

INCENTIVES IN EXPERIMENTS: A THEORETICAL ANALYSIS[†]

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ABSTRACT. Experimental economists currently lack a convention for how to pay subjects in experiments with multiple tasks. We provide a theoretical framework for analyzing this question. Assuming statewise monotonicity and nothing else, we prove that paying for one randomly-chosen problem—the *random problem selection* (RPS) mechanism—is essentially the only incentive compatible mechanism. Paying for every period is similarly justified when we assume only a ‘no complementarities at the top’ (NCaT) condition. To help experimenters decide which is appropriate for their particular experiment, we discuss empirical tests of these two assumptions.

Keywords: Experimental design; decision theory; mechanism design.

JEL Classification: C90; D01; D81.

I. INTRODUCTION

Incentivizing subjects has long been a key tenet of experimental economics. But when subjects are asked to make multiple decisions, the way in which they are incentivized can distort their actual choices. Several such distortions have been observed in previous studies. For example, if all decisions are paid, subjects can become more risk-seeking after losses and more risk-averse after gains (Thaler and Johnson, 1990; Weber and Zuchel, 2005; Ackert et al., 2006). They may also recognize that their risk is diversified when multiple lotteries are paid, driving them to choose riskier lotteries in each decision (Laury, 2005). In auctions, bidders who have won in previous rounds tend to bid less aggressively in later rounds (Kagel and Levin, 1991; Ham et al., 2005). In some

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cases, one choice is clearly used as a hedge against risk in another (Blanco et al., 2010; Armantier and Treich, 2013).

It has been suggested that paying for only one randomly-chosen decision—what we call the random problem selection (RPS) mechanism—will be incentive compatible, meaning it will prevent such distortions.¹ But that mechanism may generate new types of distortions when subjects integrate their decisions into one large lottery. This possibility was noted by Holt (1986), Karni and Safra (1987), and Segal (1988), and such distortions have been observed by Cox et al. (2014a), Freeman et al. (2016), Harrison and Swarthout (2014), and Brown and Healy (2014).

Though aware of these concerns, experimental economists have not yet settled on an accepted convention for paying subjects. Perhaps this is because different settings call for different mechanisms. Or because the appropriate mechanism depends on the theory being tested. But we find that most authors do not attempt to justify their chosen payment mechanism in their manuscript. If the mechanism choice is deliberate, its rationale is not frequently given.

To provide some evidence for these claims, we surveyed all experimental papers published in 2011 in the ‘top five’ economics journals and in the field journal *Experimental Economics*. We counted the payment mechanisms used (Table I) and the extent to which the authors discussed the incentive properties of their chosen mechanism (Table II).² Of the 32 experiments with multiple tasks, 56% pay for every decision, 25% use the RPS mechanism, and 13% pay for multiple randomly-selected decisions. The frequencies differ somewhat between individual-choice experiments and game experiments, but clearly no convention exists in either setting. By our count, 29% of papers with multiple tasks do not even mention which mechanism was used—one must look to the experiment instructions or an online appendix to see how subjects were paid. An additional 48% describe the payment mechanism, but do not justify its appropriateness in the given experiment.

¹The possibility of distortions was recognized by Wold (1952) and Savage (1954, the ‘hot man’ example on p.29). The RPS mechanism was suggested by Allais (1953), and by W. Allen Wallis in a person communication with Savage. Early applications include Becker et al. (1964, the ‘BDM’ mechanism), Yaari (1965), and Grether and Plott (1979). It is most often called the random lottery incentive mechanism (Safra et al., 1990); we adopted ‘RPS’ from Beattie and Loomes (1997) (and, indirectly, from Holt, 1986) because our framework does not require randomness to be represented by objective lotteries.

²This survey is meant to be representative, not exhaustive. It includes both lab and field experiments. Most field studies only have subjects engage in one task. If subjects play a game repeatedly against fixed partners and are paid for every period, we count it as ‘Only 1 Task’. ‘Some Random’ refers to experiments that randomly pay 2, 3, 4, or 5 decisions, with some of the paid decisions possibly being non-random. Among ‘Some Random’ and ‘All Paid’ experiments, 18 showed subjects outcomes every period, while the other six only showed outcomes at the end; all nine of the ‘Top 5’ experiments showed outcomes every period. The rank-based payment mechanism gives payments to players based on their relative hypothetical earnings summed across all decisions. The one unpaid experiment used children as subjects.

Payment Mechanism:	Only 1 Task	None Paid	One Random	Some Random	All Paid	Rank-Based	Total
Individual Choice Experiments							
‘Top Five’ Journals	7	0	3	1	3	0	14
<i>Experimental Economics</i>	3	0	1	0	2	0	6
Muti-Person (Game) Experiments							
‘Top Five’ Journals	9	0	1	0	8	0	18
<i>Experimental Economics</i>	8	1	3	3	5	1	21
Totals	27	1	8	4	18	1	59

TABLE I. Payment mechanisms used among all (lab and field) experimental papers published in the ‘top five’ journals and *Experimental Economics* in 2011. The ‘top five’ journals are *Journal of Political Economy*, *Quarterly Journal of Economics*, *Econometrica*, *American Economic Review*, and *The Review of Economic Studies*.

	Mechanism Not in Paper	Discussion of Incentives			Clearly I.C.	Total
		None	Brief	Extensive		
Individual Choice Experiments						
‘Top Five’ Journals	1	6	0	1	0	7
<i>Experimental Economics</i>	0	2	0	1	0	3
Muti-Person (Game) Experiments						
‘Top Five’ Journals	6	9	0	0	0	9
<i>Experimental Economics</i>	2	7	4	1	0	12
Totals	9	24	4	3	0	31

TABLE II. Among 2011 papers with an incentivized multiple-task experiment, the number that do not contain a description of the payment mechanism used, the extent to which the authors discuss the incentive properties of their mechanism, and whether or not the incentive compatibility of the payment mechanism is clear from the author’s assumptions.

Only 23% of authors explicitly justify their mechanism within the manuscript, and most do so only briefly.

Our goal in this paper is to develop a formal framework for the analysis of incentives in experiments, and then apply it to understand when the commonly-used payment mechanisms mentioned above are incentive compatible. Perhaps the most important insight from our analysis is that one must carefully distinguish between the set of choice objects X and the set of payment objects $P(X)$ in an experiment. For example, if subjects choose among simple lotteries and the RPS mechanism is used, then X is a set of simple lotteries and $P(X)$ is a set of compound lotteries. Subjects announce choices from X , but actually receive payments in $P(X)$. Thus, incentive compatibility depends *crucially* on their preferences over $P(X)$. But authors design experiments to learn about preferences over X , and so their theories and hypotheses rarely extend to $P(X)$. When they don’t, we cannot evaluate whether or not the experiment is incentive compatible under the author’s assumptions.

Returning to our survey of 2011 papers, we found that in *no* paper did the theory or hypotheses extend (trivially) to $P(X)$. Thus, in no case was incentive compatibility of the experiment (or lack thereof) clear from the author’s assumptions.

The framework we develop is very general. First, no structure is assumed on X . It could include consumption goods, objective lotteries, ambiguous acts, announcements of preferences, or strategies in a game. Second, we place no restrictions on how subjects evaluate gambles; our framework does not require expected utility, or even that gambles be assigned subjective probabilities (Machina and Schmeidler, 1992).

Our first result is that incentive compatibility is never free: No mechanism is incentive compatible without assumptions on preferences over $P(X)$. In other words, for a given payment mechanism, if every preference over $P(X)$ is admissible then the mechanism is not incentive compatible. Second, the RPS mechanism is incentive compatible if preferences over $P(X)$ satisfy a (statewise) monotonicity condition. This simply requires that if, in every state of the world, gamble A pays something more preferred than gamble B, then the decision maker chooses A over B. If this is our only assumption then the RPS mechanism is, in practice, the only incentive compatible mechanism. Third, we show that the pay-all mechanism is incentive compatible if a ‘no complementarities at the top’ (NCaT) condition on preferences over $P(X)$ holds. NCaT requires that if we take a person’s favorite object from several different menus and combine them into a bundle, that bundle must be their most-preferred of all possible bundles. If we assume NCaT and nothing else, then the pay-all mechanism is, in practice, the only incentive compatible mechanism.

Given these results, an experimenter choosing a payment mechanism needs only to decide whether these assumptions are likely to be valid in her setting. This decision could be based on previous experiments that test these assumptions in a similar setting. We discuss previous experimental evidence in Section V. If these sources are not sufficiently convincing, the experimenter may wish to collect new evidence on the question; we describe a procedure for doing so in Section V.

At first glance, our results appear to contradict some of the previous literature. In particular, several authors have provided examples of reasonable-looking preferences (or classes of preferences) for which the RPS mechanism is not incentive compatible. Examples include Holt (1986), Karni and Safra (1987), Oechssler and Roomets (2014), Baillon et al. (2014), and others. Given our results, it must be that these reasonable-looking preferences in fact violate monotonicity. Indeed, they do: Each of them assumes some form of a ‘reduction’ axiom (for example, reduction of compound lotteries), and it is well-known that monotonicity and reduction together imply linear indifference curves over X . If choices are over lotteries (Holt, 1986; Karni and Safra, 1987), this means the

RPS mechanism requires the independence axiom on X when reduction is assumed. If choices are over ambiguous acts (Oechssler and Roomets, 2014; Baillon et al., 2014), it requires ambiguity neutrality when reduction is assumed. But without reduction, no structure on preferences over X is required for incentive compatibility. Thus, experimenters choosing this mechanism should carefully consider whether or not reduction may be present in their setting. We discuss this in detail in Section III.

