ONLINE APPENDICES FOR "INCENTIVES IN EXPERIMENTS: A THEORETICAL INVESTIGATION" BY AZRIELI, CHAMBERS & HEALY

Appendix B. Modeling Games as Decisions

In this appendix we describe how one can move seamlessly between decision-theoretic analyses and game-theoretic analyses, justifying the application of our results to both settings. Let us start by considering an experiment in which the subject makes a *single decision* and then goes home. Suppose that this decision is a choice among uncertain prospects: A subject receives a bundle of Apples and Oranges contingent upon the color of a ball drawn from an urn. Specifically, suppose a subject needs to choose between prospects U and D defined by

	Red ball	Green ball
U	2A, 1O	3A, 2O
D	1A, 3O	2A, 3O

Here, A stands for Apples and O for Oranges. The subject has utility over bundles of Apples and Oranges, given by $u(3A, 2O)$. These bundles are the entries in the matrix. The subject also has preferences over uncertain prospects that pay a bundle of Apples and Oranges as a function of the realized color of the ball. These are the rows of the matrix (U and D). These latter preferences are obviously related to the utility over bundles as well as the subject's belief about the likelihood of each color. For example, under expected utility, $U \succeq D$ if and only if

$$p(\text{Red})u(2A, 1O) + p(\text{Green})u(3A, 2O) \geq p(\text{Red})u(1A, 3O) + p(\text{Green})u(2A, 3O).$$

But in general we need not assume expected utility, or even probabilistic beliefs. We just have some utility over bundles ($u(\cdot)$) and some preference over uncertain prospects ($\succeq$).

It doesn't really matter which preference we take as primitive. We could start with $\succeq$ and, if we know enough about $\succeq$ and we make some assumptions, we could derive the u (and p, under expected utility) that represents $\succeq$. Or we could start with u (and p if assuming expected utility) and we could derive $\succeq$. The latter approach is the standard in game theory and the usual assumption there is expected utility.[1] The former is standard in decision theory, where axioms on $\succeq$ would lead to a representation of $\succeq$ in terms of u and p, for example. But either approach is valid.

[1]In classical game theory, p would be assumed via a solution concept. But we describe below many papers in which p is taken as exogenous, modeled as part of the player's hierarchy of beliefs.

Now, what does it mean that a payment mechanism is incentive compatible in this single decision experiment? It means that, no matter what are the preferences of the subject over $\{U, D\}$, the payment he receives by choosing his favorite prospect is better (according to his preferences) than the payment he receives by choosing the other prospect. The key point is that, *what matters for incentive compatibility are the preferences of the subject over the set of objects from which he can choose, i.e. over the set* $\{U, D\}$. It does not matter what the corresponding utility over bundles of fruits would look like. And beliefs don't matter. And it does not matter whether u or $\succeq$ was taken as primitive.[2]

An incentive compatible mechanism is easy to construct in this example: The subject gets the prospect he chose. That is, when choosing U the subject will get the bundle (2A,1O) if the drawn ball is red and the bundle (3A,2O) if the drawn ball is green, and similarly when he chooses D.

Let us move on to an experiment in which subjects play a *single game* and then go home. For example, suppose they play a 2×2 game with payoff (in dollars) matrix given by

	L	R
U	\$2,\$1	\$3,\$2
D	\$1,\$3	\$2,\$3

Notice that, from the perspective of the row player, this is the same as the decision problem above, except Apples are now 'dollars for the row player', Oranges are now 'dollars for the column player', and the two states of the world (L and R) are chosen by another subject instead of an urn.

As discussed above, the primitive in standard game theory is usually the preferences of the players over strategy profiles, *i.e.* over the entries of the matrix. But what matters for incentive compatibility is the preference of each player over the objects he can choose from, *i.e.* the set of his own strategies. This preference over U and L is clearly related to the utility over strategy profiles (the entries in the matrix), as well as to the belief about the strategy of the opponent (which may depend on what he believes his opponent believes about his own play, and so on). But ultimately what matters for incentive compatibility is only whether the rows player prefers to play U or D (and, for the column player, whether she prefers L or R).

