

A set $E \subseteq X$ is surely identified if, for every $\succeq$, the choices $m = \mu(\succeq)$ reveal the $\succeq$-maximal element of E. Let $SI(D)$ be the family of surely identified sets for D.

Lemma

$$E \in SI(D) \Leftrightarrow \forall x, y \in E \;\; \exists D_i \in D, \;\; \{x, y\} \subseteq D_i \subseteq E$$

In practice, usually $SI(D) = \{D_i\}_{i=1}^{k} \bigcup \{x\}_{x \in X}$.

RSS Mechanisms

Given ϕ, denote $P^{\phi}(\omega) = \{\phi(m)(\omega)\}_{m \in M}$.

Definition

ϕ is a Random Set Selection (RSS) Mechanism if, for each $\omega \in \Omega$, $P^{\phi}(\omega) \in SI(D)$ and for every $m \in M_R$,

$$\phi(m)(\omega) = \max(P^{\phi}(\omega)|m).$$

Interpretation: I roll a die and pay you either for a real decision you made, or for a fake decision where I can *always* figure out what you would have chosen.

RPS $\subset$ RSS

One known example: Krajbich (2011)

Characterization

Theorem

(D, ϕ) is incentive compatible w.r.t. $\mathcal{E}^{\text{mon}}$ if and only if

1. *ϕ is an RSS mechanism;*
2. *Each D_i is surely identified by the sets $\{P^\phi(\omega)\}_{\omega \in \Omega}$;*
3. *$m \in M_{NR}$ implies $\phi(m) \notin \phi(M_R)$.*

Idea of Proof:

1. At each ω you get the revealed best possible element $\phi(m)(\omega) = \max(P^\phi(\omega)|m)$; thus, RSS
2. Each D_i matters for the outcome
3. Non-rationalizable messages give you something from each payment set, but can't possibly be your favorite in all sets.

Almost-Characterizing RPS

Usually $SI(D) = \{D_i\}_{i=1}^{k} \bigcup \{x\}_{x \in X}$.
(For example, if each D_i is disjoint.)

In this case, RSS = RPS + "singleton payments".

Thus, in practice, IC $\iff$ RPS + singleton payments

Incentive Compatibility (Lotteries)

- Each $\succeq$ over X extends to $\succeq^*$ over $\Delta(X)$
- $\succeq^*$ agrees with $\succeq$ on degenerate lotteries
- Let $\mathcal{E}(\succeq)$ be the set of admissible extensions of $\succeq$

Definition

An experiment (D, φ) is **incentive compatible** with respect to $\mathcal{E}$ if, for every $\succeq$ and extension $\succeq^* \in \mathcal{E}(\succeq)$, every $m^* \in \mu(\succeq)$ and every $m \in M$,

$$\varphi(m^*) \succeq^* \varphi(m)$$

and

$$\varphi(m^*) \succ^* \varphi(m)$$

whenever $m \notin \mu(\succeq)$.

Monotonicity (Lotteries)

Definition

Fix $\succeq$. The lottery f *First Order Stochastically Dominates (FOSD)* the lottery g with respect to $\succeq$ if, for every $x \in X$,

$$\sum_{\{x' \in X : x' \succeq x\}} f(x') \geq \sum_{\{x' \in X : x' \succeq x\}} g(x').$$

If there is strict inequality for at least one x then we say f *strictly FOSD* g with respect to $\succeq$.

Definition

An extension $\succeq^*$ of $\succeq$ is monotonic if $f \succeq^* g$ whenever f FOSD g w.r.t. $\succeq$ and $f \succ^* g$ whenever f strictly FOSD g w.r.t. $\succeq$.

$\mathcal{E}^{\mathrm{mon}}(\succeq)$ = The set of all monotonic extensions of $\succeq$.