Although our theory is framed as decision-theoretic, we describe in an appendix how the structure applies equivalently to game-theoretic experiments as well. If the subject plays multiple games against different opponents and receives no feedback between periods, then the RPS mechanism can be used just as in decision-theoretic settings. If, however, the subject plays multiple periods against the same opponent, then they are engaged in a repeated game. In that case it is inappropriate to attempt to view each period as independent, and therefore inappropriate to try to incentivize each period's choice as independent. Instead, the researcher should view the entire repeated game as one large decision problem.³

Feedback between decisions can also cause issues for incentive compatibility when the subject's choices can alter the feedback they receive. In individual decision problems this gives rise to multi-armed bandit problems and generates incentives for experimentation. In games, subjects might test certain strategies against early opponents to see what would be profitable against later opponents. These settings should also be analyzed as one large decision problem, making the RPS inappropriate. There is, however, a way for the experimenter to circumvent the experimentation incentive: By giving the subject feedback on what *would have* happened for every choice the subject could have made, the subject no longer can affect what they learn through their choices. In bandit problems, this means reporting what would have happened if each arm had been pulled. In games, the experimenter would use the 'strategy method' to elicit how the opponent would have reacted to each possible action of the subject, and feed back *all* of this information to the subject after each decision. With the experimentation incentive eliminated, it becomes appropriate to view each decision problem as independent and to consider using the RPS mechanism. In the appendix we show that indeed it is incentive compatible under an appropriate definition of monotonicity, as long as experimentation incentives are eliminated.

There are many related discussions of experimental incentives that are complementary to our analysis, because they focus on incentives within a single decision problem but not across problems. In one of the earliest papers on this topic, Smith (1976) describes how to use monetary payments to induce a desired utility function over fictitious

³If the subject plays multiple repeated games, then the RPS mechanism would call for one repeated game to be chosen at random for payment.

goods in a single experimental market. Similarly, Smith and Walker (1993), Camerer and Hogarth (1999), and others study the impact of increasing monetary stakes within a single decision. In a single repeated game, Charness and Genicot (2009), Fischer (2011), Chandrasekhar and Xandri (2011), Frechette et al. (2011), and Sherstyuk et al. (2011) all recognize that paying for only the final period induces the correct incentives regardless of subjects' risk preferences. All of these studies focus on proper incentives within a single decision problem, while our work shows how to provide incentives across multiple decision problems.

In terms of our theoretical contribution, we employ a standard mechanism design approach that allows for random mechanisms, so our paper is closely related to work on random mechanisms by Gibbard (1977), Barbera (1977), and Barbera et al. (1998). These authors require only weak incentive compatibility and study mechanisms where the entire preference relation is announced. Our innovations are focusing on strict incentive compatibility, and eliciting only the most-preferred items from an exogenous list of menus (or, 'decision problems'), rather than an entire preference relation.⁴

II. THE GENERAL FRAMEWORK

The set of possible *choice objects* is given by X . No structure on X is assumed; examples of possible $x \in X$ include consumption goods, lotteries, ambiguous urns, strategies in a game, behavioral strategies in an extensive-form game, labor decisions in the field, donations to charity, and streams of future consumption.

The subject has a preference relation \succeq over X . We make no assumptions on \succeq other than completeness and transitivity. Preferences need not be 'selfish', and need not conform to any decision-theoretic model like expected utility. The \succeq -*dominant* elements of any set $E \subseteq X$ are denoted by

$$\text{dom}_{\succeq}(E) = \{x \in E : (\forall y \in E) x \succeq y\}.$$

Where applicable, strict preference (the asymmetric part of \succeq) is denoted by $>$.

The researcher has an exogenously-given list of k decision problems, denoted $D = (D_1, \dots, D_k)$, where each decision problem $D_i \subseteq X$ is a finite set (or, menu) of choice objects from which the subject is asked to choose. We avoid trivial decision problems by assuming $|D_i| > 1$ for all i .⁵

⁴We also resurrect a foundational issue in revealed preference theory, dating back to Wold (1952): How can one infer an entire preference relation, if doing so requires observing multiple choices? Does revealed preference theory have empirical content? The answer is yes, but only if choices are observed under an incentive compatible payment mechanism.

⁵It is possible that such decision problems do affect preferences through a *framing* effect, discussed below.

The researcher does not know the subject's preference relation \succeq , but wants to use the decision problems to observe some properties of \succeq . For example, a researcher interested in correlating risk preferences with time discounting may have D_1 be a set of lotteries, and D_2 be a set of intertemporal choices.

Since choices are restricted to D , the choice data from the experiment can be thought of as an announced choice (or message) vector $m = (m_1, \dots, m_k)$, with $m_i \in D_i$ for each i . The space of all possible messages is $M = \times_i D_i$. For each $i \in \{1, \dots, k\}$, let

$$\mu_i(\succeq) = \text{dom}_{\succeq}(D_i)$$

be the set of \succeq -dominant elements of D_i , and define $\mu(\succeq) = \times_i \mu_i(\succeq)$. We refer to each $m \in \mu(\succeq)$ as a *truthful* message for \succeq .

We now describe $P(X)$, the space of possible payments a subject can receive. If only one decision problem D_1 is given, and if the subject is paid their chosen item $m_1 \in D_1$, then the set of possible payments is simply $D_1 \subseteq X$. When multiple decisions are given, it is possible that a subject is paid a *bundle* of items from X . Following general equilibrium notation, we write bundles as vectors that list the quantity of each good received. We assume goods are not divisible, so all quantities must be non-negative integers. Formally, let $B(X) = (\mathbb{Z}_+)^X$ be the space of non-negative integer-valued vectors of length $|X|$, with typical element b . Thus, $b_x \in \{0, 1, 2, \dots\}$ denotes the number of units of x contained in bundle b . For any $x \in X$, we let the variable x also denote the singleton bundle containing one unit of x and no units of any other good.

A randomizing device may also be used to select bundles from $B(X)$ for payment. Let Ω be the state space containing all possible realizations ω of the randomizing device. For example, if the device is a six-sided die, then $\Omega = \{1, \dots, 6\}$. We adopt the subjective uncertainty framework of Savage (1954), modeling a random payment as an *act*—a mapping from Ω into $B(X)$ —rather than an objective lottery with known probabilities. The space of all acts is $B(X)^\Omega$, so that each act assigns a bundle in $B(X)$ to each possible realization in Ω . We assume throughout that Ω is finite.

A constant act is one that pays the same bundle in every state ($f(\omega) = b$ for all $\omega \in \Omega$). Constant acts represent non-random payments. We abuse notation, letting b represent both the bundle itself, and the constant act that pays b in every state. Since x can represent a singleton bundle, it can also represent the constant act that pays that singleton bundle in every state. No confusion should result.

In an experiment, payments depend on the subject's announced choices. Formally, a (*payment*) *mechanism* ϕ takes the announced choice $m \in M$ and awards the subject with an act in $B(X)^\Omega$. We therefore have $P(X) = B(X)^\Omega$, and $\phi : M \rightarrow P(X)$. Let $\phi(m)(\omega)$ identify the bundle in $B(X)$ that is paid if the subject announces choice vector m and state ω

obtains. We refer to the pair (D, ϕ) as an *experiment*. Think of D as given exogenously by the research question at hand, and ϕ as being chosen by the experimenter. Since we view D as fixed, we often refer to experiments and mechanisms interchangeably.

The primitive preference \succeq is defined over elements of X , not $P(X)$. To study incentive compatibility, we must also assume that the subject has a complete and transitive preference \succeq^* on $P(X)$ which ‘extends’ \succeq in a sense to be made precise. To avoid confusion, we henceforth refrain from calling \succeq^* a preference relation; instead, we refer to it as an extension of \succeq . The asymmetric relation \succ^* denotes the asymmetric part of \succeq^* .

The researcher has in mind a set of *admissible* extensions for each preference \succeq ; this admissible set represents the extensions of \succeq that the researcher believes are possible. This belief would typically come from empirical evidence, but sometimes it may simply be assumed without data. We only require that all admissible extensions \succeq^* agree with \succeq on the space of constant singleton acts: if $f(\omega) = x$ and $g(\omega) = y$ for every ω , then $f \succeq^* g$ if and only if $x \succeq y$. We call this *consistency* of \succeq^* .

A successful experiment is one in which the payment mechanism always induces the subject to announce their choices truthfully, regardless of \succeq and \succeq^* . We refer to this as *incentive compatibility*.⁶

Definition 1 (Incentive Compatibility). A mechanism ϕ is incentive compatible if, for every preference \succeq , every admissible extension \succeq^* , every truthful message $m^* \in \mu(\succeq)$, and every message $m \in M$, we have that $\phi(m^*) \succeq^* \phi(m)$, with $\phi(m^*) \succ^* \phi(m)$ whenever $m \notin \mu(\succeq)$.

In other words, incentive compatible experiments induce the subject to announce truthfully a favorite item in every decision problem.

Preferences (and admissible extensions) may depend on the experiment. Incentive compatibility only requires that preferences in the current experiment be truthfully elicited. It does not require that preferences be stable across experiments. For example, suppose $D_1 = \{\text{exercise, rest}\}$, $D_2 = \{\text{salad, fish}\}$, and $D'_2 = \{\text{cheeseburger, pizza}\}$. We might find that announced choices in D_1 differ between (D_1, D_2) and (D_1, D'_2) , even when $\phi(m)(\omega) = m_1$ for every ω . The presence of such *framing effects* is not an indictment of the experiment’s incentives, but rather a suggestion that preferences over D_1 are highly sensitive to the context of the decision environment and, therefore, that observations of this decision may be difficult to generalize. We take D as fixed and do not analyze framing effects formally.