[2]Note that much information is lost in moving from utility over bundles to preferences over uncertain prospects. An experiment where a subject chooses either "U" or "D" can convey only one bit of information. It is not rich enough to learn about the subject's beliefs over Red and Green, or their utility over the four possible bundles. But this 'lost information' is not relevant for incentive compatibility. If the experimenter were interested in this information, it could be elicited via different decision problems; see Karni (2009), for example.

As in the decision problem experiment there is a simple way to incentivize subjects to reveal their preferences over their own strategies: The players get paid the dollar amounts corresponding to the chosen entry in the matrix. From the point of view of the row player, when he chooses U the outcome will be either $(\$2, \$1)$ or $(\$3, \$2)$ contingent on whether the column player chose L or R (and similarly when he chooses D).

Notice that one could assume a different model of game play if desired (for example, a more psychological or behavioral model such as Level-k), and our framework would still apply as long as the model ultimately generates a ranking over the player's own strategies.[3] In the end, this is all that matters for incentive compatibility.

To complete the discussion, consider the case of multiple games. Specifically, the subject plays a sequence of games and gets no feedback between the rounds (see the other appendix for the case with feedback). The subject has preferences over his own strategies in each game. Then, just as in the case of multiple decision problems, if the experimenter uses the RPS mechanism to determine which game will be chosen for payment, and if the subject's "extension" of their preferences (to the space of acts that select a chosen strategy from one randomly-selected game) satisfies monotonicity (as defined in the paper), then it is incentive compatible for each player in each game to choose the strategy that he prefers most in that game. Similarly, if NCaT is satisfied then the pay-all mechanism is incentive compatible. There is no need to develop a separate theory for games, since *for the purpose of avoiding distortions in choice due to complementarities*, all that matters is $\succeq$ over strategies. In other words, from the perspective of incentive compatibility, games are no different than decision problems.

One might wonder if there is a substantive difference in having the states of the world be chosen by another player, rather than by an urn. In this setting the row player can have uncertainty not just about the column player's action, but also beliefs about the column player's beliefs about the row player's action, and beliefs about those beliefs, and so on. Thus, we can construct an entire hierarchy of beliefs for the row player. Brandenburger and Dekel (1993) show how one can define a type space (often called a 'strategies-based type space', or 'epistemic type space') that coherently models all possible hierarchies of beliefs of the row player (and a similar type space for the column player). And Epstein and Wang (1996) show how it can be done even without expected utility or probabilistic sophistication. Note that in these sorts of type spaces, the primitive uncertainty is over the opponent's strategies, and beliefs are taken as exogenous rather than derived from an assumed solution concept.

Though we could construct such a type space, most of it is irrelevant for incentive compatibility. In the end, the row player's utility over bundles (u) and their first-order

[3]In the Level-k model, for example, each Level-k type would rank his own strategies based on their expected payoff given his belief that the opponent will play the Level-$(k-1)$ strategy.

belief about the column's action (p) determine a preference over U and D. It is this preference that matters for incentive compatibility.

Finally, this construction is not novel. Many authors have studied games using the strategies-based type space framework.[4] Dekel and Siniscalchi (2015) have a lengthy chapter in the *Handbook of Game Theory* that surveys many results derived using a strategies-based type space. For example, Brandenburger and Dekel (1987, 1993) and Aumann (1987) study rationalizability and correlated equilibrium in this setting.[5] Aumann and Brandenburger (1995) derive sufficient conditions on rationality and beliefs for players to play Nash equilibrium. Brandenburger et al. (2008) similarly characterize the iterated elimination of weakly dominated strategies. In extensive-form games, Ben-Porath (1997), Battigalli and Siniscalchi (1999, 2002), and Battigalli and Friedenberg (2012) study the link between common knowledge of rationality and backwards induction. Geanakoplos et al. (1989) and Battigalli and Dufwenberg (2009) use a strategies-based type space to study psychological games.