Monotonicity and Dominance (Lotteries)

Lemma

*A mechanism φ is incentive compatible with respect to $\mathcal{E}^{\mathrm{mon}}$ if and only if, for every $\succeq$ and every $m \neq \mu(\succeq)$, $\varphi(\mu(\succeq))$ FOSD $\varphi(m)$ w.r.t. $\succeq$. (**Truth FOSD's Lies**)*

The RPS Mechanism (Lotteries)

Definition

A mechanism φ is an RPS mechanism if there exists a full-support probability distribution λ over $D = (D_1, \ldots, D_k)$ such that for every alternative $x \in X$,

$$\varphi(m)(x) = \sum_{\{i \,:\, m_i = x\}} \lambda(D_i).$$

RPS and Monotonicity (Lotteries)

Proposition

If only monotonic extensions are admissible ($\mathcal{E} \subseteq \mathcal{E}^{\text{mon}}$) then any RPS mechanism is incentive compatible.

Sketch of Proof:

- Lying in any decision problem shifts probability from more to less desired objects, hence any lottery that can be obtained by lying is FOSD by the lottery obtained by truth-telling
- Now apply previous lemma

What else is IC (with $\mathcal{E}^{\text{mon}}$)?

Example:

- $D_1 = \{x, y\},\ D_2 = \{x, z\},\ D_3 = \{y, z\}$
- Consider the mechanism φ that puts probability of 0.8 on the revealed most preferred object and 0.2 on the revealed second-best (for $m \in M_R$)
- φ is IC but not an RPS mechanism (even when restricted to M_R)
- $E = \{x, y, z\}$ is SI
- $\lambda(D_1) = \lambda(D_2) = \lambda(D_3) = 0.2,\ \lambda(E) = 0.4$ generates φ

Lesson: We may put weight on surely identified sets

What else is IC (with $\mathcal{E}^{\text{mon}}$)?

Example:

- $D_1 = \{x, y\}, \ D_2 = \{x, z\}, \ D_3 = \{y, z\}$
- Consider the mechanism φ that puts probability of 0.6 on the revealed most preferred object and 0.4 on the revealed second-best (for $m \in M_R$)
- φ is IC but not an RPS mechanism (even when restricted to M_R)
- $E = \{x, y, z\}$ is SI
- $\lambda(D_1) = \lambda(D_2) = \lambda(D_3) = 0.4, \ \lambda(E) = -0.2$ generates φ

Lesson: We may put *negative* weights on surely identified sets

Note: $\lambda(D_1) = \lambda(D_2) = \lambda(D_3) = 0.6, \ \lambda(E) = -0.8$ generates a non-IC mechanism

WSS Mechanisms

Definition

A mechanism $\varphi : M \to \Delta(X)$ is a *weighted set-selection (WSS) mechanism* if there exists some $\lambda : SI(D) \to \mathbb{R}$ such that for every rationalizable $m \in M_R$ and every $x \in X$,

$$\varphi(m)(x) = \sum_{\{E \in SI(D) \ : \ \max(E|m) = x\}} \lambda(E).$$

RPS $\subset$ RSS

Switch Positivity

Definition

A WSS mechanism φ (with associated weighting vector λ) satisfies *switch positivity* if, for every $x, y \in X$ and $A \subseteq X \setminus \{x, y\}$ it holds that

$$\sum_{\{E \in SI(D)\,:\,\{x,y\} \subseteq E \subseteq A \cup \{x,y\}\}} \lambda(E) > 0$$

(provided the sum is not empty).

Characterization (Lotteries)

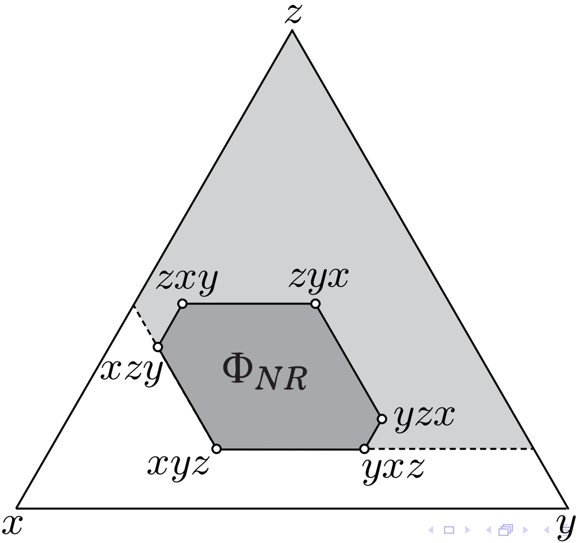
Theorem

(D, φ) is incentive compatible w.r.t. $\mathcal{E}^{\mathrm{mon}}$ if and only if

1. *φ is a WSS mechanism;*
2. *φ satisfies switch positivity;*
3. *if $m \in M_{NR}$ then $\varphi(m) \in conv(\varphi(M_R)) \setminus \varphi(M_R)$.*

'Proof'

$D_1 = \{x, y\}, \ D_2 = \{x, z\}, \ D_3 = \{y, z\}$



'Proof' (cont.)