⁶Starmer and Sugden (1991) and Bardsley et al. (2010) refer to incentive compatible experiments as *unbiased*. Cox et al. (2014b) say they satisfy the *isolation hypothesis*, while Starmer and Sugden (1991) and Cubitt et al. (1998) say they avoid *contamination*.

To visualize an application of this framework, suppose the subject is asked to play k games, each against a different opponent.⁷ In each game $i \in \{1, \dots, k\}$ the subject (denoted by index h) chooses her strategy s_h^i from the strategy space S_h^i and her opponent (indexed by $-h$) chooses a strategy s_{-h}^i from the strategy space S_{-h}^i . An outcome function g maps each strategy profile in $S_h^i \times S_{-h}^i$ to a pair of dollar payments in \mathbb{R}^2 .⁸ Thus, each strategy $s_h^i \in S_h^i$ represents an act mapping each ‘state’ $s_{-h}^i \in S_{-h}^i$ into the payment vector $g(s_h^i, s_{-h}^i) \in \mathbb{R}^2$. These strategies in S_h^i are the choice objects over which the subject chooses (meaning, $S_h^i = D_i \subseteq X$), so \succeq represents the subject’s preferences over her own strategy space. In our subjective framework, \succeq encapsulates not just a ranking over dollar payments, but also the subject’s ‘beliefs’ over S_{-h}^i .⁹ The subject’s message m is the vector of strategies (s_h^1, \dots, s_h^k) across the k games that she actually selects, given the payment mechanism. If she is paid for all games, then she receives as payment the bundle of all k of these strategies. This bundle of strategies is an object in $B(X)$. Once all s_{-h}^i are realized and revealed to the subject, this bundle of strategies maps into a realized final vector of dollar payments given by $\sum_{i=1}^k g(s_h^i, s_{-h}^i)$.

Experiments on individual decision-making under uncertainty have the exact same structure, except we would view S_{-h}^i as being chosen by nature rather than by an opponent. In experiments with no uncertainty X is simply a set of choice objects. Obviously, there are many possibilities for the structure of the choice objects. But our analysis does not depend on this structure; we simply assume that the subject has a preference over X and an extension of those preferences to $P(X)$.

To begin our analysis, we consider the case where the researcher is not willing to make any assumptions about which extensions on $P(X)$ are admissible.

Proposition 0. If every extension is admissible, then there exists an incentive compatible payment mechanism if and only if $k = 1$ (the experiment has only one decision problem).

Proofs appear in the appendix. Proposition 0 verifies that, in experiments with multiple decisions, incentive compatibility is never free: the experimenter must make some assumptions about subjects’ possible extensions. Since most authors do not make explicit assumptions on $P(X)$, we often lack sufficient information to judge whether or not their experiment is incentive compatible within their framework (see Table II).

⁷Or against the same opponent but with no feedback between games. If the subject plays the same opponent in every game and receives feedback between games then the k games should be thought of as one large multi-stage game, with subjects making only one choice (their multi-stage strategy) in the experiment.

⁸Technically, subjects play *game forms* (which assign physical outcomes to strategy profiles), not games.

⁹We do not assume those ‘beliefs’ are probabilistic, in the sense of Machina and Schmeidler (1992), or that the subject cares only about her own payment. Recall also that \succeq can depend on which games the subject faces via framing effects.

III. MONOTONICITY & THE RPS MECHANISM

One natural restriction on extensions is that they respect basic dominance relations. Given an underlying preference \succeq on X , act f is said to *dominate* act g (written $f \supseteq g$) if for each $\omega \in \Omega$, $f(\omega) \in X$ and $g(\omega) \in X$ (the acts pay only singleton bundles) and $f(\omega) \succeq g(\omega)$. If $f \supseteq g$ and $f(\omega) \succ g(\omega)$ for some ω then f strictly dominates g (written as $f \supset g$). An extension that respects the dominance relation is said to be *(statewise) monotonic*.¹⁰

Definition 2 ((Statewise) Monotonicity). The extension \succeq^* is a monotonic extension of \succeq if $f \supseteq g$ implies $f \succeq^* g$, and $f \supset g$ implies $f \succ^* g$.

Monotonicity places no restriction on how non-singleton bundles are evaluated. Wealth effects, portfolio effects, and hedging are all possible under monotonicity. Thus, it cannot guarantee incentive compatibility of any mechanism that pays in bundles. Instead, monotonicity is useful for ensuring incentive compatibility in random mechanisms that select (non-bundle) elements from X . The RPS mechanism is one such mechanism.

Definition 3 (Random Problem Selection Mechanisms). A payment mechanism ϕ is a *random problem selection* (RPS) mechanism if there is a fixed partition $\{\Omega_1, \dots, \Omega_k\}$ of Ω with each Ω_i non-empty such that, for each $i \in \{1, \dots, k\}$ and $m \in M$,

$$\omega \in \Omega_i \text{ implies } \phi(m)(\omega) = m_i.$$

In other words, if event Ω_i obtains then the subject is paid what they chose in the i th decision problem. The only difference between various RPS mechanisms is in the randomizing devices used, so we often refer to them collectively as *the* RPS mechanism.

It is well known that the RPS mechanism is incentive compatible when all admissible extensions satisfy the expected utility axioms. What is not clearly stated in the existing literature is whether the RPS mechanism is incentive compatible under more general preferences. With our framework, it is easy to see that deviating from truth-telling to a less-preferred option in some D_i results in an act that gives a less-preferred outcome in event Ω_i , and has no affect in any other event. Thus, the deviation is dominated by truth-telling. Monotonicity then guarantees that such a deviation is never preferred. This simple argument proves the following result.¹¹

Proposition 1. If all admissible extensions satisfy monotonicity, then the RPS payment mechanism is incentive compatible.

¹⁰Our definition of monotonicity implicitly assumes that all states $\omega \in \Omega$ are non-null; see Savage (1954, p.24). Otherwise it is equivalent to Savage's P3.

¹¹In our working paper, we provide a weaker condition—called ϕ -monotonicity—that is both necessary and sufficient for incentive compatibility of an RPS mechanism ϕ . It is essentially monotonicity restricted to the range of ϕ .

When the decision problems are games and the RPS mechanism is used, monotonicity implies that the subject will select their favorite strategy in each game. If, for example, all subjects in the experiment have a preference in each game that maximizes their expected dollar payoff given the actual play of others, then their announced strategies in each game will be a Nash equilibrium.¹² Of course we do not assume this model of play; subjects' preferences over their own strategies can be arbitrary. Monotonicity only restricts how those within-game preferences map into across-game choices, exactly as it does with individual-choice experiments.

Notice that Proposition 1 does not require an objective randomizing device. It holds even if the experimenter uses a randomizing device for the RPS mechanism that subjects perceive as ambiguous. Thus, experimenters need not spend time or added complexity convincing subjects that the RPS randomization device is truly objective.¹³

A Characterization Under Monotonicity

To obtain a full characterization of incentive compatible mechanisms under the monotonicity assumption, we assume in this section that only strict preferences on X are admissible. Thus, $|\mu_i(\succ)| = 1$ for each i . Furthermore, there can be messages in some experiments that cannot be truthful for any strict preference relation. For example, if $D_1 = \{x, y\}$, $D_2 = \{y, z\}$, and $D_3 = \{z, x\}$, then $m = (x, y, z)$ cannot be rationalized by any strict preference relation. We say that this m is not *rationalizable*. Let

$$M_R = \{m \in M : (\exists \succ) m = \mu(\succ)\}$$

be the set of rationalizable messages, and $M_{NR} = M \setminus M_R$ be the set of non-rationalizable messages.

To understand how incentive compatibility can extend beyond the RPS mechanism, consider again the decision problems $D_1 = \{x, y\}$, $D_2 = \{y, z\}$, and $D_3 = \{z, x\}$. From any rationalizable announcement, we can always infer the subject's most-preferred element from the set $E = \{x, y, z\}$. For example, $m = (x, y, x)$ reveals that $\text{dom}_{\succ}(E) = \{x\}$. Now consider a mechanism with four states. In states ω_1 , ω_2 , and ω_3 the subject gets paid m_1 , m_2 , and m_3 , respectively, as in the RPS mechanism. In state ω_4 the subject is paid $\text{dom}_{\succ}(E)$, where \succ is inferred from m . Clearly, this mechanism is also incentive compatible under monotonicity.

¹²If the games have multiple Nash equilibria, the RPS randomizing device could facilitate coordination. But, under monotonicity, play in each game would still be a Nash equilibrium of that game; the randomizing device could not be used to expand the set of equilibria (as in correlated equilibrium) because it is publicly observed.

¹³If the choice objects are meant to be viewed as objective lotteries, then their randomizing device does need to be perceived as objective. The one used for the RPS mechanism does not.

To construct mechanisms like this, we must first understand when and how we can infer \succ from $m \in M_R$. First, if $x, y \in D_i$ for some i and $m_i = x$, then we say that x is *directly revealed preferred* to y . Let $R(m)$ be the transitive closure of this directly revealed preferred binary relation. We say that x' is *revealed preferred* to y' under choices m if $x' R(m) y'$.¹⁴ Note that $R(m)$ may not be complete.