Appendix C. A Dynamic Model with Feedback and Learning

In this appendix we provide a generalization of our model to the dynamic setting in which the subject announces choices sequentially, receives feedback after each period, and may 'learn' (or, more generally, have changing preferences) across time. If the setting is a sequence of games, we assume the subject plays against different opponents each period; recall that a repeated game against a single opponent should be viewed as one large decision problem. Although the notation here is more complex, the result is similar to that in the paper. For brevity we only study monotonicity and the RPS mechanism; a similar generalization of NCaT and the pay-all mechanism would be fairly trivial.

We introduce our framework first through an example.

Example 1 (*Ultimatum Games: Direct Response Method*)**.** A subject ('she') will play two mini-ultimatum games, each against different opponents (both 'he'). Her options in each period $t \in \{1, 2\}$ are to offer either an Equal split or an Unequal split of a \$10 pie. Thus, each period-t decision problem is given by $D_t = \{E_t, U_t\}$. If she chooses $m_t = U_t$ then her period-t opponent can choose Accept or Reject. If he accepts then she gets \$9

[4]And, more generally, the decision-theoretic approach to game theory had a long and rich history even before strategies-based type spaces were developed. Rationalizability (Bernheim, 1984; Pearce, 1984) was motivated by studying individual-level rationality in the context of a game. Von Neumann and Morgenstern (1944) developed expected utility theory expressly for use in games, an exercise further developed by Luce and Raiffa (1957).

[5]The papers by Aumann (1987) and Brandenburger and Dekel (1987, 1993) define and study rationalizability in the context of a strategies-based type space, assuming expected utility. Epstein (1997) defines and studies rationalizability in a strategies-based type space that does not assume expected utility (Epstein and Wang, 1996); this most closely matches our framework.

θ	A_1A_2	A_1R_2	R_1A_2	R_1R_2
s_1	E_1	E_1	E_1	E_1
s_2	E_2	E_2	E_2	E_2
s'_1	U_1	U_1	U_1	U_1
s'_2	U_2	U_2	E_2	E_2

TABLE I. The Θ-acts generated by the plans s and s' in each period.

and he gets \$1. If he rejects they both get \$0. But if the subject chooses $m_t = E_t$ then the opponent is given no choice; they both get \$5 regardless.

Let $\Theta = \{A_1, R_1\} \times \{A_2, R_2\}$ be the set of states that captures the uncertainty regarding what the opponents *would* do if the subject were to offer an unequal split, where A_t indicates that the opponent in period t would accept an unequal split and R_t indicates they would reject. For example, this might represent whether each opponent has selfish or spiteful preferences. In period 1 the subject has no information about the realized $\theta \in \Theta$. But at the end of period 1 she might receive feedback about her period-1 opponent's choice, and this might affect her preferences in period 2. If she chooses $m_1 = U_1$ then she finds out if the first opponent accepts (A_1) or rejects (R_1). In other words, by choosing U_1 the subject would know whether the realized state θ is in the set $\{(A_1, A_2), (A_1, R_2)\}$ or in the set $\{(R_1, A_2), (R_1, R_2)\}$. But if she chooses $m_1 = E_1$ then she would get no information; she would still face the same uncertainty about θ as before the first period (i.e. $\theta \in \Theta$).

The subject does not pick a fixed vector of messages $m = (m_1, m_2)$; instead, she picks an entire contingent *plan* of what messages she will send, depending on what feedback she observes. Plans therefore are functions, which we will denote by s. As she executes her plan she will ultimately submit a message vector m, but what vector she submits depends on what feedback she receives during the experiment, which in turn depends on the realized state θ as well as on the subject's own previous choices. For example, one possible plan s is to choose E_1 in the first period and E_2 in the second period. Another plan s' chooses U_1 in the first period, and then chooses E_2 if R_1 was observed and U_2 if A_1 was observed.