$D_1 = \{x, y\}, \; D_2 = \{x, z\}, \; D_3 = \{y, z\}$

There is a normalized and convex 'capacity' $v : 2^{\{x,y,z\}} \to [0, 1]$ that 'represents' φ:

$$
\begin{aligned}
\varphi(\mu(\succeq))(a_1) &= v(a_1, a_2, a_3) - v(a_2, a_3) \\
\varphi(\mu(\succeq))(a_2) &= v(a_2, a_3) - v(a_3) \\
\varphi(\mu(\succeq))(a_3) &= v(a_3)
\end{aligned}
$$

$\{a_1, a_2, a_3\} = \{x, y, z\}$ and $\succeq$ ranks $a_1 \succeq a_2 \succeq a_3$.

'Proof' (cont.)

Each v can be represented uniquely by the 'unanimity capacities':

$$v(A) = \sum_{E \subseteq A} \lambda(E)$$

$$\begin{aligned}
\varphi(\mu(\succeq))(a_1) &= v(a_1, a_2, a_3) - v(a_2, a_3) = \sum_{a_1 \in E} \lambda(E) \\
\varphi(\mu(\succeq))(a_2) &= v(a_2, a_3) - v(a_3) = \sum_{a_2 \in E \subseteq \{a_2, a_3\}} \lambda(E) \\
\varphi(\mu(\succeq))(a_3) &= v(a_3) = \sum_{E \subseteq \{a_3\}} \lambda(E)
\end{aligned}$$

But this is exactly the required representation...

Note: v convex $\Leftrightarrow$ λ satisfies "switch positivity"

IC Mechanisms: Acts vs. Lotteries

- The lotteries framework can be seen as a restriction of the set of possible extensions $\succeq^*$
- The subject is indifferent between any two acts that generate the same lottery
- Incentive compatibility becomes a weaker requirement
- 'More' mechanisms are IC

IC Mechanisms: Acts vs. Lotteries

Definition

Say that $((\Omega, \mu), \phi)$ *generates* φ if, for each $m \in M$ and $x \in X$,

$$\varphi(m)(x) = \mu\left(\{\omega \in \Omega : \phi(m)(\omega) = x\}\right).$$

IC Mechanisms: Acts vs. Lotteries

Proposition

If ϕ is an IC act-mechanism (defined on some state space Ω), and μ is a full-support probability distribution on Ω, then the lotteries-mechanism φ generated by $((\Omega, \mu), \phi)$ is IC.

Proposition

Assume that φ is an IC lotteries-mechanism.

1. *If the associated weighting vector λ of φ is non-negative, then there exists an IC acts-mechanism ϕ (on some Ω) and a probability μ on Ω such that $((\Omega, \mu), \phi)$ generates φ on rationalizable messages.*
2. *If the associated weighting vector λ of φ contains negative elements, then φ cannot be generated by any IC acts-mechanism ϕ (even when restricted to rationalizable messages).*

Summary

- If paying all, need to assume no complementarities.
  - Fairness, portfolio, hedging, wealth, ...
- If RPS, need to assume monotonicity. Weak, unless 2-stage gambles.
  - Reduction & non-expected utility
  - Order Reversal & ambiguity aversion
- Other mechanisms may be IC for certain models.
- **Experimenter needs to decide for themselves!**

My (current) opinion:

- Use RPS
- Separate decisions as much as possible.
- Use separate, physical randomizing devices.

Other Issues

Other Monotonicity Violations:

- Decision Overload w/ Easy/Default Option (NCaT also questionable)
- Ex-Ante Fairness (NCaT also questionable)
- Irrational Diversification (NCaT also violated)

Issues Besides IC:

- Payment Inequality
- Payment Variance
- Confusion
- Irrational Choice

Theory is *not* explicitly dynamic! (But we can discuss.)

The End