Let

$$\text{dom}_m(E) = \{x \in E : (\forall y \in E) x R(m) y\}$$

be the set of $R(m)$ -dominant elements of E . If m does not reveal the most-preferred element of E , then $\text{dom}_m(E) = \emptyset$. Otherwise, $|\text{dom}_m(E)| = 1$. If m is truthful, then $\text{dom}_m(E) = \text{dom}_\succ(E)$ whenever it is non-empty.

Definition 4 (Surely Identified Sets). A set $E \subseteq X$ is *surely identified (SI)* if, for every $m \in M_R$, $\text{dom}_m(E) \neq \emptyset$. In other words, E is SI if, for any \succ , the message $m = \mu(\succ)$ identifies the most-preferred element of E , so that $\text{dom}_m(E) = \text{dom}_\succ(E)$.

Let $SI(D)$ be the collection of sets surely identified from the given list of decision problems D . Obviously, each D_i is in $SI(D)$, but there may be other sets in $SI(D)$. For instance, if $D_1 = \{x, y\}$, $D_2 = \{y, z\}$, and $D_3 = \{z, x\}$, then $SI(D) = \{D_1, D_2, D_3, \{x\}, \{y\}, \{z\}, \{x, y, z\}\}$.¹⁵

We wish to discuss mechanisms that choose sets in $SI(D)$ for payment; to this end, define the *payment set* of a mechanism ϕ at each state ω to be the set of bundles the subject can get in state ω by varying her message. Formally, for each ω , let

$$P(X|\phi, \omega) = \{\phi(m)(\omega)\}_{m \in M} \subseteq B(X),$$

and define the collection of all payment sets by

$$\mathcal{P}^\phi = \{P(X|\phi, \omega)\}_{\omega \in \Omega}.$$

In an RPS mechanism, $\mathcal{P}^\phi = \{D_1, \dots, D_k\}$. The following definition generalizes RPS mechanisms to allow other surely identified sets to be used as payment sets.

Definition 5 (Random Set Selection Mechanisms). A mechanism ϕ is a *random set selection (RSS)* mechanism if

- (1) $\mathcal{P}^\phi \subseteq SI(D)$, and
- (2) if $m \in M_R$ then for each $\omega \in \Omega$, $\phi(m)(\omega) = \text{dom}_m(P(X|\phi, \omega))$

The first condition requires that every payment set be surely identified. It also rules out any mechanism that pays in non-singleton bundles, since all surely identified sets

¹⁴Formally, $x' R(m) y'$ if there is a chain $x' = z_1, \dots, z_l = y'$ such that z_i is directly revealed preferred to z_{i+1} for every $i = 1, \dots, l-1$.

¹⁵Every singleton set is trivially SI. SI sets have a particularly simple characterization, which is given in our working paper.

are in X . Condition (2) requires that most-preferred elements are chosen from each payment set whenever messages are rationalizable. No restrictions are placed on the acts chosen at non-rationalizable messages. Our main theorem shows that a particular subclass of RSS mechanisms (which includes the RPS mechanism) fully characterizes the set of incentive compatible mechanisms when the experimenter assumes monotonicity (and nothing else).

Theorem 1. Suppose preferences on X are strict and all extensions satisfying monotonicity are admissible. A mechanism ϕ is incentive compatible if and only if it is a random set selection (RSS) mechanism in which

- (1) $D_i \in SI(\mathcal{P}^\phi)$ for each $i \in \{1, \dots, k\}$, and
- (2) $\phi(M_R) \cap \phi(M_{NR}) = \emptyset$.

Condition (2) requires that non-rationalizable messages pay something different than any rationalizable message, so that those payments will be strictly dominated by the truthful (rationalizable) payment. Condition (1) ensures that every decision problem is ‘consequential’: even if D_i is not a payment set, condition (1) ensures that a deviation in D_i will be reflected in the resulting payment in some state, which by the nature of RSS mechanisms implies that the resulting act is dominated.

The theorem assumes all monotonic extensions are admissible. In our working paper, we show the theorem is true under a variety of other ‘rich’ sets of extensions, such as all expected utility extensions, all probabilistically sophisticated extensions, and all multiple-priors extensions.¹⁶

In most applications, the RPS mechanism is the only one satisfying the conditions of Theorem 1. The non-RPS mechanisms require the existence of SI sets outside of $\{D_1, \dots, D_k\}$, such as $E = \{x, y, z\}$ in the above example. But in most experiments the set of decision problems is not rich enough for such SI sets to exist, so the RPS mechanism is the unique incentive compatible mechanism.¹⁷

We do not have a complete characterization of incentive compatible mechanisms when weak preferences are admissible. In that case, the RPS mechanism is still incentive compatible, and we know from examples that the set of incentive compatible mechanisms

¹⁶The theorem can also apply when \geq is restricted. The well-known Becker et al. (1964) (BDM) mechanism for eliciting the subjective value of an object (and its counterpart for eliciting probabilities, as in Grether, 1981 and Karni, 2009) is an example with such a restriction. There, $X = \{(0,0), (1,1), (1,2), \dots, (1,N)\}$, where $(1,n)$ means “receiving the object and paying $\$n$ ” and $(0,0)$ means “not receiving the object and paying $\$0$ ”. Preferences on X in this environment are assumed to satisfy *single-crossing*, so that if $(1,n) > (0,0)$, then for all $n' < n$, $(1,n') > (0,0)$. Given $>$, the *value* of the object is $n^* = \max\{n : (1,n) > (0,0)\}$. Under single crossing, announcing n^* reveals the entire preference over X . Thus, the BDM is simply an RPS mechanism in which $D_n = \{(0,0), (1,n)\}$ for each $n \in \{1, \dots, N\}$ and an announcement of n^* is interpreted as $m_n = (1,n)$ if $n \leq n^*$ and $m_n = (0,0)$ if $n > n^*$.

¹⁷In our working paper we formalize conditions on D that guarantee the RPS mechanism is unique.

must be strictly smaller. But the non-RPS mechanisms still rely on surely-identified sets, so again we conclude that the RPS mechanism is the only incentive compatible mechanism in most applications.

Objective Lotteries: Monotonicity, Expected Utility, and Ambiguity Aversion

In this section we study the RPS mechanism in settings where the subject has well-defined probabilities over the draw of which decision will be paid. In such a setting, Holt (1986) provides an example of an experiment and a non-expected utility preference for which the RPS mechanism is not incentive compatible. Similarly, Karni and Safra (1987) assume any rank-dependent utility representation for \succeq is possible and then prove that incentive compatibility for a certain class of experiments implies that \succeq must be consistent with expected utility. The apparent conclusion that expected utility of \succeq is *necessary* for the RPS mechanism to be incentive compatible seems at odds with our Proposition 1, which requires only monotonicity of \succeq^* and says nothing of \succeq .

The reconciliation of this apparent paradox is that Holt (1986) and Karni and Safra (1987) assume that subjects reduce compound lotteries to simple lotteries. This reduction axiom is strong; under reduction, any subject who violates expected utility preferences must also violate monotonicity. For such a subject, one can always find an experiment for which the RPS mechanism is not incentive compatible.

A similar apparent contradiction appears in the context of ambiguity aversion. Oechssler and Roomets (2014), Baillon et al. (2014), Kuzmics (2015), and Bade (2015) all point out that ambiguity averse subjects can use the randomness of the RPS mechanism as a hedge against ambiguity, causing a potential violation of incentive compatibility. For example, suppose a subject is asked *twice* whether she wants to bet on the home team or the away team in an upcoming football game, and a coin flip will determine whether her first or second answer is played out. In a single bet she would take the home team. But suppose in the duplicated bet she chooses the home team once and the away team once. Regardless of the outcome of the game, she would guarantee herself a coin flip between winning and losing the bet, which entails no ambiguity. If she is sufficiently ambiguity averse, she may prefer this diversified strategy over truthfully betting on the home team twice.

This example suggests that ambiguity neutral preferences are *necessary* for the RPS mechanism to be incentive compatible. But the hedging argument depends crucially on the subject reversing the order of conditioning. Our subject thought about the football game as being resolved ‘before’ the coin. If she conditions on the coin first, she faces ambiguity regardless of how the coin lands, so the mechanism provides no hedging opportunity. But the commonly-applied ‘order reversal axiom’ (Anscombe and Aumann,

1963) says that she must be indifferent between the two orders of conditioning, meaning she must recognize the hedging opportunity. Indeed, the order reversal axiom works exactly like the reduction of compound lotteries axiom above: if we assume order reversal, then any subject who violates ambiguity neutrality must also violate monotonicity. Again, one can then find an experiment for which the RPS mechanism is not incentive compatible for this subject.

In this section we present a simple framework that formalizes and unifies these two observations. But we must emphasize that these insights are not our own. The main insight dates back at least to Samuelson (1952), but follows most clearly from the original work of Anscombe and Aumann (1963).¹⁸ The contrapositive of their theorem is that, if a subject does not have personal probabilities (thus, is not ambiguity neutral) but does satisfy order reversal, then they must violate monotonicity. This is exactly the point we emphasize here. Anscombe and Aumann (1963) emphasize that order reversal is “akin in spirit” to the reduction of compound lotteries (Luce and Raiffa, 1957); we simply formalize that kinship by presenting both as a single axiom in a very general setup. These insights are also clearly expressed by Segal (1990) and Seo (2009), and emphasized more recently by Baillon et al. (2014) and Kuzmics (2015).