Notice that any plan s generates a 'Θ-act' in each period t, that is a mapping $s_t : \Theta \to D_t$. The choice object $s_t(\theta) \in D_t$ indicates what the subject would choose in period t when she plays according to plan s and the realized state is θ. The Θ-acts generated by the plans s and s' of the previous paragraph in each period are shown in Table I.

Since by choosing a plan the subject essentially chooses a Θ-act in each period, we model the primitive preferences of the subject as being defined over Θ-acts. Suppose that the subject prefers the Θ-act s_1 (the constant E_1, see Table I) over the Θ-act s'_1 (the constant U_1). This can be the case for example if her prior beliefs are that the subject in the first period is likely to reject an unequal split. Suppose also that the subject prefers s'_2 over s_2, with a possible rationale being that the subject believes that the responses of the two opponents are positively correlated, so she would be better off offering an unequal split in the second period if and only if the first opponent accepted such an offer.

A plan is truthful if it chooses an optimal Θ-act in each period from the set of feasible acts in that period. It is important to note that in the current example the set of acts available to the subject in the second period depends on her choice in the first period: If she chooses E_1 then only constant acts are available in period 2 (since there will be no feedback); but if she chooses U_1 then the set of available acts in period 2 is larger and contains acts that vary with the response of the first opponent. In period 1 the only two available acts are s_1 and s'_1, so a truthful plan must choose s_1 initially. But if she chooses s_1 then in the second period only constant Θ-acts would be available. Assuming that with no further information the subject still prefers an equal split, her optimal choice now is E_2, so the truthful plan must choose s_2. In other words, the plan s is the unique truthful plan in this example.

Assume that the RPS mechanism is used for payment. As in the static framework, there is a state space $\Omega = \{\omega_1, \omega_2\}$ and if ω_t is realized then the announced period-t choice m_t is paid. Formally, $\phi(m)(\omega_t) = m_t$. If the subject plays according to the truthful plan s then, for every θ, she receives E_1 if ω_1 is realized and E_2 if ω_2 is realized. In other words, she receives the Θ-act s_1 in state ω_1 and the Θ-act s_2 in state ω_2. If instead the subject plays according to s' then she receives the Θ-act s'_1 in state ω_1 and the Θ-act s'_2 in state ω_2.

As in the static framework, the subject has an extension $\succeq^*$ over payment acts (i.e., acts from Ω to Θ-acts) that satisfies monotonicity. Thus, incentive compatibility can only be ensured by requiring that the truthful plan generates a payment act which dominates any other payment act generated by any non-truthful plan. But notice that the RPS mechanism described in the previous paragraph does not satisfy this property: The payment act generated by the plan s' is not dominated by the payment act generated by the truthful plan s, since $s'_2 \succ s_2$. This implies that the RPS mechanism is not incentive compatible in this example.

Incentive compatibility fails in Example 1 because the feedback structure provides a clear incentive for experimentation: by choosing the less-preferred option U_1 in the first period, the subject is able to learn more valuable information on which she can condition her choice in period 2. This allows her to achieve more desirable outcomes in certain contingencies. This is similar to a multi-armed bandit problem: A subject might pull the riskier arm in earlier periods just so she can have more information when making her later-period choices.

In our view, any experiment that provides these experimentation incentives should not be analyzed as a sequence of independent decisions. Instead they should be analyzed as one large decision problem, much like a repeated game against a single opponent. In fact, multi-armed bandit problems are always modeled theoretically as one large dynamic decision problem—not a sequence of independent problems—and so any experiment involving multi-armed bandits should be analyzed as one large problem as well.

The experimentation incentives comes entirely from the subject's ability to change their feedback by changing their message. Thus, we can rule out experimentation incentives by requiring that feedback never depends on the subject's announcements. We now present a modified version of the experiment in Example 1 which eliminates the incentive to experiment.