Formally, assume that \succeq^* satisfies probabilistic sophistication (Machina and Schmeidler, 1992), so that \succeq^* identifies well-defined probability ‘beliefs’ p over Ω , which are ‘rich’ in the sense that for any $\alpha \in [0, 1]$, there is $E \subseteq \Omega$ for which $p(E) = \alpha$.¹⁹ In general, let $(p_1, x_1; \dots; p_l, x_l)$ denote the lottery that selects each x_i with probability p_i .

Now suppose the set X is convex.²⁰ Let $\sum_i p_i x_i$ denote the *mixture* (or, convex combination) of x_1 through x_l , with weights p_1 through p_l , respectively. To be clear, $\sum_i p_i x_i$ is an element of X , while $(p_1, x_1; \dots; p_l, x_l)$ is a lottery over X . The reduction of compound lotteries and Anscombe & Aumann’s ‘order reversal’ axiom both equate preferences over mixtures with preferences over lotteries. We provide here a unified version of these assumptions, which we simply call *reduction*.

Definition 6 (Reduction). Assume X is convex and extension \succeq^* of \succeq satisfies probabilistic sophistication. Then \succeq^* satisfies *reduction* if, for all $(p_1, x_1; \dots; p_l, x_l)$ and $(p'_1, x'_1; \dots; p'_n, x'_n)$,

$$(p_1, x_1; \dots; p_l, x_l) \succeq^* (p'_1, x'_1; \dots; p'_n, x'_n) \iff \sum_{i=1}^l p_i x_i \succeq \sum_{i=1}^n p'_i x'_i.$$

¹⁸Samuelson used the term *compound independence* for monotonicity. He noted that a weakened version of monotonicity suffices for the result. The mathematical fact that additivity across states comes from allowing the order to commute is also seen in Harsanyi (1955), though in a slightly different context.

¹⁹This assumption is not consistent with finite Ω , but is made here to allow for the possibility of a general objective randomization device.

²⁰More generally, the analysis applies to any mixture space, as in Herstein and Milnor (1953).

Next, we define the independence axiom of Von Neumann and Morgenstern (1944), but interpreted in the current context in which x , y , and z are not necessarily lotteries.

Definition 7 (Independence). Assume X is convex. Then \succeq satisfies *independence* if, for all $x, y, z \in X$ and all $\alpha \in (0, 1]$,

$$x \succeq y \iff \alpha x + (1 - \alpha)z \succeq \alpha y + (1 - \alpha)z.$$

As discussed above, the strength of assuming reduction in the presence of monotonicity is well-known, and gives the following result.

Fact 1. Assume X is convex. If \succeq^* satisfies monotonicity and reduction, then \succeq satisfies independence.²¹

Consider the settings of Holt (1986) and Karni and Safra (1987), in which X is itself a convex space of objective lotteries. Preferences \succeq are defined over ‘one-stage’ lotteries, while \succeq^* evaluates ‘two-stage’ lotteries. The mixture $\sum_i p_i x_i$ represents a one-stage lottery. Reduction now refers to the familiar reduction of compound lotteries. Independence refers to expected utility preferences on X . With this framework, the contrapositive of Fact 1 clearly summarizes the findings of Holt (1986) and Karni and Safra (1987):

Fact 1.1. Suppose X is a convex set of objective lotteries. If \succeq violates the independence axiom and \succeq^* satisfies reduction (of compound lotteries), then monotonicity of \succeq^* is violated. Thus, there exist experiments for which the RPS mechanism is not incentive compatible.

In other words, if reduction is assumed, one should be wary of testing models of non-expected utility preferences using the RPS mechanism.

Now we consider the argument that the RPS mechanism provides a hedge against ambiguity. Formally, suppose X is a set of acts mapping some finite state space Θ into a convex set of outcomes Y (e.g., money payments). An extension \succeq^* evaluates lotteries over acts, as in Anscombe and Aumann (1963). Mixtures of acts are performed state-wise, meaning $\sum_i p_i x_i$ is an act in X that pays $\sum_i p_i x_i(\theta) \in Y$ in state θ . We assume X is convex, meaning it is closed under the mixing operation.

In this setting, reduction now is equivalent to Anscombe & Aumann’s ‘order reversal’ axiom. And independence is equivalent to ambiguity neutrality, as defined by Schmeidler (1989). Thus, reduction and monotonicity jointly imply ambiguity neutrality of \succeq .²²

²¹The proof is simple. Suppose $\alpha > 0$. By monotonicity, $(\alpha, x; 1 - \alpha, z) \succeq^* (\alpha, y; 1 - \alpha, z)$, and so by reduction $\alpha x + (1 - \alpha)z \succeq \alpha y + (1 - \alpha)z$. The converse can just as easily be seen to hold.

²²See also Mongin and Pivato (2015). Applying Gorman’s (1968) theorem, they derive a version of the Anscombe and Aumann (1963) theorem (thus, ambiguity neutrality) using various notions of monotonicity for two-stage acts.

This insight organizes the various findings of Oechssler and Roomets (2014), Baillon et al. (2014), and others:

Fact 1.2. Suppose X is a convex space of acts mapping a finite state space Θ into a convex set Y . If \succeq is not ambiguity neutral and \succeq^* satisfies reduction (also known as ‘order reversal’), then monotonicity of \succeq^* is violated. Thus, there exist experiments for which the RPS mechanism is not incentive compatible.

Those violations of incentive compatibility will present as hedging opportunities that appeal to the ambiguity-averse subject.²³ In general, one should be wary of testing theories of ambiguity aversion with the RPS mechanism when reduction is satisfied.

Of course, all of these conclusions vanish if reduction is violated. Without reduction (or any other similar axiom), properties of \succeq are completely unrelated to the incentive compatibility of the RPS mechanism.

Feedback, Experimentation Incentives & The Strategy Method

As described, our framework is entirely static. Any uncertainty inherent in the choice objects is not resolved until after the subject has announced all of her choices. In many experiments, however, subjects receive feedback about each choice before proceeding to the next decision. We develop a fully dynamic version of our framework in the appendix, but describe here informally the main lesson from that exercise: If the feedback subjects receive can depend on their decisions, then incentive compatibility cannot be assured. But if the feedback each period is completely independent of the subject’s choices then the RPS mechanism is incentive compatible under a similarly-defined monotonicity axiom.

To see the issue with choice-contingent feedback, consider a subject playing two modified ultimatum games, one after another. In each game the subject has a \$10 endowment and must choose either an equal split of \$5 for each, or keeping \$9 for herself. Label the equal split choice by E and the unequal split by U . If the subject chooses E then the recipient has no choice; the equal split is the final payoff. But if the subject chooses U then the recipient can either accept or reject the split. If it is rejected then both players earn \$0. The subject will see the result of the first game before playing the second.²⁴

²³Baillon et al. (2014) argue that the RPS mechanism will be incentive compatible if the state ω is resolved (but not revealed) before θ , as this may force only one order of conditioning in subjects. Oechssler et al. (2016) test this idea in the lab and find support for order reversal, though they also find little evidence of ambiguity aversion in their setting.

²⁴To clarify how this example fits in our framework, think of $D_1 = \{E_1, U_1\}$ as the game played against the first opponent, and $D_2 = \{E_2, U_2\}$ as the game played against the second opponent. Each E_t is a constant act that pays \$5 to both players, while each U_t is an act that pays (\$9, \$1) if the ‘state of the world’ is that

This design offers a clear experimentation incentive. By choosing U in the first game the subject can collect data on whether opponents tend to accept or reject unequal splits. A risk-averse subject who would prefer E if the game were only played once might rationally prefer to choose U in the first period in an effort to collect this information. Not only would the RPS mechanism not be incentive compatible, but it would be inappropriate to analyze these two games as separate decision problems since the first-period action can alter the information available in the second period.

The experimenter can restore incentive compatibility of the RPS mechanism by altering the feedback structure. Consider another version of the same experiment wherein the experimenter elicits from the recipient what they *would* choose if the first mover were to choose U . This is elicited regardless of the first mover's actual choice.²⁵ The experimenter then reveals the second mover's choice to the first mover at the end of the game, even if the first mover actually chose E . Now the first mover cannot alter what feedback she receives. Eliciting a player's actions at every contingency is known as the *strategy method*; here it is useful in that it allows that experimenter to provide feedback about the entire strategy profile in an extensive-form game, rather than just the realized path of play. Thus, the feedback cannot be affected by players' chosen actions, eliminating experimentation incentives.

In the appendix we show that, once experimentation incentives are eliminated in this way, the RPS mechanism is incentive compatible under an appropriately redefined notion of monotonicity. Notice that the presence of feedback itself is not a problem for incentive compatibility, though the researcher must acknowledge that the most-preferred choices revealed in each decision problem might have been different if the feedback the subject received had differed.

IV. NCAT & THE PAY-ALL MECHANISM

According to Table I, the most-frequently used mechanism in current practice is to pay subjects their announced choice in every decision. We call this the pay-all mechanism.

Definition 8 (The Pay-All Mechanism). Mechanism ϕ is the *pay-all* mechanism if, for every $\omega \in \Omega$ and every $m \in M$, we have $\phi(m)(\omega) = \sum_i m_i$.

We now show how this mechanism may not be incentive compatible. Suppose x_1 is a safe lottery and y_1 is a risky lottery. A risk-averse subject will have $x_1 \succ y_1$. But if x_2 is a cash payment large enough to alter the subject's risk preferences, then $y_1 + x_2 \succ x_1 + x_2$,

the recipient accepts, and $(\$0, \$0)$ if the recipient rejects. Thus, $X = \{E_1, U_1, E_2, U_2\}$, which are all acts whose state space specifies whether the opponent would accept or reject U_t .

²⁵The second mover's choice is binding once the first mover's action is revealed; thus, this 'strategy method' is strictly incentive compatible as long as the second mover believes U is not a null event.

violating incentive compatibility. As another example, if each x_i is a safe lottery and each y_i is a risky lottery, then our risk-averse subject will have $x_i > y_i$ for each i . But if the lotteries are independent then the risk associated with $\sum_i y_i$ may be quite small, leading to a possible portfolio effect in which $\sum_i y_i > \sum_i x_i$.

We now describe an assumption that rules out these distortions and makes the pay-all mechanism incentive compatible. Since the mechanism pays in constant acts (there is no randomness), the required restriction is only on how subjects evaluate fixed bundles, but not gambles over bundles.

Definition 9 (No Complementarities at the Top). The extension \succeq^* is an NCaT extension of \succeq if, for every $(x_1, \dots, x_k) \in \mu(\succeq)$ and every $(y_1, \dots, y_k) \in \times_i D_i$,

$$\sum_i x_i \succeq^* \sum_i y_i,$$

with strict preference if there is at least one i for which $y_i \notin \mu_i(\succeq)$.

NCaT guarantees that any bundle of favorite elements is preferred to every other bundle the subject could receive via the pay-all mechanism. It means there are no complementarities strong enough to overwhelm the subject's top-ranked alternatives when bundled together. NCaT does allow for complementarities among lower-ranked alternatives and smaller-sized bundles, however, and is therefore weaker than assuming no complementarities anywhere. It is only complementarities 'at the top' that can distort incentives. We discuss in Section V empirical evidence related to this assumption.

Proposition 2. The pay-all mechanism is incentive compatible if every admissible extension satisfies NCaT.

The proof is immediate from the definition of incentive compatibility. We now provide a characterization of incentive compatible mechanisms when preferences are strict and only NCaT is assumed. We show that any incentive compatible mechanism must agree with the pay-all mechanism whenever the subject announces a rationalizable message. As in Theorem 1, we require that non-rationalizable messages map to payments that cannot be paid under any rationalizable message, ensuring (via NCaT) that truth-telling is strictly preferred to any non-rationalizable deviation.

Theorem 2. Suppose preferences are strict and all extensions satisfying NCaT are admissible. Let ϕ^{PA} be the pay-all mechanism. A mechanism ϕ is incentive compatible if and only if

- (1) $\phi(m)(\omega) = \phi^{\text{PA}}(m)(\omega)$ for all $m \in M_R$ and $\omega \in \Omega$,
- (2) $\phi(M) \subseteq \phi^{\text{PA}}(M)$, and
- (3) $\phi(M_R) \cap \phi(M_{NR}) = \emptyset$.

Again, most experiments have $M_{NR} = \emptyset$ because $\cap_i D_i = \emptyset$. In that case condition (1) implies condition (2), and condition (3) is vacuous, so the only incentive compatible mechanism is the pay-all mechanism.

Some authors choose to pay subjects for a randomly-selected subset of decision problems; we call this the *random multiple problem selection* (RMPS) mechanism. If we assume a generalized form of monotonicity that also operates on non-singleton bundles, and a generalized form of NCaT that restrict preferences over all bundles that might get paid (not just those of size k), then this mechanism will also be incentive compatible.²⁶

V. EMPIRICAL EVIDENCE

The main contribution of this paper is to identify the assumptions on $P(X)$ that are needed for incentive compatibility of the RPS and pay-all mechanisms. But whether or not those assumptions are valid for a particular experimental design is an empirical question, which theory alone cannot answer. Thus, we conclude with a discussion of how one might test these assumptions, and we review past laboratory studies that test them directly.

Consider an experimenter who plans on running experiment (D, ϕ) using the RPS mechanism and wants to test first whether or not monotonicity will hold in that setting. One approach would be to design a new experiment (D', ϕ') that tests monotonicity directly. This would require at least two decision problems (one to learn about \succeq , and one to learn about \succeq^*). But then (D', ϕ') would require its own assumptions for incentive compatibility, which would need to be tested by another experiment, *ad infinitum*.

Given this difficulty, we see two ways that one could proceed. The first is to use choice objects for which we are reasonably confident about \succeq (e.g., more money is preferred to less). Once \succeq is assumed, properties of \succeq^* can be then tested using a single decision problem. The other way to proceed is to study monotonicity ‘between-subjects’—or at a population level—by randomly assigning some subjects to a single decision problem about \succeq , and the rest to a single decision problem about \succeq^* . Comparing choice frequencies between groups gives a statistical test of whether monotonicity (or NCaT) holds for every subject. A problem with this approach is that the two groups will see different decision problems and thus their underlying preferences may be differentially altered by framing effects, generating a false rejection of monotonicity. A potential solution is to give each group the decision problem faced by the other group, but not pay for this added problem. In this way both groups see the exact same problems, and thus have the

²⁶Most experimenters also pay a show-up fee s in addition to earnings from the experiment. Technically, this creates a bundle, which may distort incentives. This practice can be justified, however, by assuming that, for every $x, y \in X$, $x \succeq y$ if and only if $x + s \succeq^* y + s$. We call this *show-up fee invariance*, and we believe it is a reasonable assumption when s represents a fairly small cash payment.

exact same framing effects, but one group is paid only for the question about \geq while the other is paid only for the question about \geq^* .²⁷

In fact, any experimenter can run their own between-subjects statistical test of monotonicity or NCaT in their own setting by using the design just described: simply have one group be paid as in the original experiment (giving data on \geq^*), while other groups face the exact same decision problems but are paid only for one fixed D_i (giving data on \geq). The choice frequencies can then be compared between treatments using a chi-squared or Fisher test. We describe below several papers that have used this technique. Unfortunately, the statistical power of these tests can be quite low, so a large number of subjects may be needed for the test to be sufficiently informative. Since researchers are unlikely to run this test for every experiment, we proceed with a survey of existing evidence on monotonicity and NCaT. This may provide some initial guidance in choosing which assumption is appropriate for a given environment.

Tests of Monotonicity

Several authors have run between-subjects tests of the RPS mechanism (hence, monotonicity). Most have different groups viewing different sets of decision problems, confounding monotonicity violations with framing effects. Examples include Beattie and Loomes (1997), Cubitt et al. (1998, Experiment 2), Cox et al. (2014a,b), Harrison and Swarthout (2014), and Freeman et al. (2016). The first two find almost no violations of monotonicity. The latter three show significant differences between groups. But in all of these, the subjects who faced the RPS mechanism saw a different set of problems than the subjects who only saw a single decision problem. This difference in framing may have altered preferences, causing a potential confound with the results.

There are, to our knowledge, three experiments that use the between-subjects design and have no such framing confound between groups. The first is Starmer and Sugden (1991), who use four groups to run two independent tests. In one test the null hypothesis of monotonicity is not rejected with a p -value of 0.223. In the second, a marginal p -value of 0.052 is obtained, though the tests are slightly underpowered with only 40 subjects per group.²⁸ The second is Cubitt et al. (1998, Experiment 3), who also run two tests using roughly 50 subjects per group. The first gives a p -value of 0.685, while the second is 0.120. Both of these papers study choice over specific pairs of lotteries

²⁷This argument assumes preferences are affected by the decision problems present, but not by the payment mechanism itself. In a dynamic framework this test may not work because subjects may alter their choices on unpaid problems, thus altering the feedback they receive prior to making their paid choice. This would generate a difference in feedback between groups, muddling their comparison.

²⁸Their tests pool together two groups that saw different decision problems. Breaking these apart, we find the p -values are 0.356 and 0.043, respectively.

that reveal the presence of common-consequence and common-ratio effects. The third is Brown and Healy (2014), who run a between-subjects test of the multiple price list procedure of Holt and Laury (2002). They use 60 subjects per group. There is a clear violation of monotonicity when all decision problems are shown on one screen (p -value 0.040), but not when each decision problem is shown on a separate screen and their order randomized (p -value 0.697). This suggests that presenting all decisions together may trigger reduction-like behavior, causing a monotonicity violation for non-expected-utility subjects.²⁹

Camerer (1989) uses the RPS mechanism in his experiment, and then, once the payment state is realized, surprises subjects by asking if they'd like to change their decision in the paid decision problem. Less than three percent of subjects opt to change, suggesting that the RPS mechanism is incentive compatible.³⁰ Studying lottery choices and assuming reduction, Hey and Lee (2005b) statistically test the extreme theories that subjects either treat each choice in isolation or combine them to form one large lottery. Using multiple functional forms for preferences over lotteries and two different criteria, they find the data fit better the hypothesis that each decision is treated in isolation.³¹

Loomes (1998) and Rubinstein (2002) document a violation of monotonicity caused by 'irrational diversification', or 'probability matching'. Imagine a roulette wheel for which red is more likely than black. Although betting on red every spin is the stochastically dominant strategy, many subjects choose to place some of their bets on black, as if diversifying their portfolio of bets. Rubinstein (2002) observes this when paying for all decisions, indicating NCaT is violated. But first-order stochastic dominance is also violated, suggesting that monotonicity would also be questionable in this setting. It is unknown whether a payment mechanism exists that can avoid this irrational diversification problem.

Ex-ante fairness concerns can lead to monotonicity violations that are unrelated to reduction. For example, consider a dictator who must give \$1 to either Ann or Bob. Suppose he prefers to give to Ann. If the dictator were given this same problem twice and the RPS mechanism used, he may prefer to give to Ann once, and Bob once, so that the random choice of which problem gets paid provides an *ex-ante* fair division between Ann and Bob. This was first suggested by Diamond (1967), and evidence of this kind of preference has been documented by Bolton and Ockenfels (2010) and Cappelen et al.