Example 2 (*Ultimatum Games: Strategy Method*)**.** The subject plays the same two mini-ultimatum games as in Example 1. Again, the opponent has no choice if an equal split ($m_t = E_t$) is offered, but in this version the experimenter asks the opponent what they would choose if $m_t = U_t$ *before* m_t is chosen. In the experimental literature, this is known as the 'strategy method'.[6] At the end of the period this information is revealed to the subject regardless of what she chose. Thus, regardless of what the subject chose in the first period she would know before the second period whether the state θ is in the set $\{(A_1, A_2), (A_1, R_2)\}$ or in the set $\{(R_1, A_2), (R_1, R_2)\}$.

Recall that the subject prefers the Θ-act s_1 over s_1', and suppose that s_2' is not only preferred to s_2 but also to any other Θ act that depends only on the response of the first opponent (see Table I). Then the unique truthful plan is to choose E_1 in period 1, and to condition the choice in period 2 on the (hypothetical) response of the opponent from period 1. This plan chooses s_1 in period 1 and s_2' in period 2. Note that this plan was not feasible in the direct response version of the experiment.

The RPS mechanism described in the previous example is now incentive compatible. Under the truthful plan the payment-act pays s_1 in state ω_1 and s_2' in state ω_2; any other feasible payment-act is dominated by this state-by-state. Thus, under monotonicity, the subject strictly prefers to follow the truthful plan.

We now build a general framework and prove that if experimentation is not possible then the RPS mechanism is essentially the only incentive compatible mechanism.

[6]The opponent's choice is incentivized because, if the subject actually chooses U_t, then the opponent's choice is implemented.

In each period $t \in \{1, \dots, T\}$ the subject chooses $m_t \in D_t \subseteq X$. To simplify the exposition assume that for each t and $t' \neq t$, D_t and $D_{t'}$ are disjoint.[7] Let $m = (m_1, \dots, m_T) \in M = \times_t D_t$. As in the example, there is a state space Θ which captures the uncertainty that the subject faces in the experiment (*e.g.*, the preferences of her opponents). The subject has a preference $\succeq$ over the set X^Θ of Θ-*acts*.[8]

After making a choice in period t the subject receives feedback o_t that depends on her choice m_t as well as on the realized $\theta \in \Theta$. Specifically, let $\sigma_t : D_t \times \Theta \to O_t$ be the *period-t feedback function*. A *period-t history* is a vector of the form $(o_1, \dots, o_{t-1})$ that describes all observations prior to period t. The set of all such histories is denoted H_t, with the convention that $H_1 = \{h_1\}$ where $h_1 = (\emptyset)$.

A *plan* is a vector $s = (s_1, \dots, s_T)$, where for each t the function s_t maps H_t into D_t. Given plan s, each state θ generates a sequence of messages that, with abuse of notation, we denote by $s(\theta) = (s_1(\theta), \dots, s_T(\theta))$. Formally, let $s_1(\theta) = s_1(\emptyset)$ for every θ, and for every $t \geq 2$ defined $s_t(\theta) = s_t(\sigma_1(s_1(\theta), \theta), \dots, \sigma_{t-1}(s_{t-1}(\theta), \theta))$. Notice that we can therefore view each s_t as a mapping from Θ into D_t, that is, as a Θ-act.

Given a plan s, say that two states are *indistinguishable in period* t if they generate the same sequence of observations in all periods prior to t. Formally, θ, θ' are indistinguishable in period t if $(\sigma_1(s_1(\theta), \theta), \dots, \sigma_{t-1}(s_{t-1}(\theta), \theta)) = (\sigma_1(s_1(\theta'), \theta'), \dots, \sigma_{t-1}(s_{t-1}(\theta'), \theta'))$. This is clearly an equivalence relation over Θ, and we let A_s^t be the collection of all Θ-acts with range D_t that are measurable with respect to the partition associated with it. Simply put, A_s^t is the set of Θ-acts available to the subject in period t if she followed the plan s in all past periods.