²⁹Cox et al. (2014b) do run a treatment called 'ImpureOT' that eliminates the framing difference, but in their analysis they do not directly compare this treatment to their RPS treatment.

³⁰This procedure cannot be used regularly, since forward-looking subjects would realize that their initial choices are inconsequential.

³¹Hey and Lee (2005a) find a similar conclusion when subjects are given problems sequentially and future problems are not known.

(2013), for example. In general, experimenters should be wary of this possibility when studying repeated play of games with unfair outcomes.

In developing prospect theory (Kahneman and Tversky, 1979), it was found that subjects remove common components of compound lotteries. This ‘isolation effect’ has often been used as a justification for incentive compatibility of the RPS mechanism (Cubitt et al., 1998; Wakker et al., 1994). Indeed, isolation (properly formalized) is equivalent to monotonicity (assuming transitivity), so isolation can be an appropriate justification for using the RPS mechanism.

Recall that monotonicity and reduction are quite strong when jointly satisfied. Fortunately for the RPS mechanism, reduction has found little empirical support in the literature (see Camerer, 1995, p.656 for a survey). For example, Loomes et al. (1991); Starmer and Sugden (1991); Cubitt et al. (1998, Experiment 1); and Beattie and Loomes (1997) all run experiments using the RPS mechanism in which two different messages m and m' lead to the same simple lottery if reduction is assumed. In their data, subjects choose one message significantly more often than the other, clearly indicating that m and m' are evaluated differently in many subjects’ preferences. Snowball and Brown (1979), Schoemaker (1989), and Bernasconi and Loomes (1992) also observe violations of reduction. Halevy (2007) finds that those who do perform reduction tend to satisfy independence. Thus, subjects who respect reduction seem to be rare, and seem to be exactly those for whom independence (and, therefore, monotonicity) is a reasonable assumption. Recall, however, that reduction may be triggered when decision problems are shown together (Brown and Healy, 2014).

In the ambiguity domain, Dominiak and Schnedler (2011) find that subjects who exhibit ambiguity aversion in a two-color Ellsberg-urn experiment generally do not prefer a coin flip between the two possible bets over each one separately. A plausible explanation for this result is that these subjects do not view the coin as providing a hedge against ambiguity, because they only view the coin’s randomization as being performed ‘before’ the realization of the urn draw. In other words, reduction appears not to be satisfied in this context.

Tests of NCaT

There are many settings in which the NCaT assumption appears controversial. In the introduction, we listed several instances of violations, including wealth effects (Thaler and Johnson, 1990; Kagel and Levin, 1991; Weber and Zuchel, 2005; Ham et al., 2005; Ackert et al., 2006) and portfolio effects (Laury, 2005). These possibilities are often discussed in the experimental literature. We also described above the irrational diversification result of Rubinstein (2002), in which NCaT is violated.

Ex-post fairness concerns can also lead to NCaT violations. Consider again the dictator choosing twice whether to give \$1 to Ann or Bob. In one decision he prefers giving to Ann, but if both choices are paid then he may prefer to give \$1 to each. Formally, the bundle (1,1) may be preferred to the truthful bundle (2,0). Since fairness causes issues with both the RPS and the pay-all mechanisms, the obvious solution is to avoid (when possible) experiments in which these difficult trade-offs are repeated across multiple decisions with the same recipients.

On a positive note, the isolation effect is found when all decision are paid (Tversky and Kahneman, 1981), so NCaT may be justified in some settings. Several other models of risky choice implicitly satisfy NCaT. For example, if subjects learn about payments after every period, then NCaT is satisfied if they have reference-dependent preferences with rapidly-updating reference points (Cox et al., 2014b), or separable expected utility over earned income rather than terminal wealth (see Cox and Sadiraj, 2006).

Overall, it is difficult at this point to say when NCaT will or will not be satisfied. This is because there are many different ways in which complementarities can arise; assuming away all complementarities is a blanket assumption whose interpretation can vary widely from one context to another. General guidelines will be difficult to achieve.

APPENDIX A: PROOFS

Proof of Proposition 0

For sufficiency, if $k = 1$ then the mechanism in which $\phi(m) = m$ for each $m \in M$ is clearly incentive compatible. The proof of necessity proceeds in several steps. In each, assume the hypothesis that ϕ is incentive compatible, and extensions are only restricted to be consistent with \geq on X (the space of constant acts that pay singleton bundles).

Step 1: $|\text{Range}(\phi)| > 1$.

If $x, y \in D_i$ (with $x \neq y$) then consider a preference \geq where $x > z$ for all $z \neq x$ and a preference \geq' where $y >' z$ for all $z \neq y$. Let $m = \mu(\geq)$ and $m' = \mu(\geq')$, and note that $m \neq m'$ since $m_i = x$ and $m'_i = y$. Incentive compatibility therefore requires $\phi(m) >^* \phi(m')$, which implies $\phi(m) \neq \phi(m')$. Thus, $|\text{Range}(\phi)| > 1$.

Step 2: $\text{Range}(\phi) \subseteq X$ (the space of constant acts that pay singleton bundles).

First, suppose there is some $m' \in M$ such that $\phi(m')$ is not a constant act. Using step 1, let $m \neq m'$ be such that $\phi(m) \neq \phi(m')$, and then pick any \geq such that $m \in \mu(\geq)$. Pick an extension \geq^* of \geq such that $\phi(m') >^* f$ for every act $f \neq \phi(m')$. But then $\phi(m') >^* \phi(m) \in \mu(\geq)$, contradicting incentive compatibility.

Next, suppose $\phi(m)$ is a constant act for every m , but there is some $m' \in M$ such that $\phi(m') = b'$ is not a singleton bundle. As before, pick some $m \neq m'$ where $\phi(m) = b \neq b'$,

and some \succeq such that $m \in \mu(\succeq)$. Pick an extension \succeq^* of \succeq such that $b' \succ^* b''$ for every $b'' \neq b'$. But then $\phi(m') \succ^* \phi(m) \in \mu(\succeq)$, contradicting incentive compatibility.

Thus, every $\phi(m)$ is a constant act paying a singleton bundle.

Step 3: $\text{Range}(\phi) \subseteq \bigcap_i D_i$.

Suppose not. Then there is some $x' \in \text{Range}(\phi)$ (by step 2) and some D_j where $x' \notin D_j$. Suppose $x, y \in D_j$ ($x \neq y$). Now pick a preference \succeq where $x' \succ x \succ z$ for every $z \in X \setminus \{x', x\}$, and a strict preference \succeq' where $x' \succ' y \succ' z$ for every $z \in X \setminus \{x', y\}$. Let $m = \mu(\succeq)$ and $m' = \mu(\succeq')$, and note that $m_j = x$ and $m'_j = y$, so $m \neq m'$. Incentive compatibility requires that $\phi(m) = x'$ and $\phi(m') = x'$. But our strict notion of incentive compatibility also requires that $\phi(m) \succ^* \phi(m')$ for any extension \succeq^* of \succeq , which is a contradiction.

Step 4: $\text{Range}(\phi) = D_1 = D_2 = \dots = D_k$.

Suppose not. Then there is some D_i and some $x' \in D_i$ such that $x' \notin \text{Range}(\phi)$. Let \succeq be a preference where $x' \succ z$ for every $z \neq x'$, and let $m = \mu(\succeq)$. Let $x'' = \phi(m)$, and note that $x'' \in D_i$ by step 3. Now let \succeq' be a strict preference where $x'' \succ' z$ for every $z \neq x''$, and let $m' = \mu(\succeq')$. Since $m'_i = x''$ and $m_i = x'$, we have $m' \neq m = \mu(\succeq)$. Therefore, incentive compatibility requires that $\phi(m) \succ \phi(m')$. But incentive compatibility also requires that $\phi(m') = x''$, so that $\phi(m') = \phi(m)$, a contradiction.

Step 5: $k = 1$.

(If we assume no two decision problems are identical, then this step is unnecessary.) Suppose not. By step 4, we have $D_1 = D_2 = \text{Range}(\phi)$. Pick any m' such that $m'_1 \neq m'_2$, and let $x = \phi(m')$. Now consider the preference \succeq where $x \succ z$ for every $z \neq x$, and let $m = \mu(\succeq)$. Since $m_1 = m_2 = x$, we have that $m \neq m'$. Incentive compatibility requires that $\phi(m) \succ \phi(m')$, but also that $\phi(m) = x = \phi(m')$, a contradiction.

Proof of Theorem 1

For each \succeq , let $\mathcal{E}(\succeq)$ be the set of admissible extensions consistent with \succeq , and $\mathcal{E}^{\text{mon}}(\succeq)$ be the set of all possible monotonic extensions of \succeq . Our main results concern the case where all admissible extensions are monotonic ($\mathcal{E}(\succeq) \subseteq \mathcal{E}^{\text{mon}}(\succeq)$). Here, a sufficient condition for incentive compatibility is that acts resulting from truth-telling messages dominate all acts resulting from any other message, with strict dominance whenever the other message is not truthful.

Definition 10 (Truth Dominates Lies). A mechanism ϕ has the *truth-dominates-lies (TDL) property* if, for every \succeq , every $m^* \in \mu(\succeq)$, and every $m \in M$, we have that $\phi(m^*) \supseteq \phi(m)$, with $\phi(m^*) \sqsupset \phi(m)$ whenever $m \notin \mu(\succeq)$.

Recall that $\phi(m^*) \supseteq \phi(m)$ for all m implies that the range of ϕ is X .

Lemma 1. If $\mathcal{E}(\succeq) \subseteq \mathcal{E}^{\text{mon}}(\succeq)$ for every \succeq and ϕ has the TDL property then ϕ is incentive compatible with respect to \mathcal{E} .

Proof. Fix a preference \succeq , a truthful message $m^* \in \mu(\succeq)$, and an arbitrary message m . If $\phi(m^*)$ dominates $\phi(m)$ under \succeq then, under monotonicity, $\phi(m^*) \succeq^* \phi(m)$ for any extension $\succeq^* \in \mathcal{E}(\succeq) \subseteq \mathcal{E}^{\text{mon}}(\succeq)$ (with strict orderings when $m \notin \mu(\succeq)$). Since this holds for all \succeq , the experiment is incentive compatible. \square

Lemma 2. Suppose $\mathcal{E} = \mathcal{E}^{\text{mon}}$. A mechanism ϕ is incentive compatible (with respect to \mathcal{E}) if and only if it has the TDL property.

Proof. Lemma 1 gives sufficiency of the TDL property, so we prove necessity here. Assume ϕ is incentive compatible. Let \succeq be a preference, and let $m^* \in \mu(\succeq)$. We claim that $\phi(m^*)$ dominates $\phi(m)$ under \succeq . We know from incentive compatibility that for all $\succeq^* \in \mathcal{E}(\succeq)$ and $m \in M$, $\phi(m^*) \succeq^* \phi(m)$. Because $\mathcal{E} = \mathcal{E}^{\text{mon}}$, we also know that $\bigcap_{\succeq^* \in \mathcal{E}(\succeq)} \succeq^* = \sqsupseteq$ (see Szpilrajn, 1930 or lemma 2 in our online appendix). In particular, if $f \succeq^* g$ for every $\succeq^* \in \mathcal{E}(\succeq)$, then $f \sqsupseteq g$, which also means that neither f nor g pay in non-singleton bundles. Thus, $\phi(m^*) \sqsupseteq \phi(m)$. Now, suppose that $m \notin \mu(\succeq)$. Then, for all i , it follows by definition that $m_i^* \succeq m_i$, and that there exists j for which $m_j^* \succ m_j$. Therefore, for all $\succeq^* \in \mathcal{E}(\succeq)$, we have $\phi(m^*) \succ^* \phi(m)$. Consequently, by our hypothesis that $\bigcap_{\succeq^* \in \mathcal{E}(\succeq)} \succeq^* = \sqsupseteq$, we know that $\phi(m) \sqsupseteq \phi(m^*)$ is false. Further, we know that $\phi(m^*) \sqsupseteq \phi(m)$ is true. Conclude that $\phi(m^*) \sqsupset \phi(m)$. \square

We now begin the proof of Theorem 1 by showing that any RSS mechanism that satisfies conditions (1) and (2) is incentive compatible (with respect to \mathcal{E}^{mon}). Let \succeq be arbitrary, $m^* = \mu(\succeq)$ and let \succeq^* be some (monotonic) extension of \succeq . We claim that $\phi(m^*) \succeq^* \phi(m')$ for any $m' \neq m^*$. This follows since, for each ω , $\phi(m^*)(\omega) = \text{dom}_{m^*}(P(X|\phi, \omega)) \in X$ and $\phi(m')(\omega) \in P(X|\phi, \omega) \subseteq X$, so $\phi(m^*)(\omega) \succeq \phi(m')(\omega)$. Since $m' \neq \mu(\succeq)$, we must also show that there exists $\omega \in \Omega$ for which $\phi(m^*)(\omega) \succ \phi(m')(\omega)$. Suppose not, so that $\phi(m^*)(\omega) \sim \phi(m')(\omega)$ at each ω . Because \succeq is a linear order, this implies that $\phi(m^*) = \phi(m')$. Recalling condition (2) of the hypothesis, this implies that $m' \in M_R$, so there exists \succeq' for which $m' = \mu(\succeq')$. Since $\phi(m^*) = \phi(m')$, both acts pick the same elements from every $P(X|\phi, \omega)$. Condition (1) requires $D_i \in SI(\mathcal{P}^\phi)$ for every i , so that $\mu_i(\succeq) = \mu_i(\succeq')$ for every i . But $\mu(\succeq) = \mu(\succeq')$ contradicts $m^* \neq m'$.

Conversely, let ϕ be an incentive compatible mechanism for (D_1, \dots, D_k) . Recall that, for each $\omega \in \Omega$, $P(X|\phi, \omega) = \phi(M)(\omega)$. Let $m^* \in M_R$, and let \succeq such that $m^* = \mu(\succeq)$. By incentive compatibility (recall Lemma 2), it follows that for all $m \in M$, we have $\phi(\mu(\succeq)) \sqsupseteq \phi(m)$. In particular, this implies that $\text{Range}(\phi) \subseteq X$ and, for all $\omega \in \Omega$, $\phi(\mu(\succeq))(\omega) \succeq \phi(m)(\omega)$ (by definition of \sqsupseteq), or $\phi(m^*)(\omega) \succeq \phi(m)(\omega)$. That is, $\phi(m^*)(\omega) \succeq y$ for all $y \in$

$P(X|\phi, \omega) \subseteq X$. Since \succeq was arbitrary, this establishes both that $P(X|\phi, \omega) \in SI(D)$, and that $\phi(m)(\omega) = \text{dom}_m(P(X|\phi, \omega))$ whenever $m \in M_R$. Hence, ϕ is an RSS.

We claim now that for all i , $D_i \in SI(\mathcal{P}^\phi)$. If not, then by definition, there exists D_i , and preferences \succeq, \succeq' for which for all $\omega \in \Omega$, $\text{dom}_{\succeq} P(X|\phi, \omega) = \text{dom}_{\succeq'} P(X|\phi, \omega)$, but for which $\text{dom}_{\succeq} D_i \neq \text{dom}_{\succeq'} D_i$. Hence, $\mu(\succeq) \neq \mu(\succeq')$. Since ϕ is an RSS mechanism, for all $\omega \in \Omega$, $\phi(\mu(\succeq))(\omega) = \text{dom}_{\succeq} P(X|\phi, \omega) = \text{dom}_{\succeq'} P(X|\phi, \omega) = \phi(\mu(\succeq'))(\omega)$. Consequently, $\phi(\mu(\succeq)) = \phi(\mu(\succeq'))$, but $\mu(\succeq) \neq \mu(\succeq')$. In particular, since $\mu(\succeq)$ and $\mu(\succeq')$ are each single-valued, incentive compatibility implies that there exists $\omega \in \Omega$ for which $\phi(\mu(\succeq))(\omega) > \phi(\mu(\succeq'))(\omega)$, a contradiction.

Finally, suppose that there are $m \in M_R$ and $m' \in M_{NR}$ such that $\phi(m) = \phi(m')$. Let \succeq be such that $\mu(\succeq) = m$. Incentive compatibility requires that $\phi(m) \sqsupseteq \phi(m')$ with respect to \succeq , which contradicts $\phi(m) = \phi(m')$.

Proof of Theorem 2

Assume NCaT and incentive compatibility. Clearly, $\phi(M) \subseteq \phi^{\text{PA}}(M) \cup X$. To see this, note that random payments cannot be used since no assumption is made on non-trivial gambles. Furthermore, if $\phi(m) = b$ (meaning $\phi(m)(\omega) = b$ for all ω), where $b \in B(X)$ but $b \notin \phi^{\text{PA}}(M) \cup X$, then pick some preference \succeq' where $m \notin \mu(\succeq')$ and let b be the most-preferred bundle according to \succeq' . Clearly incentive compatibility fails for \succeq' .

Next, we argue that either $\phi(M) \subseteq \phi^{\text{PA}}(M)$ or $\phi(M) \subseteq X$. Suppose not, so $\phi(m) = b \in \phi^{\text{PA}}(M)$ and $\phi(m') = x' \in X$. Pick any \succeq such that $m \notin \mu(\succeq)$, and pick an extension \succ^* such that $b \succ^* x$ for all $x \in X$. This does not violate NCaT, but incentive compatibility fails.

If $\phi(M) \subseteq X$ then NCaT places no applicable restrictions on \succeq^* , and the proof of Proposition 0 (steps 3–5) shows that we have $k = 1$, in which case $\phi \equiv \phi^{\text{PA}}$, $\phi^{\text{PA}}(M) = X$, and $M_{NR} = \emptyset$, proving the theorem.

Consider instead the case where $\phi(M) \subseteq \phi^{\text{PA}}(M)$. Pick any $m \in M_R$ and \succeq such that $m = \mu(\succeq)$. Suppose $\phi(m) = b \neq \sum_i m_i$. Let \succeq^* be an extension of \succeq in which b is the lowest-ranked bundle in $\phi^{\text{PA}}(M)$. This does not violate NCaT, since NCaT only requires $\sum_i m_i$ be top ranked. But the subject will strictly prefer to announce any $m' \neq m$ where $\phi(m') \neq \phi(m)$, violating incentive compatibility.³² Thus, $\phi(m) = \sum_i m_i$ for all $m \in M_R$. Furthermore, it is clear that $\phi(M_{NR}) \cap \phi(M_R) = \emptyset$; otherwise strict incentive compatibility would be violated.

³²Clearly, incentive compatibility will also fail if $\phi(m') = \phi(m)$ for every m' .

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