Definition 1 (**Truthful Plan**)**.** A plan s^* is truthful if $s_t^* \succeq a$ for every $t = 1, \dots T$ and for every $a \in A_s^t$. Let $S^*(\succeq)$ be the set of all truthful plans at $\succeq$.

As in the static framework, a payment mechanism is a mapping $\phi : M \to X^\Omega$, where Ω is the state space of the mechanism's randomizing device. But notice that here the messages m would typically depend on the feedback received by the subject, which in turn depends on the realized θ. More specifically, a subject which follows a plan s would be paid $\phi(s(\theta)) \in X^\Omega$ when θ is realized. Put differently, if the realized state of the randomization device is ω then the subject is paid the Θ-act $\phi(s(\cdot))(\omega)$, which pays $\phi(s(\theta))(\omega)$ in each state θ. We call a mapping $f : \Omega \to X^\Theta$ a *payment act*, and we denote by $\mathcal{F}$ the set of all such acts. For example, $\phi(s(\cdot)) \in \mathcal{F}$. As in the static framework we assume that the subject has an extension $\succeq^*$ over $\mathcal{F}$ of her underlying preference $\succeq$.

[7] If the same object x appears in r different decision problems, redefine X to contain r copies of x.

[8] To make the connection to our static framework clearer and to save on notation we use as primitive the preferences of the subject over X^Θ. One could instead start with a collection of 'conditional preferences' $\{\succeq_E\}_{E \subseteq \Theta}$, where $\succeq_E$ is a preference over X^E representing the subject's ranking conditional on learning that event E occurred.

In particular, $\succeq^*$ is used to rank payment acts and, therefore, determines whether the truthful plans are most-preferred.

We can now give the definitions of incentive compatibility, dominance, and monotonicity adapted to our dynamic framework.

Definition 2 (Incentive Compatibility). A mechanism ϕ is incentive compatible if, for every preference $\succeq$, every admissible extension $\succeq^*$, every truthful plan $s^* \in S^*(\succeq)$, and every plan s, $\phi(s^*(\cdot)) \succeq^* \phi(s(\cdot)))$, with $\phi(s^*(\cdot)) \succ^* \phi(s(\cdot)))$ whenever $s \notin S^*(\succeq)$.

Definition 3 (Dominance). Given $\succeq$, payment-act f dominates payment-act g (denoted $f \sqsupseteq g$) if $f(\omega) \succeq g(\omega)$ for every $\omega \in \Omega$. If $f \sqsupseteq g$ and there is some $\omega \in \Omega$ such that $f(\omega) \succ g(\omega)$ then f strictly dominates g (denoted $f \sqsupset g$).

Definition 4 (Monotonicity). Given $\succeq$, an extension $\succeq^*$ satisfies monotonicity if $f \sqsupseteq g$ implies $f \succeq^* g$, and $f \sqsupset g$ implies $f \succ^* g$.

Next, we recall the definition of the RPS mechanism.

Definition 5 (RPS Mechanisms). Payment mechanism $\phi : M \to X^\Omega$ is a *random problem select (RPS)* mechanism if there is a fixed partition $\{\Omega_1, \ldots, \Omega_T\}$ of Ω with each Ω_t non-empty such that, for each $t \in \{1, \ldots, T\}$ and $m \in M$,

$$\omega \in \Omega_t \text{ implies } \phi(m)(\omega) = m_t.$$

In the static framework it was possible that $D_t \cap D_{t'} \neq \emptyset$. Thus, we had to deal with the possibility of non-rationalizable messages and the concept of surely-identified sets. This led us to define the family of RSS mechanisms, and we found that the incentive compatible mechanisms were a subset of that class. In the dynamic framework each D_t is disjoint, meaning there are no non-rationalizable messages and the only surely identified sets are the decision problems themselves plus the singleton sets. Thus, the appropriate notion of an RSS mechanism in the dynamic framework simply boils down to an RPS mechanism that can also pay fixed payments.

Definition 6 (RPS Mechanisms with Fixed Payments). A payment mechanism ϕ is a *random problem selection mechanism with fixed payments* (RPS-FP) if there is a set of k choice objects $\{x_1, \ldots, x_k\} \subseteq X$ (called the *fixed payments*, with the possibility that $k = 0$) and a partition $\{\Omega_1, \ldots, \Omega_{T+k}\}$ of Ω with each Ω_t non-empty such that, for each $t \in \{1, \ldots, T+k\}$ and $m \in M$,

$$\omega \in \Omega_t \;\; \text{implies} \;\; \begin{cases} \phi(m)(\omega) = m_t & \text{if} \quad t \leq T \\ \phi(m)(\omega) = x_{t-T} & \text{if} \quad t > T \end{cases}$$

If $k = 0$ then an RPS-FP mechanism reduces to an RPS mechanism.

Finally, we need to formalize the idea that the subject cannot influence the information she receives through her own choices.

Definition 7 (No Experimentation). The feedback functions $\{\sigma_t\}_t$ *prohibit experimentation* if $\sigma_t(m_t,\theta) = \sigma_t(m'_t,\theta)$ for every t, every θ, and every $m_t, m'_t \in D_t$.

Theorem 1. Consider an experiment in which the feedback functions prohibit experimentation. If only monotonic extensions are admissible then any RPS mechanism is incentive compatible. Furthermore, if the underlying preferences $\succeq$ over X^Θ are strict and all monotonic extensions are admissible then a mechanism is incentive compatible if and only if it is an RPS-FP mechanism.

Proof. Since the feedback functions prohibit experimentation, the set A^t_s of acts available to the subject in period t is independent of the plan s. We can therefore omit s from the notation and denote by A^t the set of feasible acts in period t.

Notice that we are now in a very similar set-up to the static framework of the paper: The subject chooses an element of A^t in each period t; truthfulness means choosing a maximal element in each period; extensions are preferences over payment acts that map Ω into choice objects; and incentive compatibility means that being truthful results in a payment act that is preferred to any payment act obtained by lying for every admissible extension. However, the difference from the static framework is that the sets A^t and the choices from these sets are only in the subject's mind; the experimenter only observes the realized choices m_t made from the sets D_t, and so the mechanism can only depend on these choices.

Let us call the static framework in which the subject chooses from the sets $\{A^t\}$ and the mechanism pays based on these choices the "hypothetical static framework". We say that a mechanism ϕ in the dynamic framework is equivalent to a mechanism ϕ' in the hypothetical static framework if $\phi(m_1,\ldots,m_T) = \phi'(s_1,\ldots,s_T)(\theta)$ whenever $m_t = s_t(\theta)$ for all t. It is easy to see that ϕ is incentive compatible in the dynamic framework if and only if it is equivalent to an incentive compatible mechanism ϕ' of the hypothetical static framework.

Now, it follows from Proposition 1 in the paper that any RPS mechanism in the hypothetical static framework is incentive compatible when the subject's extension satisfies monotonicity. Also, any RPS mechanism of the dynamic framework is equivalent to an RPS mechanism of the hypothetical static framework. This proves the first part of the Theorem.

To prove the second part notice that since the sets $\{D_t\}$ are disjoint, the sets $\{A^t\}$ are disjoint as well. It follows that the only surely identified sets (see Definition 4 in the paper) in the hypothetical static framework are the sets $\{A^t\}$ themselves and the singletons. Thus, we know from Theorem 1 in the paper that if all monotonic extensions are admissible then any incentive compatible mechanism in the hypothetical static framework is an RSS mechanism that in each state ω pays either the chosen element from A^t or some fixed Θ-act that does not depend on the choices made by the subject. Again, it is easy to check that the mechanisms in the dynamic framework that are equivalent to such mechanisms in the hypothetical static framework are exactly RPS-FP mechanisms. This concludes the proof. □

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